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**A SIMULATED CDA FINANCIAL
MARKET WITH A BEHAVIORAL
PERSPECTIVE**

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Standard finance people are modelled as “rational”,
whereas behavioural finance people are modelled as normal

Statman (1999, p. 20)

INTRODUCTION	4
1. DECISIONS UNDER RISK: A BEHAVIORAL APPROACH	7
1.1 ASYMMETRY ON RISK PERCEPTION	7
1.2 THEORY OF RATIONAL CHOICES	8
1.2.1 ALLAIS PARADOX	10
1.3 FROM EXPECTED UTILITY THEORY CRITIQUES TO PROSPECT THEORY	12
1.3.1 CERTAINTY EFFECT	13
1.3.2 ISOLATION EFFECT	14
1.3.3 FRAMING EFFECT	16
1.4 PROSPECT THEORY	17
1.4.1 VALUE FUNCTION	18
1.4.2 PROBABILITY WEIGHTING FUNCTION	19
1.5 CUMULATIVE PROSPECT THEORY	20
1.5.3 FURTHER DEVELOPMENT: TWO-PARAMETER WEIGHTING FUNCTION	25
1.6 FROM LAB TO EMPIRICAL EVIDENCES	27
2. AGENT-BASED MODEL	29
2.1 INTRODUCTION	29
2.2 SUPPORTING ABM IN ECONOMICS	30
2.2.1 COMPLEXITY MATTERS	30
2.2.2 FILLING THE THEORY-DATA GAP	32
2.2.3 CAUSALITY, UNCERTAINTY AND EX-ANTE EVALUATIONS	33
2.2.4 RESTORING HETEROGENEITY	34
2.2.5 MULTI-DISCIPLINARITY	35
3. FROM REAL TO SIMULATED CDA FINANCIAL MARKETS	36
3.1 FINANCIAL MARKETS: FROM OUT-CRY TO ELECTRONIC TRADING SYSTEMS	36
3.2 TECHNICAL AND FUNDAMENTAL ANALYSIS	39
3.2.1 FUNDAMENTAL ANALYSIS	39
3.2.2 TECHNICAL ANALYSIS	39
3.2.2.1 TECHNICAL INDICATORS: AN EYE ON BOLLINGER BANDS.	41
3.3 CONTINUOUS DOUBLE AUCTION MODEL	43
3.4 EXPERIMENTS WITH HUMAN TRADERS: THE WORK OF THE NOBEL PRIZE VERNON SMITH AS A PIONEER	43
3.5 REPLACING HUMAN TRADERS WITH ZERO INTELLIGENT ONES	47
3.6 HERD BEHAVIOR FROM NATURE TO FINANCIAL MARKETS	48
3.7 WHEN MISTAKES TAKE PLACE: GAMBLER'S FALLACY	51
3.7.3 HEURISTICS' ROLE IN THE GAMBLER'S FALLACY	52

4. MODEL MAKING	55
4.1 MODEL 1.0: BASIC CDA WITH ZI TRADERS	55
4.1.4 INTERFACE	55
4.1.5 SETUP	56
4.1.6 THE GO PROCEDURE AND CDA IMPLEMENTATION	58
4.1.7 CONCLUSIONS AND REMARKS ON MODEL 1.0	61
4.2 MODEL 2.0: ZI TRADERS SAPIENS:	63
4.2.8 INTERFACE AND VARIABLES	63
4.2.9 NLS EXTENSIONS FOR NETLOGO	65
4.2.10 HERD BEHAVIOR IN NETLOGO	65
4.2.11 BOLLINGER BANDS IN NETLOGO	68
4.2.12 A SIMPLE STRATEGY FOR ZI SAPIENS BASED ON BOLLINGER'S BANDS	69
4.2.13 GAMBLER'S FALLACY IN NETLOGO	71
4.2.14 ADDITIONAL PROCEDURES	72
4.2.15 WEAKNESSES OF MODEL 2.0	73
4.3 MODEL 3.0	74
4.3.1 INTERFACE	74
4.3.2 THE CODE STRUCTURE	76
4.3.3 AGENT'S DECISION PROCESS	77
4.3.3.1 ZI PRICE EVALUATION	78
4.3.3.2 TECHNICAL ANALYSIS STRATEGY	79
4.3.3.3 IMITATION STRATEGY	80
4.3.3.4 GAMBLER'S FALLACY	82
4.3.4 PROSPECT THEORY IN THE GAME	83
4.3.5 STOCK VARIABILITY	86
4.3.6 NETLOGO AND R	87
5. EXPERIMENTS AND RESULTS	88
5.1 ZI	89
5.1.1 FIRST SCENARIO: GODE AND SUNDER	89
5.1.2 SECOND SCENARIO: STOCK VARIABILITY	93
5.2 IMITATIVE BEHAVIOR	98
5.2.1 FIRST SCENARIO: STANDARD CASE	98
5.2.2 SECOND SCENARIO: THE AGENT – FISH SCHOOLING	104
5.3 GAMBLER'S FALLACY	110
5.3.1 FIRST SCENARIO: STANDARD CASE	110
5.3.2 SECOND SCENARIO: HIGH GAMBLING TEMPTATION	115
5.4 TECHNICAL STRATEGY	120
5.4.3 FIRST SCENARIO: STANDARD CASE	120
5.5 COMPLETE SYSTEM	125
5.5.1 FIRST SCENARIO: STANDARD CASE	125
5.5.2 SECOND SCENARIO: STOCK VARIABILITY AND RISK-ATTRACTIVENESS	134

CONCLUSIONS	145
APPENDIX	148
A. PARAMETERS' RELATION OF THE TPWF	148
B. STATISTICAL AND ECONOMETRIC TOOL: THEORY AND R COMMANDS	149
C. THE SOFTWARE TOOLBOX: NETLOGO, R AND RSERVE	154
D. THE CODE	156
REFERENCES	178
ACKNOWLEDGEMENTS	189

INTRODUCTION

The heart of the matter in this work is how human being deals with economical and financial choices. What saving plan is more convenient when individuals can not know in advance what their income, health and preferences will be in the future? Which level of prices, kind of products and strategies are going to be more profitable for a firm facing an unknown future demand? Most economical questions are deeply entangled with uncertainty since the nitty-gritty is about making choices today for an unknown tomorrow.

Two disciplines have been exploring these issues: Psychology and Economy. Both of them with different perspectives and relying on different research agendas. Psychologists in fact underscored the difference between implicit and explicit processes. In particular implicit process forces to consider heuristics and non-actions in the domain of the individual choices. The implicit decision of doing nothing is therefore a meaningful decision. Conversely explicit processes reflect the actions arising from individual strategies.

On the other side Economics mainly focuses on explicit processes when studying human (rational) decisions. The representative agent characterizes the neo-classical approach of Economics. Such approach models ideal phenomena as it happens for other sciences such as physics and other natural sciences. Despite the critiques the representative agent is an important tool in the domain of Economics. It provides the chance to analyze aggregated choices of agents where multiple decisions are conceived as the result of a collective mind.

This work gathers both contributions thanks to the Agent-Based Modelling (ABM) allowing us to investigate a complex system such as a (simulated) financial market in the domain of Behavioral Finance. Behavioral finance explains markets failures considering actual behavior as the starting point, rather than postulating rationality as the main driving force of individual behavior and it provides insights into two sets of problems:

- The individual-collective relationship. When considering the market as the neutral expression of collective actions, it is important to draw the attention on how the “Social Output” arises from single and independent individuals who operate as in isolation according to their expectations, knowledge, emotions, rationality etc... It is widespread to conceive financial markets with its own memory (on past performances) and also with its own “feelings” (like the Bull and Bear jargon) as if it were a unique Social Mind.
- Explaining economical mistakes. This can be done moving away from the rational paradigm of mainstream economics and moving instead into a cognitive perspective that opens the black-box of human mind highlighting the mechanism behind human decisions and its biases.

These insights have been fruitfully taken into account in this work. Chapter one investigates the Economical and Psychological contributions in the domain of Human decision under risk. Starting from the theory of rational choices we present the pioneer work of Daniel Bernoulli in 1738 and the well-known axiomatizations of Morgenstern and von Neumann two centuries later. From here we highlighted the main contradictions and failures of the expected utility theorem as the Allais Paradox. These weaknesses have been overtaken by the work of Economics Nobel¹ awarded Kahneman² and Tversky (1979, 1992) in the prospect theory. Prospect theory is presented taking account of its two key ingredients: the value function and the weighting-function. In particular the latter has been implemented in the two parameters form proposed by Gonzalez and Wu (1999) that provides two useful psychological and economical concepts such as the risk-attractiveness and the probability discriminability. Over the chapter the Cumulative prospect theory has been introduced in the light of both theoretical development and empirical findings.

In Chapter 2 instead we analyzed the main reasons to support ABM as a research tool to be added in the Economical toolbox. Referring to Behavioral Finance contributions it is worth to notice that ABM goes in the same investigative direction when dealing with the individual-collective relationship. In fact by modelling agents and their individual strategies an aggregate output naturally arises such as the price movements over time.

Chapter 3 better defines the financial world we would like to simulate highlighting its main features. We proposed the Continuous Double Auction (CDA) model as a valid mechanism to match bid and ask offers while determining the prices at which trades are executed. This model has already been successfully applied in the literature. Historically the first relevant case relied on real interactions among individuals in an experimental setting proposed by the Nobel awarded Smith (1962). Gode and Sunder (1993) instead employed the CDA mechanism in their famous simulation model where agents were designed as Zero-Intelligence algorithmic traders whose bid and ask orders are randomly produced and subject only to minimal constraints.

It is interesting to notice that this chapter follows the implicit-explicit dichotomy that characterizes Behavioral Finance. In fact while some explicit processes of traders' decisions have been explored as the investment strategies adopted we did not forget to recall the implicit components that affect human behavior. That is why we draw the attention on heuristics' and rational fallacy' role on human decisions.

Chapter 4 instead tracks the development of the final model using Netlogo and R starting by the Gode and Sunder model in Netlogo proposed by Professor Terna. Over the chapter have been described both intuition and formula behind the strategies and rational errors implemented in the final model 3.0. In its final version the model comprehends both the Cumulative prospect theory contributions: the value function and the weighting function. Additionally some agent's strategies are at disposal of agents. Those are an imitative strategy, a technical analysis strategy based on

¹ In this work when I refer to Nobel prize in Economics I intend the Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel, which is awarded by the Sweden's central bank since 1969. In fact it is worth to point out that in its original form the Nobel price is awarded since 1901 for outstanding contributions for humanity in chemistry, literature, peace, physics, medicine or physiology; Economics was not one of them.

² Unfortunately, Tversky died before the prize was awarded, and the rules state that only live people can get it. Kahneman has stated that he considers it a joint prize for the two of them

Bollinger's Band and the Gambler's fallacy. Those are chosen by the agents under specific rules and conditions explained in the chapter.

The last chapter simulates the impact of the strategies and mistakes in the CDA financial market where agents can behave randomly or not according to the research settings. The intention here is twofold based on two different levels. From an agent-micro perspective we want investigate whether agents are successful or not in the market, under which circumstances and if their success is arbitrary or the result of some specific conditions. On the other side, from the macro perspective we investigate whether the new behavior patterns at agent's disposal affect the aggregate picture in terms of price series.

1. DECISIONS UNDER RISK: A BEHAVIORAL APPROACH

1.1 Asymmetry on risk perception

In financial market risk and reward are the names of the game. The risk is commonly referred to the fact that no matter which econometrics or quantitative tools are used the future is still unpredictable. The only thing we are sure about tomorrow it is that is going to be different than today. Such unpredictability is a risk for investments which are rewarded by gains or losses according to what really happens.

Economists have developed tools and techniques to calculate and check such risk that is linked to fluctuations in the value over time and measured as standard deviations. In order to measure the extent of risks, they rely on long time series similar to those used by insurance companies to calculate the amount of premium to ensure by specific risks the customer's assets.

If people would rely on a similar measurement, even if more or less accurate, and using intuitively this information to calculate the extent of risks, they would be symmetrical to descent and ascent movements of asset's value. The sorrow of the loss would be counter-balanced from the identical happiness of previous gains (and there would not need to be grieving, which is strictly speaking an incomprehensible phenomenon from the point of view of rationality traditional economic). According to rational paradigm, a person when choosing between a certain event and an uncertain one with the same expected value he will prefer to go for the certain one. People are likely to display a wise risk-aversion.

Consider for instance one of Legrenzi's examples (Legrenzi 2006). We are playing a game where the two options available are to win at 50% 200,000 EUR or to receive immediately with no risk 100,000 EUR. A smart player may conclude that these two would be equivalent options only if the game were repeated many times. However since you play it once, the increase subjective utility from 0 (vacuum pack) to 100,000 (safe option) is greater than the increase of utility in moving from 100,000 to 200,000 under uncertainty. Additionally participants are also tempted to accept smaller offer, i.e. 90,000 euro, compared to 50% of 200,000. It is all about the breakeven point: if a person is prudent (and his prudence depends on its financial resources and, therefore on, the marginal utility of income), he will be happy to get a "certain" gain whose value is less than the expected value of the uncertain option. Since each player can participate only once, they often accept values which are much lower than the expected value uncertain option.

Consider now a hypothetical complementary situation. In this case the presenter gives you 200,000 Euros as part of a game to which you are forced to play at least twice. You face a sure loss of 100,000 euro or, alternatively, a 50% chance of losing 200,000 and 50% do not miss anything. In this case, people prefer this second option: the uncertain. If you choose the sure loss, within the two mandatory rounds you lose for sure the 200,000 you received at the beginning. Alternatively with the uncertain option you can lose everything at once during the first game but if you're lucky you can hope to save your money twice. In this case, you return at home with 200,000 Euros.

This asymmetric way to deal with risk is easily understandable in hostile environments or in extreme circumstances where you should take greater risks to survive. Maybe many of the tribes of hunter gatherers from which we descended sacrificed themselves facing mortal dangers deciding to go for riskier options but some managed to escape because they dared extreme options. The asymmetry in dealing with the risks is represented by a function value that worth for Daniel Kahneman, in 2002, the Nobel Prize in Economics. According to the value function people have different ways to perceive and evaluate gains and losses. Simply put considering the same amount of money lost or gained: loosing hurts more than winning feels good.

1.2 Theory of rational choices

Most economical questions are deeply entangled with uncertainty since the nitty-gritty is about making choices today for an unknown tomorrow. A well-known approach in the domain of economics and decision theory is represented by the expected utility (EU) hypothesis. The really first pioneer was Daniel Bernoulli back in 1738. The main features of this model can be well understood from the following example

Somehow a very poor fellow obtains a lottery ticket that will yield with equal probability either nothing or twenty thousand ducats. Will this man evaluate his chance of winning at ten thousand ducats? Would he not be ill-advised to sell this lottery ticket for nine thousand ducats? To me it seems that the answer is in the negative. On the other hand I am inclined to believe that a rich man would be ill-advised to refuse to buy the lottery ticket for nine thousand ducats. If I am not wrong then it seems clear that all men cannot use the same rule to evaluate the gamble. The rule established in §1 must, therefore, be discarded. But anyone who considers the problem with perspicacity and interest will ascertain that the concept of value which we have used in this rule may be defined in a way which renders the entire procedure universally acceptable without reservation. To do this the determination of the value of an item must not be based on its price, but rather on the utility it yields. (...)Meanwhile, let us use this as a fundamental rule: *If the utility of each possible profit expectation is multiplied by the number of ways in which it can occur, and we then divide the sum of these products by the total number of possible cases, a mean utility [moral expectation] will be obtained, and the profit which corresponds to this utility will equal the value of the risk in question* (Bernoulli, 1954, p. 24, emphasis added)

With subjective utility Bernoulli took the distance from the expected value approach according to which sizes and probability of gains and losses are enough in determining individual's decisions. In particular the expected value theorem was not able to explain why in front of the same payoff people react differently. Decisions are, in Bernoulli's mind, aimed at maximizing utility rather than expected value. He placed emphasis on the psychology behind people judgments describing a concave utility function and by identifying decreasing marginal utility. In his view 1 € has a great impact to someone who owns nothing and a negligible effect to those with 100 € aside.

Bernoulli's theory was a deep historical step since

For the first time in history Bernoulli is applying measurement to something that cannot be counted. Bernoulli defines the motivations of the person who does the choosing. This is an entirely new area of study and body of theory. (Bernstein, P.L. 1996, *Against the Gods*, p106)

Bernoulli's theory contains both sensible descriptive elements melt together with normative implications: risk-aversion is here not *per se* but rather a by-product that has its roots on people attitude to the value of payoff (McDermott, 2001). Two centuries later Morgenstern and von Neumann (1947), as an appendix to their work on game theory, took a step further from Bernoulli introducing rigorous axioms:

- ❖ Completeness Individuals have always a clear idea of what they prefer since they have well-defined preferences
For every options A and B either $A \geq B$ or $B \leq A$
- ❖ Transitivity: requires that individuals display consistency in their choice, therefore:
For every options A, B and C if $A \geq B$ and $B \geq C$ then $A \geq C$
- ❖ Independence argues that preferences over two options mixed with a third one hold when the two are presented independently of the third one
If $A \geq B$ and $t \in (0, 1]$; then $tA + (1-t)C \geq tB + (1-t)C$
- ❖ Continuity states that if $A \leq B \leq C$ then there exists a probability $p \in [0,1]$, such that
 $pA + (1-p)C = B$

They overturned Bernoulli's principles by using preferences to derive utility.

In Bernoulli's model, utility was used to define preference, because people were assumed to prefer the option that presented the highest utility. In the von Neumann and Morgenstern model, utility describes preferences; knowing the utility of an option informs an observers of a player's preferences. Von Neumann and Morgenstern's axioms do not determine an individual's preference ordering, but they do impose certain constraints on the possible relationships between the individual's preferences. In von Neumann and Morgenstern's theory, as long as the relationship between an individual's preferences satisfies certain axioms such as consistency and coherence, it became possible to construct an *individual* utility function for that person; such a function could then be used to demonstrate a person's pursuit of his maximum subjective utility. (McDermott, 2001, p-17)

In their model there is not a clear distinction between normative and descriptive aspects. In fact the axiomatic expected utility is an attempt to explain not only how people should behave but also how they do actually behave.

It follows that those who do not behave according to the axioms cannot be conceived as rational agents.

John von Neumann and Oskar Morgenstern (1947) showed that the expected utility hypothesis could be derived from a set of apparently appealing axioms on preference. Since then, numerous alternative axiomatizations have been developed, some of which seem highly appealing, some might even say compelling, from a normative point of view (see for example Peter Hammond 1988).² To the extent that its axioms can be justified as sound principles of rational choice to which any reasonable person would subscribe, they provide grounds for interpreting EU *normatively* (as a model of how people ought to choose) and *prescriptively* (as a practical aid to choice). My concern, however, is with how people actually choose whether or not such choices conform to a priori notions of rationality. Consequently, I will not be delayed by questions about whether particular axioms can or cannot be defended as sound principles of rational choice, and I will start from the presumption that evidence relating to actual behavior should not be discounted purely on the basis that it falls foul of conventional axioms of choice. (Camerer, 2004, p.106)

1.2.1 Allais paradox

One the first and most famous critique to the expected utility theorem was proposed by Maurice Allais (1953). Thought real observations he put on test the consistency of the expected utility theorem by revealing two different paradoxes. In his work he proposed to participants two different hypothetical scenarios. In the first they were asked to choose one of the following gambles

Gamble A		Gamble B	
Winnings	Prob	Winnings	Prob
100 million	100%	500 milion	10%
		1 milion	89%
		Nothing	1%

In the second scenario instead choices were:

Gamble C		Gamble D	
Winnings	Prob	Winnings	Prob
100 million	11%	500 milion	10%
Nothing	89%	Nothing	90%

Allais observed that when asked to choose between A and B, the most would prefer A gamble rather than B, but at the same time they were more likely to play D rather than C. Just to make everything clearer it is easy to see how the payoffs associated to each gambles are A=100, B=149; C=11 and D = 50.

The paradox arises from the fact that under expected utility theorem those who prefer A over B should also go for C instead of D. In fact A is less risky than B which instead has a higher payoff but this is not consistent with the further decision on D which corresponds to the riskier but richer payoff. Even if in general people are risk-averse they may opt for the uncertainty they just rejected in the first case. According to Allais EUT does not seem to be applicable to real world other conclusions can here be drawn. In particular here the dependence axiom does not hold. Allais refers to this paradox as the *common consequence effects* but it has been renamed after him and it is commonly referred in the literature as the Allais paradox.

Another paradox known as *common-ratio effect* has been investigated by Allais and it is based on the experimental design based on two next scenarios. In the first one people were asked to choose between the two following options:

Option A		Option B	
Winnings	Prob	Winnings	Prob
€ 3.000	100%	€ 4.000	80%

In the second one the choice instead was:

Option C		Option D	
Winnings	Prob	Winnings	Prob
€ 3.000	25%	€ 4.000	20%

In this case most people prefer option A in the first scenario and option D in the second one which violates EUT. In fact, by normalizing utility by $U(4000) = 1$ and $U(0) = 0$, it follows that Option A means that $U(3000) > 0.8$ while Option D means that $0.2 > 0.25 * U(3000)$, which is of course a contradiction.

It would, of course, be unrealistic to expect any theory of human behavior to predict accurately one hundred percent of the time. Perhaps the most one could reasonably expect is those departures from such a theory be equally probable in each direction. These phenomena, however, involve systematic (i.e., predictable) directions in majority choice. As evidence against the independence axiom accumulated, it seemed natural to wonder whether assorted violations of it might be revealing some underlying feature of preferences that, if properly understood, could form the basis of a unified explanation. (Cramerer, 2004, p.110).

1.3 From expected utility theory critiques to Prospect Theory

The prospect theory (PT), proposed at the end of the seventies by Daniel Kahneman and Amos Tversky (1979) is based on the observation that the probability distributions perceived by individuals who make decisions under conditions of uncertainty are not invariant with respect to environmental conditions. In particular prospect it is

a critique of expected utility theory as a descriptive model of decision making under risk, (...). Choices among risky prospects exhibit several pervasive effects that are inconsistent with the basic tenets of utility theory. In particular, people underweight outcomes that are merely probable in comparison with outcomes that are obtained with certainty. This tendency, called the certainty effect, contributes to risk aversion in choices involving sure gains and to risk seeking in choices involving sure losses. In addition, people generally discard components that are shared by all prospects under consideration. This tendency, called the isolation effect, leads to inconsistent preferences when the same choice is presented in different forms. An alternative theory of choice is developed, in which value is assigned to gains and losses rather than to final assets and in which probabilities are replaced by decision weights. The value function is normally concave for gains, commonly convex for losses, and is generally steeper for losses than for gains. Decision weights are generally lower than the corresponding probabilities, except in the range of low probabilities. Overweighting of low probabilities may contribute to the attractiveness of both insurance and gambling. (Kahneman and Tversky, 1979, p 263)

In their works (1979, 1992) the two authors investigated decision making processes under uncertainty by presenting different hypothetical gambles or prospects to their samples. The submitted problems were in contradiction with the fundamental principles of the expected utility theory and they kept undermining the validity of expected utility theory as a descriptive theory of human choice behavior. In particular the two authors put the emphasis on three important psychological phenomena, actually linked each other:

- Certainty effect: the individual disutility for losses is greater than the utility he would perceive by gaining the same amount. This principle probably follows a kind of survival instinct.
- Isolation effect: When facing multiple-stages games individuals tend to isolate consecutive probabilities instead of treating them together. This mistake leads to inconsistent preferences since according to the authors in order to simplify the choice people often ignore the probabilistic alternatives.
- Framing effect: the frame that is the context in which the individual has to make a choice, has a real impact on the choice itself. In particular the way the problem is formulated affects the way in which individual perceives their starting point (or "status quo"), and the final possible outcomes.

1.3.1 Certainty effect

Following Allais (1953) they investigated how people overweight certain outcomes compared with mere probable ones and they call this effect as *certainty effect* and their results were consistent with Allais critique. An interesting methodological aspect it is that the *certainty effect* has been proved to be solid both in situation with monetary and non-monetary outcomes. They proposed symmetrical hypothetical prospect in order to check for consistency in people decisions when dealing with positive and negative payoffs. They summarized the prospects proposed and their findings in this table:

TABLE I
PREFERENCES BETWEEN POSITIVE AND NEGATIVE PROSPECTS

	Positive prospects		Negative prospects	
Problem 3: N = 95	(4,000, .80)	< (3,000).	Problem 3': N = 95	(-4,000, .80) > (-3,000).
	[20]	[80]*		[92]* [8]
Problem 4: N = 95	(4,000, .20)	> (3,000, .25).	Problem 4': N = 95	(-4,000, .20) < (-3,000, .25).
	[65]*	[35]		[42] [58]
Problem 7: N = 66	(3,000, .90)	> (6,000, .45).	Problem 7': N = 66	(-3,000, .90) < (-6,000, .45).
	[86]*	[14]		[8] [92]*
Problem 8: N = 66	(3,000, .002)	< (6,000, .001).	Problem 8': N = 66	(-3,000, .002) > (-6,000, .001).
	[27]	[73]*		[70]* [30]

(Kahneman and Tversky, 1979, p 268)

As we can see, the problem 3 and 3' have the same probability and general structure but payoffs are positive in the first case and negative in the second one. Problem 3 was the same presented by Allais (1953) where the players had to choose between a sure gains equals to 3'000 € or try to win 4'000 € with 80% chance. Its negative versions proposes to player a sure lost equals to 3'000 € or a higher lost (4'000 €) but with 80% chance. N is the number of subjects who undertook the test while the numbers in the square brackets are the resulting probability of choice among them.

What arises from the data is that preferences with positive prospects are opposite to the ones with negative prospects. They called this *reflection effect*. There are three main implications with these findings:

1. When dealing with negative payoffs the majority behaves as risk-seekers as already was noticed by Williams (1966), Markowitz(1952) and Fishburn and Kochenberger (1979)
2. –Expected utility theory is empirically invalid here. The negative prospects are inconsistent with its assumptions and the same hold for the positive case. In the presence of positive payoffs the certainty effects lead subjects to be risk-averse by preferring sure win. Diametrically oppose under negative prospects subjects turn into risk-seekers.
3. Someone may argue that preferences are still consistent with EUT invoking that variance of outcomes leads choices. Consider for instance problems 3, data show that (3'000) is highly preferred compared to (4'000, 0.80) therefore individuals go for the prospect with higher variability but lower expected value. However when prospects are reduced as in problem 4 the difference in variance between (3'000, 0.25) and (4'000, 0.20) cannot compensate expected return difference. This is even clearer by looking at problem 3' where people should choose (-3'000) since it brings both lower variance and higher volatility but they instead go for the other option (-4'000, 0.80).

Our data are incompatible with the notion that certainty is generally desirable. Rather, it appears that certainty increases the aversiveness of losses as well as the desirability of gains. (Kahneman and Tversky, 1979, p 269)

1.3.2 Isolation effect

As already noticed by Tversky (1972) isolation effect is a strategy used by individuals that produces inconsistency with the EUT. When dealing with multiple choice alternatives, people tend to ignore the elements that alternatives have in common while placing more emphasis on those aspects which are different. Kahneman and Tversky (1979) to investigate more precisely this phenomenon proposed a two-stage game, named Problem 10, where:

In the first stage:

- There is a chance equals to 0.75 that the game ends
- Or a chance equals to 0.25 to move to the second-stage where participants have to choose one of the following gambles:
 - A = (4000, 0.80; 0, 0.20) [22%]
 - B = (3000, 1.) [78%]

The participants had to choose either A or B before the game starts. The values in the square brackets correspond to the sample's decisions. This problem can be summarized as a decision tree, where the square nodes are the ones in which players decide while circle nodes denote chances.

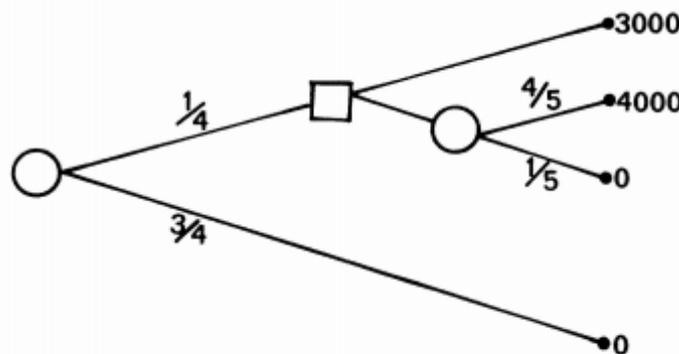


FIGURE 2.—The representation of Problem 10 as a decision tree (sequential formulation).
(Kahneman and Tversky, 1979, p 272)

By computing the compound probabilities from both stages the gambles participants are asked to decide upon are:

- (4000, 0.20; 0, 0.80) as the compound gamble for A
- (3000, 0.25; 0, 0.75) as the compound gamble for B

We can easily notice that Problem 10 is the compound version of the sequential problem 4 where options proposed were identical. Problem 4 presented two different options:

- A : (4000, 0.20) [65%]
- B : (3000, 0.25) [35%]

The data shows a reversal of preferences between the compound and sequential version of the same problem.

The reversal of preferences due to dependency among events is particularly significant because it violates the basic supposition of a decision-theoretical analysis, that choices between prospects are determined solely by the probabilities of final states (Kahneman and Tversky, 1979, p. 272).

Moreover we can notice that participants' decisions on the compound problem reflect the same preferences displayed in Problem 3 where they had to choose between:

- A (4000, 0.80; 0, 0.20) [80%]
- B (3000, 1.0) [20%]

The strategy adopted by people was to ignore the first stage of the problem since the outcomes are shared by both prospects while focusing on those aspects that are different.

At the end they considered only the second stage of the problem displaying the same dominant preferences as for Problem 3.

1.3.3 Framing effect

A first example of the framing effect is the fact that uncertain gains and losses are evaluated in relation to the wealth owned. The expected utility of the subjects is therefore not based on the monetary values according to the probability distribution, but rather on the deviation of these values from the status quo, which corresponds to the wealth or endowment of the individual.

Different levels of wealth can therefore produce contradictory orders of preferences on the same pair of goods. This idea is not totally new for Economics it has already been proposed in the late forties by James Duesenberry (1949) and his theory of relative income.

A popular example was proposed by Kahneman Tversky (1981) and known as the Asian disease. In this hypothetical framework individuals have to deal with life and death choices rather than monetary payoffs.

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows: (Kahneman Tversky, 1981, p 453)

A first group of respondents had to decide between the two following alternatives:

- | | |
|--|-------|
| Program A, if chosen it would lead to the safety of 200 persons. | [72%] |
| Program B if chosen would lead to | [28%] |
| ○ 1/3 chance that 600 persons will be saved | |
| ○ 2/3 chance that nobody will be saved | |

A second group of respondents instead faced the two following alternatives

- | | |
|---|-------|
| Program C, if chosen it would lead to the death of 400 persons. | [28%] |
| Program B if chosen would lead to | [72%] |
| ○ 1/3 chance that nobody will die | |
| ○ 2/3 chance that 600 persons will die | |

The two scenarios are totally identical but the way they are presented. In fact the amount of people who die or live is the same in both cases. The responses however took this frame effect into account by displaying risk adverse decision when dealing with gains and risk taking attitude under losses.

1.4 Prospect Theory

Kahneman and Tversky (1979) highlight the psychological motivations of perceptual distortions, showing how the understanding of the economic decisions cannot be separated from the analysis of the functioning of mental processes.

The loss aversion is a second important element of the prospect theory. This different attitude to gains and losses it is in clear contradiction with the theory of rational choice. Empirical evidences show that the incurring monetary losses trigger reactions in individuals proportionally more intense than obtaining gains of the same size.

Based on these observations, Kahneman and Tversky (1979) built a new decision-making function, with the purpose of offering an inductive representation of the cognitive processes that determine economic choices. In this sense, according to Innocenti's analysis (2009) the prospect theory can be considered as a descriptive model, without regulatory or prescriptive implications.

They argue that when decisions are taken the overall value of prospect, V , is defined in reference of two different scales: π and v .

The first scale, π , associates with each probability p a decision weight $\pi(p)$ which reflects the impact of p on the over-all value of the prospect. However, π is not a probability measure, and it will be shown later that $\pi(p) + \pi(1 - p)$ is typically less than unity. The second scale v , assigns to each outcome x a number $v(x)$, which reflects the subjective value of that outcome. Recall that outcomes are defined relative to a reference point, which serves as the zero point of the value scale. Hence, v measures the value of deviations from that reference point, i.e., gains and losses. (Kahneman and Tversky, 1979, p. 275)

Therefore the overall value of a prospect is described by these two functions. Consider a regular prospect as the ones described in the previous pages ($x, p ; y, q$) where either $p + q < 1$, or $x \leq 0 \leq y$, or $x \geq 0 \geq y$, then the overall value on the prospect can be described as:

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

An interesting case occurs when prospects presented lead to strictly positive or strictly negative outcomes. In fact in the decision-making process these prospects are broken into two components:

- A riskless part accounting for the minimum gain/ loss certain to be obtained/ paid
- A risky part accounting for the additional gain/loss actually in game

Formally if $p + q = 1$ and either $x < y < 0$ or $x > y > 0$, then the value of the prospect can be formulated as:

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)]$$

It means that the value of a strictly positive or negative prospect is equal to the value of the riskless component $[v(y)]$ plus the difference in terms of value between the outcomes $[v(x) - v(y)]$ multiplied by the weight associated with the more extreme outcomes $[\pi(p)]$. The crucial point here is to notice that the weighted function just applied to the risk component of the prospect but not to the riskless

one. This second equation corresponds to the first one if and only if $\pi(p) + \pi(1 - p) = 1$ which according to the authors is not usually satisfied.

The idea to re-define the utility theory based on gains and losses rather than on final asset positions was already suggested by Markowitz (1952) who also underlined the change of risk attitude in the presence of positive or negative prospects. Moreover Markowitz (1952) introduced a utility function displaying convex and concave regions in both the negative and positive domains. One of the weaknesses of Markowitz's solution is that it was still based on the expectation principle then he was not able to account for EUT inconsistencies described so far. The weighting function has instead its roots on the pioneer work of Edwards (1962), Fellner (1961) and van Dam (1975).

1.4.1 Value function

Prospect theory puts more emphasis on relative changes in wealth or welfare rather than on the final state. This can be applied in perceptions and judgments over sensory stimuli. In fact, physical stimuli such as brightness, loudness, temperature are evaluated by human minds according to the past - present difference. The present-past divergence is taken into account also for non-sensory perceptions as for instance when attributes to judge upon are the individual's wealth, health or prestige. Something that already Bernoulli (1738) noticed, back in the 18th century, is that the same level of income or money can be perceived negligible by some and significant for others.

The value function therefore does not ignore the impact that the initial position has. In fact it is a function based on two arguments:

1. The reference point
2. The magnitude of change, both in positive and negative cases, from that reference point.

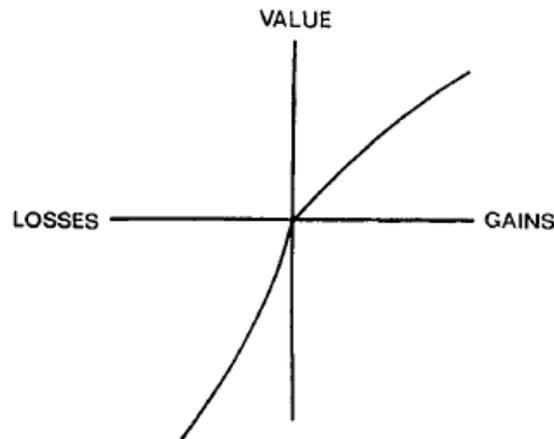
The Value function assumed by Kahneman and Tversky (1979) has some specific geometrical characteristics:

1. the function is concave for wealth changes above the reference point but convex for wealth changes below the reference point
2. small variations close to the starting point (in both regions) have a greater impact on individual utility rather than big changes far away from the same point
3. the curve has a greater slope in the region of the losses

These features reflect one-to-one important regularities observed:

1. People attribute different value to the expected probability distributions discriminating between losses or gains.
2. They show a diminishing marginal sensitivity to changes in both directions.
3. A gain and a loss in absolute terms do not have the same effect on individual, but a loss has a proportionately greater impact. (consistent with the loss aversion)

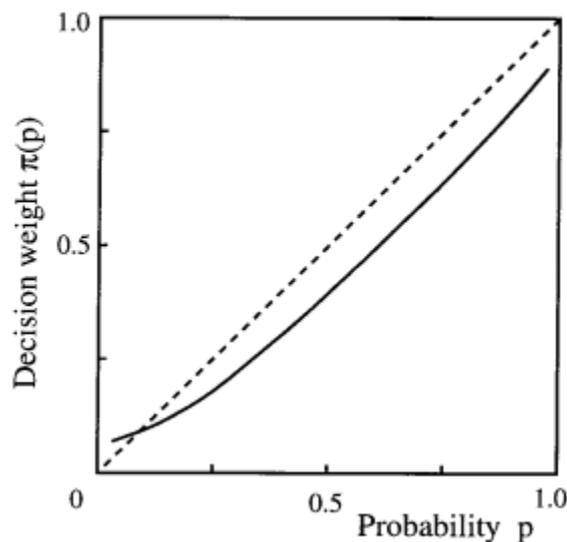
As it is clear from the picture the value function appears non-linear and passes through the origin which represents the starting point or status quo of the decision maker.



A hypothetical value function (Kahneman and Tversky, 1979, p. 279)

1.4.2 Probability weighting function

The probability weighting function $\pi(\cdot)$ assumes that individuals overestimate small probabilities as if they were higher than their actual values. At the same time large probabilities are instead underestimated relative to their numerical value. The two extreme cases 0 and 1 are not perceived as impossible and certain events that is why they are excluded from the weighting function, which presents a discontinuity close to the ends. As represented in its former formulation by Kahneman and Tversky (1979) the weighted function looks like:



Weighting function on its first description. (Kahneman and Tversky, 1979, p.283)

However this first version of weighting function has been revised by Kahneman and Tversky (1992) and the improvements turn prospect theory into the cumulative prospect theory (CPT).

1.5 Cumulative prospect theory

Cumulative prospect theory (CPT) holds the main assumptions and observations proposed in its first version while improving it.

At the centre of attention there is the weighting function π and its improvements due to both theoretical and empirical reasons.

The weighting scheme used in the original version of prospect theory and in other models is a monotonic transformation of outcome probabilities. This scheme encounters two problems. First, it does not always satisfy stochastic dominance, an assumption that many theorists are reluctant to give up. Second, it is not readily extended to prospects with a large number of outcomes. These problems can be handled by assuming that transparently dominated prospects are eliminated in the editing phase, and by normalizing the weights so that they add to unity. Alternatively, both problems can be solved by the rank-dependent or cumulative functional, first proposed by Quiggin (1982) for decision under risk and by Schmeidler (1989) for decision under uncertainty. Instead of transforming each probability separately, this model transforms the entire cumulative distribution function. (Kahneman and Tversky, 1992, p. 299)

Risk aversion and risk seeking were determined in PT by the utility function while in CPT, risk-preferences are the jointly result of individual attitude on the outcomes (represented by the v value function) and of individual attitude towards probabilities (represented by the π weighting function). Therefore these two functions determine risk attitude on individuals.

In its initial description PT presented many weaknesses that have been fixed in the CPT.

- π was a monotonic transformation that increases small probabilities while reduce high probabilities. However in the first formulation it is possible to notice from the chart that π looks relatively insensitive to changes for probabilities in the middle-region while it is relatively sensitive close to the end point (0, 1).
- π violated stochastic dominance.
- π cannot be easily implemented when the prospect's outcomes are more than two. The CPT instead expands its applied validity for any finite prospect. Moreover it can be applied not only for uncertain prospects but also to risky ones.

As remarked by Fennema and Wakker (1997):

The problem of the calculation of decision weights in PT, with regard to the generalization to many (different) outcome prospects, can be illustrated by the following example. Suppose we have a prospect with many different outcomes as follows: (-10; 0:05; 0; 0:05; 10; 0:05; 20; 0:05; 30; 0:05; 180; 0:05). If $\pi(0.05)$ is larger than 0.05 (as is commonly found) then each outcome is over weighted, and for the common value functions the prospect will be valued higher than its expected value \$85 for sure. It is very implausible that people will prefer the prospect to its expected value for sure. This anomaly is a consequence of the overweighting of all outcomes, a phenomenon that also underlies the violations of stochastic dominance (Fennema and Wakker, 1997, p 56-57),

The cumulative prospect theory presents a new value function that applies Quiggin's (1982) rank-dependent functional independently to gains and losses considering two different weighting functions w^+ that accounts for probabilities associated with gains and w^- that accounts for probabilities associated with losses. The introduction of these two weighting functions enables to shape two different attitudes towards gains and losses probabilities. The value function v has still the same characteristics as for the original PT except for the mentioned decision weights.

Consider a prospect with multiple outcomes as $(x_1, p_1; \dots; x_n, p_n)$ then the CPT value of such prospect is given by the following formula:

$$\sum_{i=1}^k \pi_i^- v(x_i) + \sum_{i=k+1}^n \pi_i^+ v(x_i)$$

The decision weights π_i^- and π_i^+ are defined as:

$$\pi_1^- = w^-(p_1), \quad \pi_i^- = w^-(p_1 + \dots + p_i) - w^-(p_1 + \dots + p_{i-1}) \quad 2 \leq i \leq k$$

$$\pi_n^+ = w^+(p_n), \quad \pi_i^+ = w^+(p_i + \dots + p_n) - w^+(p_{i+1} + \dots + p_n) \quad k + 1 \leq i \leq n - 1$$

Considering the special case where $w(p) = p$ for all p then probabilities here stay put ($\pi_i = p_i$) and the CPT value function will turn to be the traditional expected utility formula. Therefore CPT corresponds to a generalization of expected utility formula (Fennema and Wakker, 1997).

To explain the above formula, let us first repeat that in PT probabilities for the receipt of separate outcomes were transformed, i.e. each probability p_i for receiving the separate outcome x_i was transformed into the decision weight $\pi(p_i)$. In the above formula, 'cumulative probabilities' are transformed for gains, and 'decumulative probabilities' for losses. We first consider the case of gains. A cumulative probability describes the probability for receiving an outcome or anything better than that outcome. For instance, $p_i + \dots + p_n$ is the cumulative probability for an receiving outcome x_i or anything better. Decision weights for gains are obtained as differences between transformed values of cumulative probabilities. Similarly, for losses decision weights are obtained as differences between transformed values of consecutive decumulative probabilities, i.e. probabilities describing the receipt of an outcome or anything worse than that outcome. (Fennema and Wakker, 1997, p. 56).

The value function does not change in its essence between PT and CPT.

As in the original version of prospect theory, we assume that v is concave above the reference point ($v''(x) \leq 0, x \geq 0$) and convex below the reference point ($v''(x) \geq 0, x \leq 0$). We also assume that v is steeper for losses than for gains $v'(x) < v'(-x)$ for $x \geq 0$. The first two conditions reflect the principle of diminishing sensitivity: the impact of a change diminishes with the distance from the reference point. The last condition is implied by the principle of loss aversion according to which losses loom larger than corresponding gains (Kahneman and Tversky, 1991). (Kahneman and Tversky, 1992, p.303)

Kahneman and Tversky extended the diminishing sensitivity principle also to the weighting functions.

people become less sensitive to changes in probability as they move away from a reference point. In the probability domain, the two endpoints 0 and 1 serve as reference points in the sense that one end represents “certainly will not happen” and the other end represents “certainly will happen.” Under the principle of diminishing sensitivity, increments near the end points of the probability scale loom larger than increments near the middle of the scale. Diminishing sensitivity also applies in the domain of outcomes with the status quo usually serving as a single reference point. Diminishing sensitivity suggests that the weighting function has an inverse-S-shape—first concave and then convex. That is, sensitivity to changes in probability decreases as probability moves away from the reference point of 0 or away from the reference point of 1. This inverse-S-shaped weighting function can account for the results of Preston and Baratta (1948) and Kahneman and Tversky (1979). Evidence for an inverse-S-shaped weighting function was also found in aggregate data by Camerer and Ho (1994) using very limited stimuli designed to test betweenness, by Kahneman and Tversky (1992) using certainty equivalence data, by Wu and Gonzalez (1996) using series of gambles constructed to examine the curvature of the weighting function, and by Abdellaoui (1998) using a nonparametric estimation task. (Gonzalez and Wu, 1999, p. 136 – 137)

Kahneman and Tversky (1992) estimated the weighting functions for gains $w^+(p)$ and losses $w^-(p)$ starting from the following equations:

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \qquad w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}}$$

These linear equations have the advantage to rely on only one unique parameter able to implement the convex and concave regions according to the theoretical implications described so far. Moreover it does not require $w(0.5) = 0.5$ and it is consistent with both aggregate and individual data.

In order to obtain estimates for γ and δ they presented a two-outcome monetary prospects procedure by computer to 25 graduate students recruited and paid to undertake the experiment. Half of the prospects were based on positive outcomes while the other half on negative ones. The median values and estimates for γ and δ coincided respectively with 0.61 and 0.69. Moreover parameters' estimators of the value function were consistent with loss aversion and diminishing sensitivity.

Both estimated $w^+(p)$ and $w^-(p)$ are plotted in the following chart:

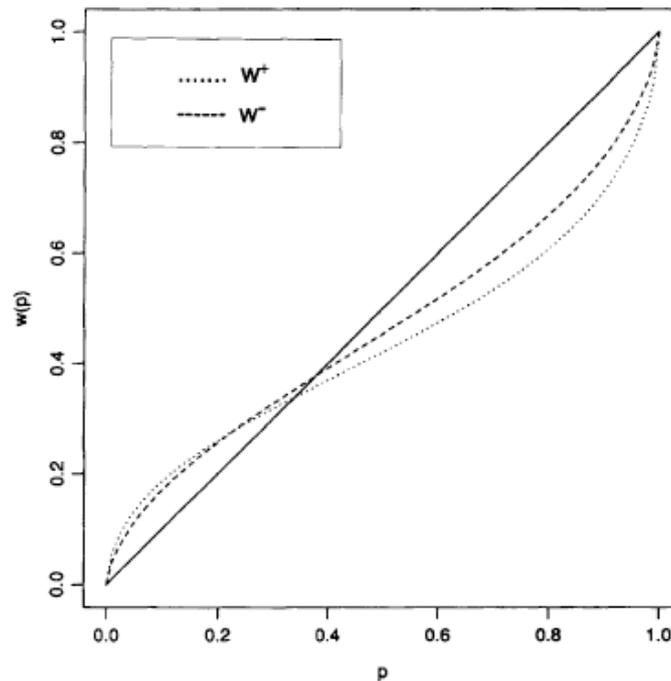


Figure 3. Weighting functions for gains (w^+) and for losses (w^-) based on median estimates of γ and δ in equation (12).

(Kahneman and Tversky, 1992, p. 313)

For both positive and negative prospects, people overweight low probabilities and underweight moderate and high probabilities. As a consequence, people are relatively insensitive to probability difference in the middle of the range. Figure 3 also shows that the weighting functions for gains and for losses are quite close, although the former is slightly more curved than the latter (i.e., $\gamma < \delta$). Accordingly, risk aversion for gains is more pronounced than risk seeking for losses, for moderate and high probabilities. (Kahneman and Tversky, 1992, p. 312)

CPT theory demonstrated how decisions under uncertainty share the main characteristics with decisions under risk do. Of course it is important to notice that these decisions have a different basis. In fact it has been demonstrated how people prefers to gamble on uncertain events on their own area of expertise rather than gambling on risky bets with chance devices even if the former are more ambiguous than the latter.

As already said the CPT incorporated the main features of the previously presented Prospect Theory. Additionally it provides a mathematical representation of the weighting functions and can be considered as a generalization of the expected utility theory. According to Kahneman and Tversky (1992):

Theories of choice are at best approximate and incomplete. (pg 317)

They say so since they know that the decision weights are sensible to how the problem is submitted to the decision-maker, since they are aware of how the formulation, the numbers and the level of outcomes may produce different curvatures in the representation of the weighting functions (Camerer, 1992).

From a theoretical standpoint

Prospect theory departs from the tradition that assumes the rationality of economic agents; it is proposed as a descriptive, not a normative, theory. The idealized assumption of rationality in economic theory is commonly justified on two grounds: the conviction that only rational behavior can survive in a competitive environment, and the fear that any treatment that abandons rationality will be chaotic and intractable. Both arguments are questionable. First, the evidence indicates that people can spend a lifetime in a competitive environment without acquiring a general ability to avoid framing effects or to apply linear decision weights. Second, and perhaps more important, the evidence indicates that human choices are orderly, although not always rational in the traditional sense of this word. (Kahneman and Tversky, 1992, p. 317)

1.5.3 Further development: Two-parameter weighting function

We have seen how the Weighting Function developed from its original formulation (Section 1.4.2) to its cumulative version (Section 1.5) (Kahneman and Tversky, 1979; 1992).

However this is not the end of the story. For simulation purposes we prefer to adopt the Weighting Function presented by Gonzalez and Wu (1999). In Kahneman and Tversky formulation (1979, 1992) there exists two Weighting Functions one for loss prospects and the other for gain prospects.

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{\frac{1}{\gamma}}} \qquad w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{\frac{1}{\delta}}}$$

As we can see, from the formula above both positive and negative cases are based on the same formula but with different parameters for the different kinds of prospect they apply. These parameters have been then estimated from a sample as described in section 1.5 .

Gonzalez and Wu (1999) instead proposed a two-parameter weighting function with two parameters.

where each parameter is given a psychological interpretation: one parameter measures how the decision maker discriminates probabilities, and the other parameter measures how attractive the decision maker views gambling. (pg. 129)

Their two-parameter weighting function has the following formula :

$$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$$

Where:

- The parameter γ controls the degree of the curvature and it refers to discriminability.
- The parameter δ controls the elevation of the function and it refers to attractiveness.

The nice additional feature of this weighting function is that parameters refer to two logical psychological attributes of each individual.

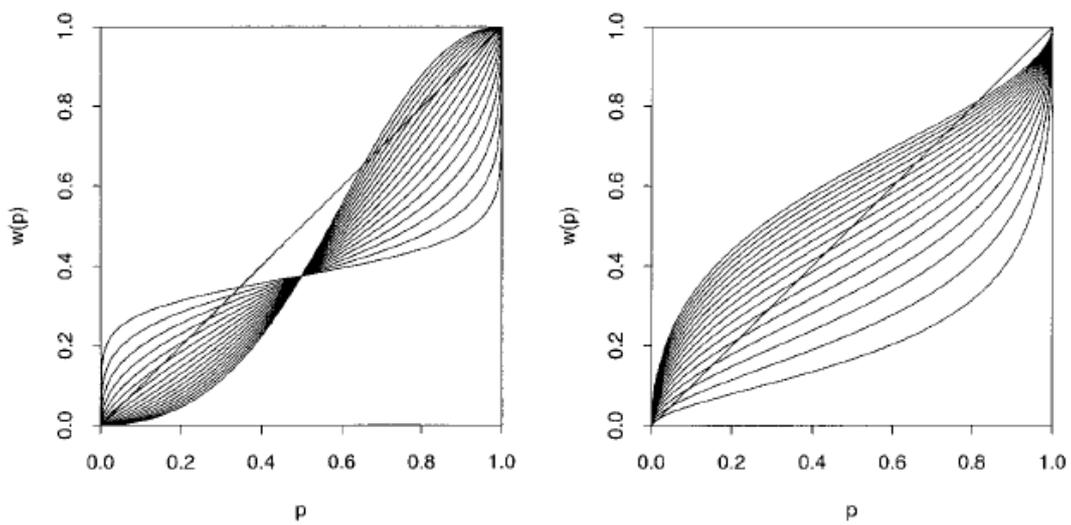
What is called discriminability refers to how the single individual discriminates the probabilities in the interval $[0, 1]$ and this varies both between individuals (interpersonal) and within the same individual on different contexts (intrapersonal). To make clearer what discriminability means for Gonzalez and Wu (1999):

The step function shows less sensitivity to changes in probability than the linear function, except near 0 and 1. A step function corresponds to the case in which an individual detects “certainly will” and “certainly will not,” but all other probability levels are treated equally (such as the generic “maybe”). [...] In contrast, a linear weighting function exhibits more (and constant) sensitivity to changes in probability than a step function. (Gonzalez and Wu, 1999, pp. 138)

Furthermore it is also possible to observe intra-personal differences on discriminability. As for example “an option trader may exhibit more discriminability for gambles based on options trading rather than gambles based on horse races” (pp. 138)

Attractiveness instead refers to the degree of over-under weighting of the function and can be conceived as the gambler’s attraction for the game where the higher is attractiveness the higher is the weighted probability function. Also here both interpersonal and intrapersonal differences have been highlighted. For instance someone could be more “attracted” to a chance domain rather than another preferring to bet on Sport results rather than political elections (or vice versa).

Graphically discriminability and attractiveness have different impacts on the weighting functions which may be clearer by looking at the following charts.



(Gonzalez and Wu, 1999, pp. 140)

In the first chart δ is equal to 0.6 while γ takes values in the interval [0.2, 1.8]. In the second chart instead γ is equal to 0.6 and δ varies between [0.2, 1.8]

Those aspects come at handy for the simulation purposes especially the observed interpersonal differences. In fact it allows us to add in the simulation different kind of agents starting by their discriminability and attractiveness values.

1.6 From lab to empirical evidences

One of most common critique to behavioral economics points at the fact that findings since have been obtained in a lab may not hold outside the experimental context.

This reasonable doubt has been addressed also to prospect theory (Kahneman and Tversky 1979; Kahneman and Tversky 1992). Many researches in the literature have been able to demonstrate how the prospect theory can explain the contradiction of human decisions from rationality under controlled experimental settings.

At the same time these findings help to understand those discrepancies that are difficult to reconcile with the traditional approach (Benartzi and Thaler 1995; Barberis et al. 2001; Barberis and Huang 2001; Barberis and Xiong 2009). Considering also that most experimental studies rely on student sample one may have an additional reason to underestimate the findings around prospect theory.

Therefore a good empirical strategy may be to investigate whether or not financial professionals behave according to prospect theory (Abdellaoui et al., 2001). Moreover if they do so there exist one unique version of prospect theory applicable to both students and professionals? Or instead two different versions of prospect theory are needed, one for the students in the experimental context and the other for the professionals in the real world? In fact while students face one-shot independent choices, financial professionals also deal with a systematic feedback on their decisions and are periodically evaluated on their performance.

Another good point is that professionals are well qualified, trained and with solid experience in order to successfully diversify risks. In their empirical study Abdellaoui et al. (2001) involved 46 financial professionals most of them were private bankers while some others were fund managers used to handle between \$20 million and \$1 billion with a \$295 million mean. They behaved according to the prospect theory and then violating the expected utility paradigm. They were consistent with the overall prospect theory but they displayed a less evident degree of loss aversion compared with the student samples.

Similar results have been also found by Kliger and Levy (2009) they assessed the performance of the three prevailing approach of choice under risk models: expected utility, rank-dependent expected utility, and the prospect theory. It is important to highlight that they took one step further by using real information from financial asset markets rather than using another abstract experimental design. In particular they focused on the call options of the index Standard & Poor's 500 (S&P 500). Their empirical results displayed loss aversion, non-linear probability weighting and diminishing marginal sensitivity which are the typical features of prospect theory.

Similarly Gurevich et al. (2009) adopted an empirical method by using US stock option data.

Option prices contain information about actual investors' preferences, which are possible to elicit using conventional option analysis and theoretical relationships. The options data in this study are used for estimating the two essential elements of CPT, namely, the Value Function and the Probability Weight Function. The main part of the work focuses on the functions' simultaneous estimation under the original parametric specifications used in Kahneman and Tversky (1992). Qualitatively, the results support the central principles of the model: the shapes and the properties of the estimated functions are in line with the theory. Quantitatively, the estimated functions are both more linear in comparison to those acquired in laboratory experiments, and the utility function exhibits less loss aversion than was obtained in the laboratory. (Gurevich et al., 2009, p. 1226)

The Probability Weighting Functions found by Gurevich (2009) are more linear than the ones proposed in the original formulation of CPT. It seems that professionals are less prone to rational mistakes in both gain and losses situations. Moreover the weighting function associated with gains (PWF gains) a less pronounced S-shape trend compared to the weighting function associated with losses (PWF losses)

Gurevich (2009, p 1225)

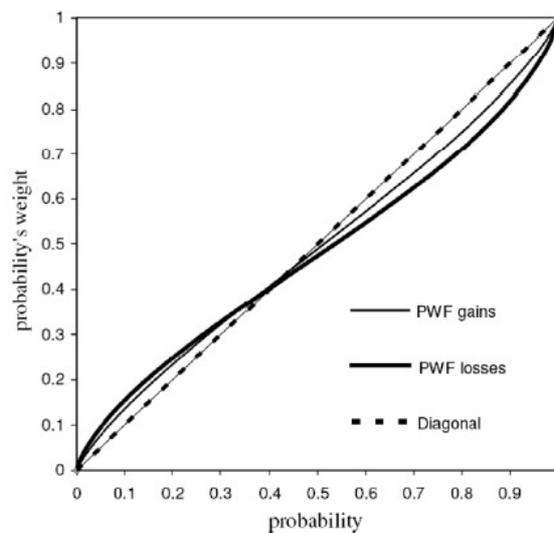


Fig. 1. PWF for gains and losses according to median estimation results.

The idea that professionals face a more linear weighting function is supported by several studies. In the two weighting function framework a more linear weighting function is in fact associated with a high discriminability among probabilities.

some experts possess relatively linear weighting functions when gambling in their domain of expertise: the efficiency of parimutuel betting markets suggests that many racetrack betters are sensitive to small differences in odds (see, e.g., Thaler & Ziemba, 1988), and a study of options traders found that the median options trader is an expected value maximizer and thus shows equal sensitivity throughout the probability interval (Fox, Rogers, & Tversky, 1996) (Gonzalez and Wu, 1999, pp. 138)

2. AGENT-BASED MODEL

2.1 Introduction

Agent-Based model (ABM) refers to a class of computational models that simulates actions and interactions of self-governing agents in order to evaluate their effects on the system as a whole (Gilbert, 2008).

Agents can be both individual and collective entities according to the researcher's model and simulation purposes. ABM can be considered as a micro-scale model since it is based on individuals' own actions and interactions among agents with the aim of re-creating a (macro) complex event. The macro phenomena then arise from the lower (micro) level as a bottom-up process (Tesfatsion, 1997). This generative approach can be found on the words of Epstein (2006):

Given some macroscopic explanandum – a regularity to be explained – the canonical agent-based experiment is as follows: Situate an initial population of autonomous heterogeneous agents in a relevant spatial environment; allow them to interact according to simple local rules, and thereby generate – or ‘grow’ – the macroscopic regularity from the bottom up [...]. In fact, this type of experiment is not new and, in principle, it does not necessarily involve computers. However, recent advances in computing, and the advent of large-scale agent-based computational modelling, permit a generative research program to be pursued with unprecedented scope and vigour (Epstein 2006, pg. 7).

It is possible to identify several key guidelines for those who want to move in the domain of ABM:

- K.I.S.S. (Keep It Simple Stupid) which can be considered as the computational version of Occam's razor, for which one goal of model designer is to achieve simplicity by avoiding useless complexity.
- Agents have bounded rationality so that they act following their own interests using heuristics, simple decision-making rules and also being mistaken.
- In ABM the whole is regarded as more than just the sum of the parts (Simon, 1962). This concept goes back up to Aristotle's tradition

At the actual state of art:

Agent-based modeling is no longer a completely new approach, but it still offers many exciting new ways to look at old problems and lets us study many new problems. In fact, the use of ABMs is even more exciting now that the approach has matured: the worst mistakes have been made and corrected, agent-based approaches are no longer considered radical and suspicious, and we have convenient tools for building models (Railsback and Grimm, 2011, p.11).

2.2 Supporting ABM in Economics

The fruitful union of ABM methodologies in the domain of economics is often referred as Agent-based Computational Economics (ACE) (Tesfatsion, 2002, 2006).

Agent-based computational economics (ACE) is the computational study of economies modeled as evolving systems of autonomous interacting agents. Starting from initial conditions, specified by the modeler, the computational economy evolves over time as its constituent agents repeatedly interact with each other and learn from these interactions. ACE is therefore a bottom-up culture-dish approach to the study of economic systems. (Tesfatsion, 2003)

It is possible to identify many good reasons in support of ABM as an important and valid tool for economic analyses able to overcome the limitations that the traditional economists' toolbox carries. In the next pages are going to be presented the main advantages of ACE as well as its typical features

2.2.1 Complexity matters

On the reasons why economists should implement ABM in their toolbox we can read on Luna and Stefansson (2000) how mathematical formalisms are not enough when dealing with complex systems.

Economists have become aware of the limitations of the standard mathematical formalism. On the one hand, when dealing with real world phenomena, it is often impossible to reach a "closed form" solution to the problem of interest. One possible approach is to simplify the problem so that an elegant closed-form solution is synthesized. The implicit assumption is that the simplification process has spared all relevant elements and discarded only unnecessary ornaments. In case this a priori seems too strong, the empirically oriented researcher may want to employ a simulation to study the dynamical properties of the system. (Luna and Stefansson, 2000, p. XXIV)

It is worth to mention Anderson (1972) and what is generally considered as the manifesto of complexity:

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted without questions. [...] The main fallacy in this kind of thinking is that the reductionist hypothesis does not by any means imply a "constructionist" one: The ability to reduce everything to simple fundamental laws does not imply the ability to start from those laws and reconstruct the universe. [...] The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new

properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. (Anderson, 1972, p. 393).

According to Anderson (1972) modern economics can be represented as a complex social system of different and many layers. At each level more and more agents are in play: consumers, policy-makers, entrepreneurs, firms, institutions, multinationals, governments, international institutions ... and even more.

The agent – system relation can be described following Terna's (2013) analogy:

People make economies, but each of us is as far from understanding and controlling the economic system as a humble ant with respect to the anthill. Economics, as a science, has been simply ignoring that "detail" for about two hundred years. In Anderson's words, complexity is the big trap generating the current paranoiac situation for which the crisis (2007-2012, at least) has no room in perfect models, but ... it exists in the actual world. How to work with the lens of complexity? We need models, for us and ... for the ants (Terna, 2013, p. 167)

Furthermore time is an additional crucial variable that makes social systems even more complex to formalize with a simple equation:

One of the traditional problems of social sciences is to have methods and tools to understand the evolving nature of social structures and institutions. Every social scientist acknowledges the process nature of social phenomena, but, for sake of tractability or for lack of appropriate modelling tools, he/she uses theories and models that do not seriously reflect this belief. Computer simulation is a crucial means to put process, change and long-term dynamics at the very core of the social science research. Thanks to its capability of reproducing, synthesizing and visualizing space and time, it allows social scientists to thinking social phenomena in terms of processes that emerge from agent interaction and change over time (Squazzoni, 2010, p. 202-203)

In this light ABM becomes an operational tool for economists who do not want to ignore complexity and are brave enough to leave behind them the need for a deterministic and reductionist approach. A new way of doing research is then available to re-shape old questions and to open the door to new ones. ABM has been argued to be a third way to do science, in Axelrod's words (1997), one of the pioneers of ABM:

Agent-based modelling is a third way of doing science. Like deduction, it starts with a set of explicit assumptions. But unlike deduction, it does not prove theorems. Instead, an agent based model generates simulated data that can be analyzed inductively. Unlike typical induction, however, the simulated data come from a rigorously specified set of rules rather than direct measurement of the real world. Whereas the purpose of induction is to find patterns in data and that of deduction is to find consequences of assumptions, the purpose of agent-based modelling is to aid intuition. Agent-based modelling is a way of doing thought experiments. Although the assumptions may be simple, the consequences may not be all obvious. (Axelrod, 1997, p.3-4)

2.2.2 Filling the theory-data gap

One common critique to economics even nowadays is the distinction between theoretical and empirical approaches. Despite many cases with consistent integration there still exist many other cases in which this gap occurs. On one hand we face theoretical models more focused on a deep inner consistency rather than on empirical persistency. On the other hand many empirical evidences are not analyzed in deep since they do not fit with available theory.

The lack of flexibility in the toolbox at disposal of economists is one of the reasons why it occurs. A well-known example of this gap is the well-known unrealistic assumption of utility-maximizing agents. In fact the formal model is not able to account for the empirical behavioral evidences, economists' alternative is then to back up even today on the as-if principle following Friedman's (1953) legacy.

The behavioral models based on empirical evidences rather than on rigorous formalization are not universally accepted today creating a new gap between mainstream economics and not. An interesting gap arises since even if there exists a general agreement on the as-if unrealistic nature there is not an alternative which fit with the traditional approach. The economist's toolbox forces those who use it to face a trade-off between realism and tractability. Differently ABM has the great advantage to escape from this trade-off allowing a convergence between theories and empirical evidences (Squazzoni and Boero, 2005).

ABM is the relatively new tool for economists to deal both with abstract theoretical questions and with technical ones that requires a big amount of data. Moreover researchers can implement empirical findings in theoretical analysis. In this domain ABM provides two great advantages. First of all, it fills the gap between theory and data providing continuous communication between these two fields. Secondly it helps researches to properly answer many questions that arise in contemporary world.

2.2.3 Causality, uncertainty and ex-ante evaluations

As already mentioned ABM allows a greater flexibility in economical models. In particular it allows researchers to choose the degree of realism they want to achieve when modelling. The real boundary here is not the old technical strict formalism but the knowledge and data available on the subject of interest. Flexibility is not only something good *per se* but it has an important impact on causality worth to be analyzed here. In fact, realistic and complex models are able to underlying the fundamental relations among the variables in play.

The researcher is free to investigate both the relationships among variables as well as those between the variables and the macro phenomenon recreated. Therefore it is possible with ABM to check for causality by directly looking at the simulated phenomenon and its variables rather than looking for causality in spurious correlation data. Together with adequate validation techniques ABM has the power to disclose causality in social and economic contexts. However, ABM is not only fruitful in the domain of theory development and valuation but can play an important role dealing with predictive analysis.

This comes at handy for policy-makers and institutions on evaluating the effect of a new policy. The great feature of ABM in this context is the possibility to perform ex-ante investigation in order to check for the consequences and the weakness arising from the introduction of the new policy. In this way it is possible to develop highly effective policies since the ex-ante investigation provides how and where to operate with changes.

The possibility of an ex-ante valuation particularly when the policy-maker intervention has not similar pre-existing examples provides a higher level of confidence in the model itself. Similarly agent-based models can better investigate the level of uncertainty connected with the phenomenon reproduced. In fact, ABM provides a deep investigation of how uncertainty spreads within the model. From here it is possible for researchers to distinguish between the uncertainty that arises from the imperfection of the model and the aleatory counterpart due to the undeniable uncertainty of human behavior.

2.2.4 Restoring Heterogeneity

One methodological critique often addressed to many economical theories is the concept of representative agent. This principle traces back to the late 19th century where we can find Edgeworth (1881) and his "*representative particular*" as well as Marshall (1890) and his "*representative firm*". In its most basic form, the choices and decisions of an heterogeneous group in one sector (consumers, firms ...) are considered as the choice of a "representative" standardized utility (or profit) maximizing agent. In order to describe the reality in its most simplified version, the observed heterogeneity is sacrificed. The aggregated effect of many different consumers, firms, households turn to be the sum of their standardized counterpart. According to Kirman (1992), this way of reasoning once applied leads to both misleading conclusions and fallacy of composition.³

ABMs are instead able to replicate the heterogeneity observed in reality. In fact researchers are here not constrained on the degree of similarity among the agents involved but they can freely choose the degree of heterogeneity they prefer. Actually, it is possible to model each single agent with different behaviors, characteristics and preferences even implementing millions of different and differences.

The Object Oriented Programming (OOP) language behind many ABM software packages suit well with agents' description. In fact it is possible to define agents considering many classes and objects providing the chance to model a virtually infinite number of different economic agents. Furthermore dealing with different type of agents rather than with a unique representative one forces research to re-consider interactions' role insofar almost denied. At the best, they were considered as a two-way communication where the representative consumer interacts with the representative firm. Again ABM produces a higher degree of realism also on social interactions among the agents. In this work ABM potential has been fruitful exploited in order to design a Continuous Double Auction model able to simulate a financial market where many traders place their bid or ask prices whose bargains determine the resulting daily price.

³ A fallacy of composition lead to conclude that something is true for the whole from the fact that it is true for some part of the whole or even of every proper part of the whole.

2.2.5 Multi-disciplinarity

The agent's interactions lead to the born and development of social networks. This social dimension has been completely denied in mainstream economics since it is not possible to formalize it in mathematical terms inside the agents' utility functions or constraints. AMBs are instead able to account for a large variety of social dynamics considering the behavioral and economical context in which they occur. In this context economics has little or nothing to offer in order to shape social dynamics. On the other hand the high flexibility offered by ABM provides the chance to the researcher to open his mind to new and fascinating contributions which may be to him unknown.

In particular in this work Economics met with Psychology in order to develop a more realistic financial market based on the findings of Kahneman and Tversky (1979, 1992). The emulation of cognition is something feasible on ABMs even though highly challenging. With Sun (2006, p.17)

What makes computational social simulation, especially computational cognitive social simulation (based on detailed models of cognitive agents), different from the long line of social theories and models (such as utility theory and game theory) includes the fact that it enables us to engage with observations and data more directly and test various factors more thoroughly. In analogy with the physical sciences (...), good social theories may be produced on the basis of matching theories with good observations and data. Cognitive agent based computational social simulation enables us to gather, organize, and interpret such observations and data with cognition in mind. Thus, it appears to be a solid means of developing social-cognitive theories (p. 17).

Interesting researches can then be undertaken integrating economics with finding of other disciplines. In ABMs agents do not have to behave just and only according to a well-defined utility function. On one hand they can be modeled free to act according to the neo-classical utility function. On the other hand it is possible to include (as well as to rely only on) other drivers for human actions.

In equation-based models this is far from possible and then the real complexity of the system is denied. Findings in the domain of decision-making in organizations, production organization and supply chain do not fit easily on an equation. In contrast the flexibility provided by ABMs is able to include those on a more realistic perspective. Furthermore ABM is a gravity train providing the opportunity to include both quantitative and/ or qualitative data as well as linear and/ or non-linear relationships.

3. FROM REAL TO SIMULATED CDA FINANCIAL MARKETS

3.1 Financial markets: from out-cry to electronic trading systems

Financial market is a broad term that identifies any market where buyers and sellers trade on financial assets such as equities, bonds, currencies and derivatives. Broadly speaking the term *market* refers to the aggregate output arising from the transactions between buyers and sellers. However it is often used to refer to exchanges either physical (like the New York Stock Exchange) or virtual (like the NASDAQ). Moreover it is possible to identify financial markets according to the assets exchanged, so we refer to Stock market for securities, to the foreign exchange market for currencies and to the derivatives market for futures and derivatives.

In particular, the first purpose of financial markets is to guarantee an efficient allocation of financial resources between borrowers (Individuals, Companies, Central Governments, and Public Corporations) and lenders (Individuals, Companies). In addition to that financial markets guarantee the transactions of financial assets thank to the transparency on the price formation and to the speed of transactions.

It is easily understood that financial market performances are deeply linked to the economy of the country they refer to. In fact a *bull* stock market reflects the strengthening of the listed companies and it is perceived as a positive sign for the whole economic growth. On the other hand a *bear* stock market could be the signal of an economical crisis that may affect some sectors or the whole economic system.⁴ Asset's prices are a mixture of available information at the moment and future expectations that affect investor's decisions. It may happen that prices do not reflect the real asset's value while it is instead the result of investor's behaviors leading to irrational sentiments and the raising of financial bubbles leading to crashes as for the dot-com bust of 2000–2002.

It is important for the aim of this work to highlight here the mechanisms of price formation and how exchanges take place in real Stock markets in order to be able to implement it successfully on the model. Traditionally transactions were managed face-to-face in an open-outcry environment. There was a real language to be known in order to place trades which was based on a complex hand-sign system. However every trade had to be registered in order to be effective.

Nowadays physical market and outcry methods have been replaced by electronic systems that manage all the relevant information in order to place orders in real times. In Italy the electronic trading system was been adopted since the 25th November 1991 for some financial instruments and extended to all listed instruments by Borsa Italiana S.p.A. in 1996. This provides the same level of information to all potential traders who can invest on real time.

⁴ Bull and bear terms are commonly used to describe market's trends respectively for increasing and decreasing trend in asset's price. The metaphor comes from the way the two animals attack in nature. While the bull points its horns up into the air the bear swipes its paws down.

The electronic trading system has led to important implications for the overall financial markets:

- Reduced cost of transactions for the investors who are able to increase their trading volumes avoiding significant increased costs.
- Greater liquidity in the market is provided since companies and individuals can trade regardless their geographical position. The greater liquidity is not something good only *per se* but it leads to a higher efficiency in the market.
- Greater competition is provided since with the abolition of barriers in the financial sector and more globalized world investors are free to trade all over the world at their own will.
- Increased transparency, now market is less opaque since it is easier to get asset's prices while they are shared around the globe simultaneously.
- Lower spreads⁵ are provided to the investors thanks to more competition and transparency in the market

The electronic trading system provides the trade offers available for the financial instruments inside a *book* which appears on operators' terminals consisting in a sequence of pages. The offers are divided into buy and sell prices and are ordered according to the prices they are ascending for sells and decreasing for buys. Moreover it reports for orders the operators' indications, the code of traded instruments and all the instructions for the trade to take place: quantity, time and price conditions. If an offer already sent is revised by the trader in terms of quantity or entry price then it loses the acquired time priority.

There are different ways in which offers can be made according to the preferred investor's conditions to price, size and timing. On price, it is possible to distinguish among:

- Orders at the market: it is the basic form of an offer according to which the investor decides to open or close his trade at the available price. This form of order is the fastest one to be executed depending on the availability of a counterpart which whom to trade.
- Orders with limit price: when a limit price specifies the maximum price to buy or the minimum price to sell the instrument.

With respect to quantity it is possible to identify

- All or None (AON) orders ensure that investors get either the entire quantity of stock requested or none at all. This is typically problematic when a stock is very illiquid or a limit is placed on the order.
- Execute the specified minimum amount. The order is executed at least for the minimum quantity chosen if available. If not, the order is deleted.

⁵ The spread of a financial instrument corresponds to the difference between the best bid price and the best ask price being quoted.

With respect to time we can find:

- Good Till Canceled (GTC) orders will remain active until traders decide to cancel it. If the time length is not specified the order will stay in the book up to the end of the trading day. Traders can decide the length of it in terms of minutes, hours or days. It can be varied according to traders' strategy to expire in the future.

All buy and sell offers compatible each other in terms of price and quantities are executed automatically by the electronic trading system through an automatic matching process. A potential buyer announces a bid price according to his own expectation that prices will increase over time. Conversely, at the same time any potential seller is free to announce an offer-price (commonly referred to as an "ask"), with the expectation that prices will decrease over time.

Therefore the process of the electronic trading system matches the *ask* prices with the lower or equal *bid* prices available in the market while *bid* prices are matched with the higher or equal *ask* prices available.

The CDA mechanism introduced in the next pages takes into account all these aspects and replicates those underlying mechanism in the price formation of the assets from the spontaneous transactions among the players involved.

3.2 Technical and fundamental analysis

Forecast the future is not fascinating per se but it is even more interesting for those operating in financial markets. Although none has a crystal ball to predict precisely what is going to happen tomorrow investors rely on two basic analytical models: fundamental and technical analysis. Despite they use different tools and are shaped on different ideas they can be used either separately or complementary. For instance a fundamental trader may decide to use technical guidelines to decide his entry and exit points whereas a technical one may rely on fundamentals in order to invest only on stocks of reliable and “good” companies.

3.2.1 Fundamental Analysis

Fundamental analysis is based on the belief that in the short-run market may misrepresent the real value of a security that will be corrected in the long-run by the market itself. For instance investor fundamental analysis could perceive that an asset is underestimated in the short-run providing a good opportunity to place a buy order and then making profits on the long-run adjustment process.

It focuses on different information depending on the nature of the asset. Firms’ shares lead investors to analyze the overall business’s financial statements in terms of assets, liabilities and earnings as well as the competitors and sector in which the firms operate. Differently in the foreign exchange market investors take into account the general economy of states considering instead interest rates, GDP, earnings, unemployment, housing, manufacturing and management. Of course the aim is to forecast and beat the market by determining which stocks to buy or sell and at which prices.

3.2.2 Technical Analysis

While fundamental investors believe that the market will adjust misprices, technical ones hold that all relevant information is already included in the available price. For a technical analyst the market discounts everything. Therefore they prefer to analyze trends and to focus on market’s sentiment as a driver able to predict future changes. In particular they use chart patterns, historical trends and technical indicators to perform forecasts. In Murphy’s words (1999), one of most popular American technical analyst:

Technical analysis is the study of market action, primarily through the use of charts, for the purpose of forecasting future price trends. The term “market action” includes the three principal sources of information available to the technician: price, volume, and open interest. (Murphy, 1999, pp.1-2)

Technical analysis is considered as a pseudo-science by many academics and researchers. This line of thinking is shared by Princeton economist Burton Gordon Malkiel (1996) and his popular work *A Random Walk down Wall Street*. According to him

"under scientific scrutiny, chart-reading must share a pedestal with alchemy."
(Malkiel, 2011, p. 157)

In defense of technical analysis is worth to report here the findings of Andrew W. Lo et al. (2000), director at MIT of the Laboratory for Financial Engineering.

Technical analysis, also known as "charting", has been a part of financial practice for many decades, but this discipline has not received the same level of academic scrutiny and acceptance as more traditional approaches such as fundamental analysis. One of the main obstacles is the highly subjective nature of technical analysis – the presence of geometric shapes in historical price charts is often in the eyes of the beholder. In this paper, we propose a systematic and automatic approach to technical pattern recognition using nonparametric kernel regression, and apply this method to a large number of U.S. stocks from 1962 to 1996 to evaluate the effectiveness of technical analysis. By comparing the unconditional empirical distribution of daily stock returns to the conditional distribution – conditioned on specific technical indicators such as head-and-shoulders or double-bottoms – we *find that over the 31-year sample period, several technical indicators do provide incremental information and may have some practical value*. (Lo, Mamaysky and Jiang, 2000, pp. 1705, emphasis added)

More recently it has also been found that

Modern studies indicate that technical trading strategies consistently generate economic profits in a variety of speculative markets at least until the early 1990s. Among a total of 95 modern studies, 56 studies find positive results regarding technical trading strategies, 20 studies obtain negative results, and 19 studies indicate mixed results. Despite the positive evidence on the profitability of technical trading strategies, most empirical studies are subject to various problems in their testing procedures, e.g. data snooping, ex post selection of trading rules or search technologies, and difficulties in estimation of risk and transaction costs. (Irwin and Park, 2007, pp. 786)

Despite it is a controversial topic in the financial literature, it is worth to highlight that most large investors and institutions typically hold both a technical and fundamental analysis team.

3.2.2.1 Technical indicators: an eye on Bollinger bands.

At the core of technical analysis there are technical indicators. In essence they are formal calculation on past data which are graphically represented on price charts. They are used in order to help to understand and predict in which direction the market is going to move.

Additionally they may provide useful information on what is happening in the market as the case for oversold or overbought conditions.⁶ During the years many technical indicators have been developed, modified and re-shaped according to investors' needs. Most popular ones have been grouped here according to their purpose:

- Support and resistance indicators: Pivot points, Fibonacci retracements.
- Trend indicators: Average directional index (ADX), Moving average (MA) Moving average convergence/divergence (MACD).
- Momentum indicators: Relative strength index (RSI).
- Volume: On-balance volume (OBV).
- Volatility indicators: Bollinger Bands (BB).

This classification is really simple and considers only a relatively small number of indicators. However the purpose here it is not to describe all aspects of technical indicators in details but to provide the main essences that have been considered in the model.

In particular this work and the model presented on next pages focused on Bollinger Bands together with Moving Averages. They have been preferred among the other indicators for their ability to provide useful information on both direction and volatility of the market based on simple but reliable statistical data.

Bollinger Bands is a technical analysis tool created by Bollinger J. in the 1980s (Murphy, 1999; Bollinger, 2001) and they have quickly become one of the most commonly used tools in technical analysis. They are considered a volatility indicator and consist of:

- a N-period moving average (MA) together with
- an upper band (UB) that corresponds to K times the N-period standard deviation above the moving average ($MA + K \sigma$)
- a lower band (LB) that corresponds to K times the N-period standard deviation below the moving average ($MA - K \sigma$)

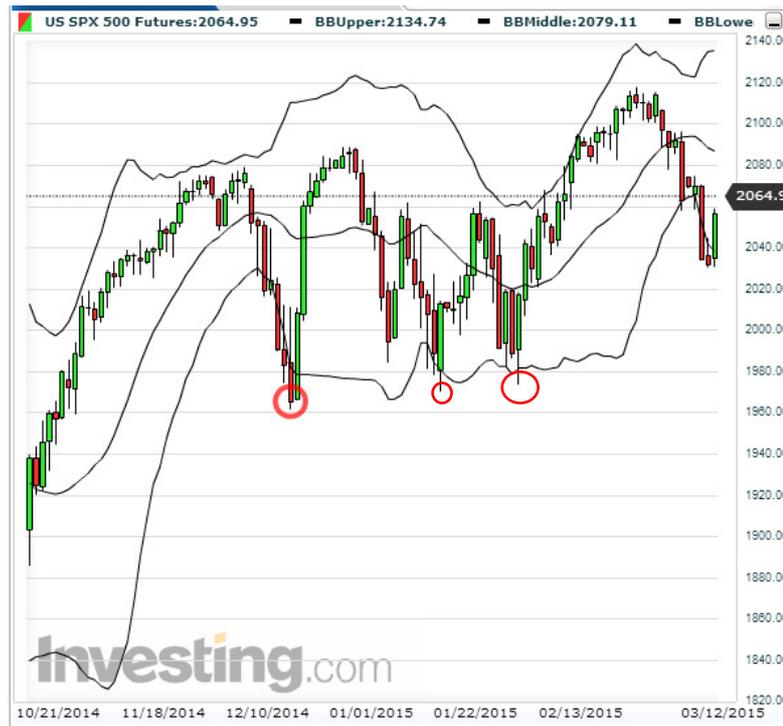
The typical values for N and K are 20 and 2, respectively and those values have been used in the simulation. Graphically they appear as a channel that encloses in the two bands four standard deviations.

Of course real and professional traders would not base their strategy uniquely on a single indicator. However I made up a simple strategy designed on the bands to make simulated traders decide as if they had some professional insight about the market.

⁶ Overbought refers to a situation where an asset's demand is so high that pushes prices up. This is may lead the asset to become over evaluated. On the opposite hand over-sold arises when prices start falling sharply (panic selling) leading the asset to be under-evaluated.

This basic strategy, implemented in the simulation, exploits heavy selling (oversold) and heavy buying (overbought) conditions that correspond to the breaks of lower or upper bands respectively. Usually, once a lower band breaks due to oversold, the asset's price reverts back above. Conversely once an upper band breaks due to overbought, the price reverts back below.

In the following chart of S&P 500 in a daily timeframe, we can see how after the break of the lower band (highlighted with a red circle) the trend of the S&P 500 started to raise up above the Moving Average.



3.3 Continuous Double Auction model

The Netlogo model presented in the next pages relies on an underlying Continuous Double Auction (CDA) mechanism. In a CDA a potential buyer announces a bid price according to his own expectation that prices will increase over time. Conversely, at the same time any potential seller is free to announce an offer-price (commonly referred to as an “ask”), with the expectation that prices will decrease over time. In this model there is no room for an auctioneer since at any time, any buyer or seller is free to accept any counterpart’s offer or bid respectively with the main purpose of choosing the best bid or offer available at that moment in the market.

The model can replicate the start and end of the trading day by using a clock running. However the “continuous” feature of the CDA is such that any agent is totally free to join, trade or leave the market whenever they want since their decisions are not coordinated by any auctioneers. At first glance the CDA has a great practical interest since it replicates almost every major financial market in the World. On one hand the model is able to resemble closely financial indexes and intra-day trading time series while on the other hand it is not able to successfully replicate volume-related statistical properties.

Laboratory experiments have shown a quick convergence to a competitive equilibrium and an efficient allocation as demonstrated by Smith (1962) and Smith et al (1982). The CDA mechanism is particularly suited to replicate modern financial markets since it is the basis of almost all automated trading systems that have been implemented in the last twenty years. These systems utilize an electronic “order book”, where not executed or partially executed orders are stored and displayed while awaiting execution.

3.4 Experiments with human traders: the work of the Nobel Prize Vernon Smith as a pioneer

Smith (1962) demonstrated how prices and allocations in a CDA simulation converged quickly to the competitive equilibrium. He was awarded with the 2002 Nobel Memorial Prize in Economic Sciences “*for having established laboratory experiments as a tool in empirical economic analysis, especially in the study of alternative market mechanisms*”.⁷ In order to build a simplified version of the real financial markets Smith opted for a minimalistic approach, scaling the problem down. The subjects of his experiments were human agents (typically undergraduate and postgraduate student) who were randomly assigned to trade either as buyers or sellers.

Every seller owned one item of “stock” to sell in the simulated market with no real monetary value behind it. Instead every buyer had an amount of money that was valueless play-money from a board-game like Monopoly; such amount was concealed to other players and known only by the researchers and that buyer himself. Of course buyers could not spend more than what they received as endowment in this way it was possible to control the limit price of each buyer, i.e. the maximum price level above which they could not purchase any stock.

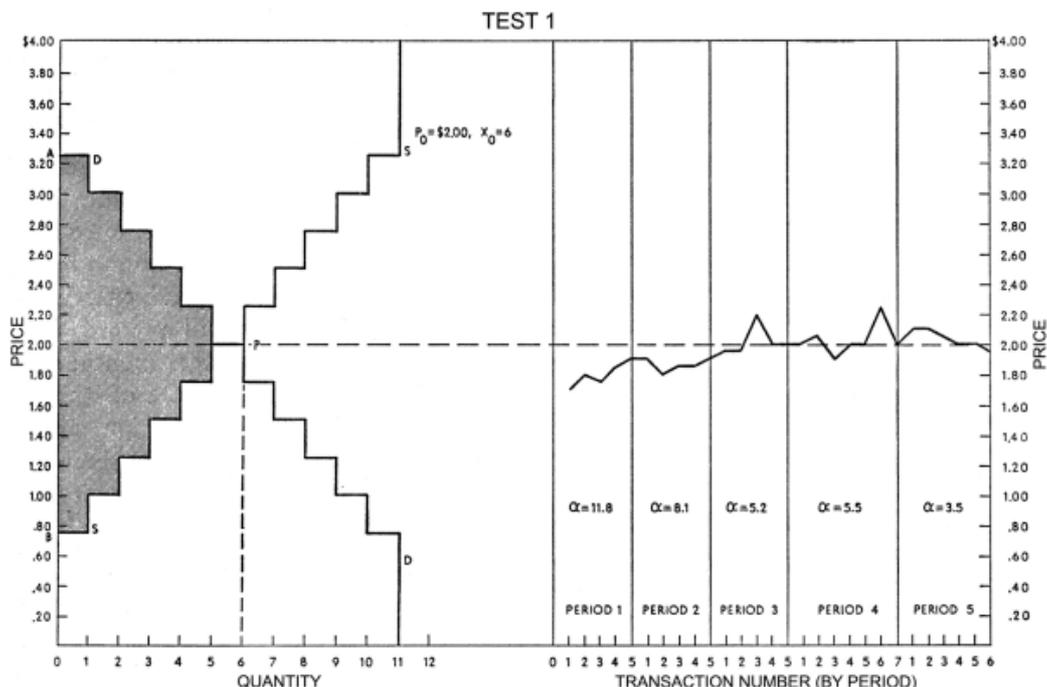
⁷ http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/

Conversely, any seller was privately told a limit price below which they should not sell their stocks. All the traders, regardless of their buyer or seller role, were told that they had to make a deal with their counterparts but under a time constraint. For this purpose a clock was used to replicate an experimental “trading day” lasting for five to ten minutes or less whether no-one wanted to trade anymore. The subjects even if dealing with valueless money and assets were motivated to make a deal in the market since no reward would have been granted to them if they had not entered into a transaction.

Moreover the real reward they would have received depended on the difference between the transaction and their own limit prices – their utility gained, in terms of profit for the sellers and of saving for the buyers. For instance in a transaction between a buyer with limit price \$1.50 and a seller with limit price \$0.20 that agreed to trade at a price of \$1.00, the buyer’s reward would have been proportional to her \$0.50 saving ($\$1.00 - \$1.50 = -\0.50) while the seller’s reward would have been proportional to his \$0.80 profit ($\$1.00 - \$0.20 = -\0.80). Therefore the seller would enjoy a greater utility since she did better out of the deal than her counterpart.

With these rules and instructions, the clock and the trading day was started and the subjects traded under the CDA mechanism in which traders were free to announce a quote at any time according to their goal. Smith noted the time of each quote, who made it, its value and if anyone accepted it or not. Since the amount of stock and money provided allowed the players to enter only into one transaction each trading day did not last more than few minutes at the end of the initial conditions were rebuilt and a new trading day started. This process was repeated for several consecutive days, typically ten or less.

The result from one of Smith experiments is summarized in the picture below.



(Smith, 1962, Chart 1, p. 113)

In the left-hand panel are drawn the demand and supply schedules resulting by sellers and buyers interactions. The demand and supply schedules intersect at $p = 2.00$, which is then the competitive equilibrium price of the experiment. In the right-hand are reported the prices of the five subsequent trading periods. There are also represented the prices' distribution of standard deviation for each period as a percentage of the theoretical equilibrium price (the number α in the diagram).

It can be easily seen that most trading prices were close to the theoretical prediction and therefore the standard deviation fell over time. Smith concluded that:

“...there are strong tendencies for a ... competitive equilibrium to be attained as long as one is able to prohibit collusion and to maintain absolute publicity of all bids, offers, and transactions. ... Changes in the conditions of supply and demand cause changes in the volume of transaction per period and the general level of contract prices. These latter correspond reasonably well with the predictions of competitive price theory.” (Smith, 1962, p. 134).

Further similar studies have been carried out by Smith and other researchers in order to check the consistency of this result under different research approaches. However those experiments agreed with Smith's original results. As Plott and Smith (1978) work in which the same general result was confirmed by also pointing out the role of market institutions. In their experiment they introduced price rigidity. In fact traders were not allowed to change prices continuously during the day as it was for Smith's original design. Instead they had to announce a unique price for the whole trading day. Such rigidity slowed down the convergence of prices toward the theoretical equilibrium level. In these experimental designs one main issue, originally addressed by Chamberlin (1948), is the fact that when agents are trading the subjects' evaluations of gains and losses are not directly observable by the researcher. Overcome the problem is possible as suggested by Chamberlin himself by providing the right monetary incentives. Such method also known as *induced-value* has been explored and developed successfully by Smith (1973, 1976) and it is nowadays a standard tool for any experimental economics. The goal of this method is to induce experimental subjects to express a certain demand function D . For instance consider the role of a buyer in an experimental market context with a unique homogenous good. What is unknown to the researcher is the subject's utility of wealth, $u(w)$. The *induced-value* method proposed by Smith induces the desired demand function by rewarding the subject with $R(q) - pq$ in terms of real money, for any quantity q bought at price p ($q = D(p)$), where R is a chosen reward function. It follows that, according to theory, the subject aim at choosing q in order to maximize $R(q) - pq$. This will lead him to choose the amount of good q such that $R'(q) = p$, where the marginal benefit of an additional good in terms of reward is equal to its marginal cost. . This is possible only considering an increasing and concave utility function u that coincides with the desired demand function if for any p the inverse derivative of the reward function R is equal to the desired demand function, $(R')^{-1}(p) = D(p)$. It is worth to summarize what said so far about CDA mechanism and notice that it is an efficient institutional design, able to lead almost in any situation to the market equilibrium level.

Smith's results are robust and hold also under the following assumptions (Novarese and Rizzello, 2004):

1. When agents do not have available information on the general price and cost structure and they only know the price at which they would buy or sell. This is an important point since according to the neoclassical theory instead perfect information is required to achieve efficiency. According to CDA model based findings, Hayek (1937) suggested that market efficiency is achieved especially when agents are characterized by idiosyncratic knowledge. In particular Hayek underlines how individuals and firms have limits on their own knowledge and that they develop skills, knowledge and experiences which are specific and unique to each singular agent. In this framework the market becomes a neutral mechanism that provides signals through prices highlighting the existence of diversity in the way of producing the goods. In this way the actors have indications about which is the relevant knowledge.
2. When in the markets there are only few buyers and sellers. Again this is opposite to the neoclassical approach requiring a large number of agents in order to achieve an efficient equilibrium.
3. When agents involved are inexperienced then after few rounds they are able to operate in the market without any need of a formal Economical background.
4. When real human agents are substituted by artificial agents designed to behave randomly and to perform with a Zero Intelligence approach. This branch of research has been explored in the last twenty years using autonomous software trader agents as proxies or replacements of human agents. Nowadays the experimental artificial study of markets is a mature and well-developed field populated by such "robot" traders and to which also this work belongs to.

3.5 Replacing human traders with Zero Intelligent ones

With modern technology arise of and new software possibilities experimental studies taken the chance to substitute their human actor with virtual agents or “robot traders”. One of the most controversial studies in this direction is the Gode and Sunder (1993) in which they:

... report market experiments in which human traders are replaced by "zero-intelligence" programs that submit random bids and offers. Imposing a budget constraint (i.e., not permitting traders to sell below their costs or buy above their values) is sufficient to raise the allocative efficiency of these auctions close to 100 percent. Allocative efficiency of a double auction derives largely from its structure, independent of traders' motivation, intelligence, or learning. Adam Smith's invisible hand may be more powerful than some may have thought; it can generate aggregate rationality not only from individual rationality but also from individual irrationality.(Gode and Sunder, 1993, p.1)

The allocative efficiency achieved in their software version of CDA mechanism is achieved by the researches without imposing any learning, intelligence or profit motivation on the traders. In fact their virtual agents are known as Zero Intelligent (ZI) traders. This result is in contrast with evolutionary theory of Alchian (1950) since even natural selection does not play a role here. Moreover

The absence of rationality and motivation in the traders seems to be offset by the structure of the auction market through which they trade. The rules of this auction exert a powerful constraining force on individual behaviour. Becker (1962) showed that price changes alter the opportunity set of consumers in such a way that even if they choose randomly from this set, the expected demand function is downward sloping; the assumption of utility maximization is not necessary to generate a downward slope. Our results are analogous to Becker's in the sense that the convergence of price to equilibrium and the extraction of almost all the total surplus seem to be consequences of the double-auction rules. (Gode and Sunder, 1993, p.135)

From a pure theoretical standpoint it can be noticed how Gode and Sunder findings:

...may help reconcile the predictions of neoclassical economic theory with its behavioral critique. Economic models assume utility-maximizing agents to derive market equilibria and their welfare implications. Since such maximization is not always consistent with direct observations of individual behavior, some social scientists doubt the validity of the market-level implications of models based on the maximization assumption. Our results suggest that such maximization at the individual level is unnecessary for the extraction of surplus in aggregate. Adam Smith's invisible hand may be more powerful than some may have thought: when embodied in market mechanisms such as a double auction, it may generate aggregate rationality not only from individual rationality but also from individual irrationality. (Gode and Sunder, 1993, p.135 - 136)

their decision on resources or assets is based on how much they think others will value them in the future.

The original selfish herd explanation plays a role too in financial markets. In fact instead of protect one's integrity from predators traders try to protect their wealth from uncertainty. Therefore herding in financial markets is a way to obtain resources and minimize risks. In fact it is often perceived safer to herd than to individually act. However things can turn to the worse when expectations are too sanguine and at a systemic level the market is a dangerous cocktail of too much optimism and uninformed imitation among traders. In these cases the crashes are just behind the door.

Ironically the bubbles that are so risky for market systems are mainly caused not by reckless and risk-taking individuals but by those who adopted herding as the least risky strategy. Luckily not all depend on human irrationality as summarized by Redhead (2008)

Kindleberger and Aliber (2005) argued that bubbles typically begin with a justifiable rise in stock prices. The justification may be a technological advance or a general rise in prosperity. Examples of technological advance stimulating stock price rises might include the development of the automobile and radio in the 1920s, and the emergence of the internet in the late 1990s. Cassidy (2002) suggested that this initial stage is characterized by a new idea or product, which causes changes in expectations about the future. Early investors in companies involved with the innovation make very high returns, which attract the attention of others. The early investors can be seen as making information trades, which are share trades arising from new information. (Redhead, 2008, p 3).

More and more empirical evidences have been collected on such phenomena as stated by Count and Bouchard (2000):

Empirical studies of the fluctuations in the price of various financial assets have shown that distributions of stock returns and stock price changes have fat tails that deviate from the Gaussian distribution (...) The heavy tails observed in these distributions correspond to large fluctuations in prices, "bursts" of volatility that are difficult to explain only in terms of variations in fundamental economic variables. The fact that significant fluctuations in prices are not necessarily related to the arrival of information (Cutler, 1989) or to variations in fundamental economic variables (Shiller, 1989) leads us to think that the high variability present in stock market returns may correspond to collective phenomena such as crowd effects or "herd" behavior. Although herding in financial markets is by now relatively well documented empirically, there have been few theoretical studies on the implications of herding and imitation for the statistical properties of market demand and price fluctuations. (Count and Bouchard, 2000, p 170 - 171)

Herding behavior then appears with its irrational emotive feature such that bubbles are lead by greed and crashes by fear. In the latter case traders join the crowd in a rush to get in or out of the market (Brunnermeier, 2001). However herding as pointed out by Hamilton has a rational background. Various rational explanations have been in fact considered in economics and finance. According to this view as the selfish guppies' traders decide to react to information about the behavior of other's market agents or traders regardless any fundamental analysis.

As Devenow, Welch (1996) well summarized:

efficient markets hypothesis (EMH) was so successful because it seemed to dispel the previously dominant notion of an irrational market driven by herds. Keynes (1936) famous adage was that the stock market was mostly a beauty contest in which judges picked who they thought other judges would pick, rather than who they considered to be most beautiful. The perceptions of MacKay (1841), Kindleberger (1989) and Galbraith (1993) were that there was convincing evidence of 'bubbles,' of mass errors caused by the fickle nature of herds. (...). With our better understanding of the importance of efficient markets, academic research has turned back towards re-examining the remaining empirical puzzles not easily addressed in parsimonious strong-form EM models: First, many financial markets phenomena display either waves and/or a certain fragility. For example, mergers and IPOs come in waves that are seemingly more amplified than possible waves in underlying fundamentals. (From our perspective, such pricing patterns are an indication but not clear evidence that investors herd.) Second, consensus among market participants seems to be low, not based on private information and still localized (e.g., Shiller et al., 1995), indicating that independent decision-making across all market participants is a fiction. Third, in conversations, many influential market participants continuously emphasize that their decisions are highly influenced by other market participants. To explain these phenomena, a 'rational herding' literature has recently been emerging in financial economics. These models are typically built on one or more of three effects: Payoff externalities models show that the payoffs to an agent adopting an action increases in the number of other agents adopting the same action (example: the convention of driving on the right side of the road). Principal-agent models show that managers, in order to preserve or gain reputation when markets are imperfectly informed, may prefer either to 'hide in the herd' not to be evaluable, or to 'ride the herd' in order to prove quality (Devenow, Welch, 1996, p 605).

3.7 When mistakes take place: Gambler's fallacy

On August 18, 1913, at the casino in Monte Carlo, black came up a record twenty-six times in succession [in roulette]. ... [There] was a near-panicky rush to bet on red, beginning about the time black had come up a phenomenal fifteen times. In application of the maturity [of chances] doctrine [the gambler's fallacy], players doubled and tripled their stakes, this doctrine leading them to believe after black came up the twentieth time that there was not a chance in a million of another repeat. In the end the unusual run enriched the Casino by some millions of francs. (Huff, 1959, pp. 28-29)

The Montecarlo or Gambler's Fallacy is based on a failure to understand statistical independence between two events, according to which the occurrence of one does not affect the probability of the other. Similarly, two random variables are independent if the realization of one does not affect the probability distribution of the other.

Just to clarify things consider a coin tossing example. We know that the probability of 21 heads in a row is 2^{-21} and it is calculated from $1/2^n$ with n equals to 21. However, by applying Bayes' theorem the probability of flipping a head after having already flipped 20 heads in a row is simply $1/2$. Gambler's fallacy has already been noticed by Pierre-Simon Laplace at the beginning of 19th century in his A Philosophical Essay on Probabilities. Laplace reported the ways in which many men calculate their probability of having sons

I have seen men, ardently desirous of having a son, who could learn only with anxiety of the births of boys in the month when they expected to become fathers. Imagining that the ratio of these births to those of girls ought to be the same at the end of each month, they judged that the boys already born would render more probable the births next of girls (Laplace 1951, p.162)

It means that the expectant fathers were afraid that if more sons were born in their nearby community, then they would be less likely to have a son.

Thus the extraction of a white ball from an urn which contains a limited number of white balls and of black balls increases the probability of extracting a black ball at the following drawing. But this ceases to take place when the number of balls in the urn is unlimited, as one must suppose in order to compare this case with that of births (Laplace 1951, p.162-163).

Even if some researches predicted that the birth sex depends on living conditions such that "good" living conditions are correlated with more male children and poorer living conditions with female children (Trivers and Willard, 1973), the chance of having a child of either sex is still generally regarded as equal.

These mistakes arise out of a wrong belief in a "law of small numbers" based on the erroneous idea that small samples are good representative of the larger population Even if it may seem far away from financial markets these mistakes are widespread and it has been observed in many several Stock markets as the Lahore Stock Exchange (Amin, Shoukat, Khan, 2009) and the Mumbai Stock Exchange (Rakesh, 2013)

3.7.3 Heuristics' role in the Gambler's fallacy

Kahneman and Tversky (1974) were the first to interpret gambler's fallacy as a cognitive bias from a psychological heuristic based on representativeness. Their research program studied how people make real-world judgments and under which conditions such judgments are wrong. In their work they presented different experimental situation to their subjects in order to underline what lead the subjects to their mistakes:

In considering tosses of a coin for heads or tails, for example, people regard the sequence H-T-H-T-T-H to be more likely than the sequence H-H-H-T-T-T, which does not appear random, and also more likely than the sequence H-H-H-H-T-H, which does not represent the fairness of the coin. Thus, people expect that the essential characteristics of the process will be represented, not only globally in the entire sequence, but also locally in each of its parts. A locally representative sequence, however, deviates systematically from chance expectation: it contains too many alternations and too few runs. Another consequence of the belief in local representativeness is the well-known gambler's fallacy. After observing a long run of red on the roulette wheel. For example, most people erroneously believe that black is now due, presumably because the occurrence of black will result in a more representative sequence than the occurrence of an additional red. Chance is commonly viewed as a self-correcting process in which a deviation in one direction induces a deviation in the opposite direction to restore the equilibrium. In fact, deviations are not "corrected" as a chance process unfolds, they are merely diluted. Misconceptions of chance are not limited to naive subjects. A study of the statistical intuitions of experienced research psychologists revealed a lingering belief in what may be called the "law of small numbers," according to which even small samples are highly representative of the populations from which they are drawn. (Kahneman and Tversky, 1974, p 1125-1126)

At this point someone could argue that psychological heuristics mainly jeopardize rational human decisions. However this is not the case. It is worth to say that the word "Heuristic" derives from the Greek εὐρίσκειν which means "serving to discover" (Baron, 2000, p. 50). They were originally introduced by the Nobel laureate of 1978, Herbert A. Simon whose primary object of research was problem-solving. He showed that human beings operate within bounded rationality and coined the term satisficing (Simon 1947, 1956), to denote all those situations where people look for solutions or accept choices and judgments which are good enough for their purposes, but could be optimized. In his view:

Decision making processes are aimed at finding courses of action that are feasible or satisfactory in the light of multiple goals and constraints (Simon, 1947, p. 274.)

Heuristics are mental short-cut that allow people to quickly decide and make real-world judgments by usually focusing on only one aspect of the complex situation they have to deal with while ignoring others They can be thought as thumb-rules which have the great advantage to be quickly processed.

Under most circumstances they work well but they often lead to systematic deviations from rational choice theory. When mistakes occur they are called cognitive biases as for the Gambler's fallacy. Tversky and Kahneman work (1974) challenges the rational theory on Human decisions and it provides a new complete framework able to explain what is behind the cognitive biases observed. In their famous paper they presented three different main heuristics:

1. Representativeness, to which it belongs the gambler's fallacy.

Many of the probabilistic questions with which people are concerned belong to one of the following types: What is the probability that object A belongs to class B? What is the probability that event A originates from process B? What is the probability that process B will generate event A? In answering such questions, people typically rely on the representativeness heuristic, in which probabilities are evaluated by the degree to which A is representative of B, that is, by the degree to which A resembles B. For example, when A is highly representative of B, the probability that A originates from B is judged to be high. On the other hand, if A is not similar to B, the probability that A originates from B is judged to be low. For an illustration of judgment by representativeness, consider an individual who has been described by a former neighbor as follows: "Steve is very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail." How do people assess the probability that Steve is engaged in a particular occupation from a list of possibilities (for example, farmer, salesman, airline pilot, librarian, or physician)? How do people order these occupation from most to least likely? In the representativeness heuristic, the probability that Steve is a librarian, for example, is assessed by the degree to which he is representative of, or similar to, the stereotype of a librarian. (Kahneman and Tversky, 1974, p-1124)

2. Availability occurs when

People assess the frequency of a class or the probability of an event by the ease with which instances or occurrences can be brought to mind. For example, one may assess the risk of heart attack among middle-aged people by recalling such occurrences among one's acquaintances (Kahneman and Tversky, 1974, p-1127)

Availability heuristics has an impact on real financial market as demonstrated by some recent research on Behavioral Finance. In particular it affects stock market and how investors react to company-specific events. (Kliger and Kudryavtseva, 2010). Availability also affects growth forecasts' analysis and then investments' decision since when assessing probabilities most analysts give more weight to current or more easily to recall information rather than processing all relevant information (Byunghwan; O'Brien; Sivaramakrishnan, 2008).

3. Adjustment and anchoring occurs whenever

People make estimates by starting from an initial value that is adjusted to yield the final answer. The initial value, or starting point, may be suggested by the formulation of the problem, or it may be the result of a partial computation. In either case, adjustments are typically insufficient. That is, different starting points yield different estimates, which are biased toward the initial values. We call this phenomenon anchoring (...). A study of intuitive numerical estimation illustrates this effect. Two groups of high school students estimated, within 5 seconds, a numerical expression that was written on the blackboard. One group estimated the product

$$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

while another group estimated the product

$$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8$$

To rapidly answer such questions, people may perform a few steps of computation and estimate the product by extrapolation or adjustment. Because adjustments are typically insufficient, this procedure should lead to underestimation. Furthermore, because the result of the first few steps of multiplication (performed from left to right) is higher in the descending sequence than in the ascending sequence, the former expression should be judged larger than the latter. Both predictions were confirmed. The median estimate for the ascending sequence was 512, while the median estimate for the descending sequence was 2,250. (Kahneman and Tversky, 1974, p-1128)

Anchoring heuristic has been studied in many different contexts outside of laboratory-experimental researches. Among the different markets in which these researches focus on, we can find auditing processes (Joyce and Biddle, 1981), real estate evaluations (Northcraft and Neale, 1987) and stock markets (Mussweiler and Schneller, 2003).

It also affects people estimates of risk and uncertainty (Wright and Anderson, 1989), estimation of future performance (Switzer and Snizek, 1991) consumer's decisions on quantity to purchase (Wansink et al., 1998) and legal decision making (Englich and Mussweiler, 2001; Englich et al., 2006). One may argue that anchoring would not affect experts since they should rely on more complex and deep analytical decision processes.

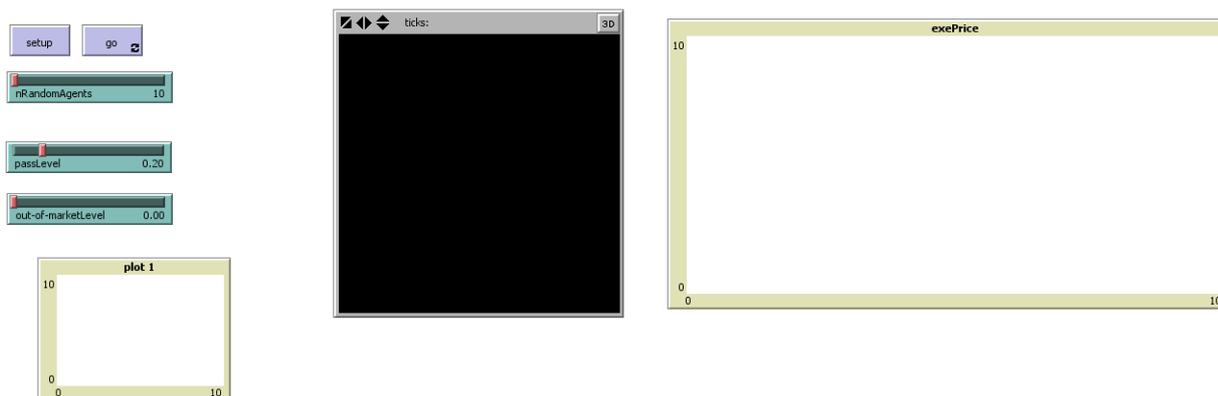
However this does not seem the case. In fact experts of different fields have been studied to show whether or not they were affected by anchoring bias. It has been demonstrated that auditors (Joyce and Biddle, 1981), realtors (Northcraft and Neale, 1987), school teachers (Caverni and Pris, 1990) automobile mechanics (Mussweiler et al., 2000), investors (Mussweiler and Schneller, 2003), and judges (Englich and Mussweiler, 2001; Englich et al., 2006) are still biased by anchoring. Moreover some studies demonstrated that there is not a huge difference between experts' and no-experts' evaluation but experts seem much more confident (Englich et al., 2006).

4. MODEL MAKING

4.1 Model 1.0: basic CDA with ZI traders

4.1.4 Interface

The “virtual” world in which I started to move the first steps relies on CDA mechanisms where ZI agents randomly decide whether to buy or to sell.



From the interface we can see that the only available commands for the observer are: a setup procedure to initialize the model and a go procedure to run all transactions in a trading day.

The model is based on the following variables:

- nRandomAgents: it is an integer number set by the observer through the first slider. It represents the number of traders in the market in a range from 1 to 1000
- logB: is a list including all submitted bid prices and who announced them. They are sorted in decreasing order since according to the model if you are selling the stock you receive the bid price (the lower price). It follows then the first price is the bid at that tick.
- logS: is a list including all submitted ask prices and who announced them. They are sorted in increasing order since according to the model if you are buying the stock you pay the ask price (the higher price). It follows then the first price is the bid at that tick.
- exePrice: is list that stores all executed tick prices for the whole go procedure.
- dailyPrice: a list that stores all daily prices, the last exePrice of each day, of the history of the model
- passLevel: it is floating number between 0 and 1 , set by the observer through the second slider. It corresponds to the probability for traders of opting out of transactions for the day.

The traders' variables are:

- pass: a dummy variable reporting whether or not the trader has opted out of trading for the day
- buy: a dummy variable reporting whether or not the trader is a buyer.
- sell: a dummy variable reporting whether or not the trader is a seller.
- price: a floating number reporting the trader's bid or ask price for every tick.
- cash: a floating number set to 1000 every setup which represents the individual's monetary endowment to trade. It is allowed to have negative cash for the individual.
- stocks: an integer number set to 0 every setup and which represents the amount of stocks owned by each individual. A positive amount of stock reflects a long position, a negative one a short one.

4.1.5 Setup

In the setup are defined the variables of the model and their initial conditions while specifying Netlogo world.

```
breed [randomAgents randomAgent]
```

```
randomAgents-own[out-of-market buy sell pass price cash stocks n_stock]
```

```
globals [logB logS exePrice dailyPrice]
```

```
to setup
```

```
clear-all
```

```
set exePrice [100]
```

```
set dailyPrice [100]
```

```
set logB []
```

```
set logS []
```

```
reset-ticks
```

```
create-randomAgents nRandomAgents
```

```
let side sqrt nRandomAgents
```

```
let step max-pxcor / side
```

```
ask randomAgents
```

```
    [set shape "person"
```

```
    set out-of-market False
```

```
    set size 2
```

```
    set stocks 0
```

```
    set cash 0
```

```
    set n_stock (random 10 + 1) ]
```

```
let an 0 let x 0 let y 0
```

```
while [an < nRandomAgents]
```

```
    [if x > (side - 1) * step
```

```
        [set y y + step          set x 0      ]
```

```
    ask randomAgent an
```

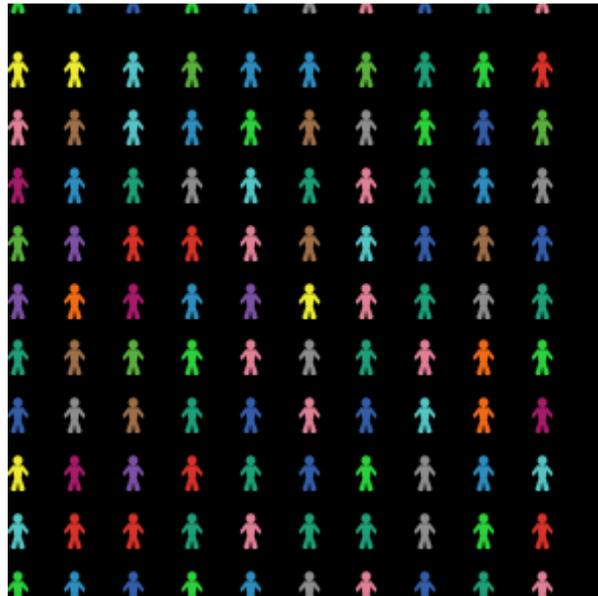
```
        [setxy x y]
```

```
    set x x + step
```

```
    set an an + 1 ]
```

```
end
```

The model setup initializes the global and trader's variables. Moreover it forces each trader to be at the same distance *step* each other and it constructs a virtual square of *side* x *side*. Since both *side* and *step* formula are based upon the number of randomAgents it then follows that the Netlogo world will appear more nicely ordered when randomAgents' root is an integer number. Here it follows the initial world at tick 0 just after the Setup procedure occurs in a world with one hundred agents:



4.1.6 The Go procedure and CDA implementation

The go procedure has two complementary parts. The first one can be interpreted as the decision process in which the ZI trader randomly decides whether to buy or sell and at which price. The second one represents the matching process in which sellers and buyers agree on trading with their counterparts.

to go

ask randomAgents

```
[
  ifelse out-of-market [set color white]
    [ifelse random-float 1 < passLevel      [set pass True]
      [set pass False]
    ifelse not pass
      [ifelse random-float 1 < (.6 - (.2 / 200.) * last exePrice) [set buy True set sell False]
        [set sell True set buy False] ]
      [set buy False set sell False]

  if pass      [set color gray]
  if buy       [set color red]
  if sell      [set color green]

  let my-price ((last exePrice) + (random-normal 0 10))
  set price my-price
]
```

The traders which are *not out-of-market* may decide not to trade according to a random choice depending on the *pass* probability. If instead, the agent participates at the negotiations he has the same probability to be either a seller or a buyer. Each trader then sets his own bid or ask price according to the last execution price summed to a random value extracted from a Normal distribution with zero mean and a deviation standard of one hundred. Therefore the trader's price can either be higher or lower than the last price. Traders now display different colors based on their role in the market:

- White whether the trader is out of the market
- Grey whether the trader passes
- Red whether the trader is a buyer
- Green whether the trader is a seller

The matching process is based on the logB and logS globals that allow the traders to make a deal with their potential counterpart without the need of introducing an auctioneer consistently with the pioneer work of Smith (1962).

```

set logB []
set logS []
tick

ask randomAgents
  [if not pass and not out-of-market
    [let tmp[]
      set tmp lput price tmp
      set tmp lput who tmp
      if buy [set logB lput tmp logB]
      set logB reverse sort-by [item 0 ?1 < item 0 ?2] logB
      if (not empty? logB and not empty? logS) and
      item 0 (item 0 logB) >= item 0 (item 0 logS)
        [
          set exePrice lput (item 0 (item 0 logS)) exePrice
          let agB item 1 (item 0 logB)
          let agS item 1 (item 0 logS)
          ask randomAgent agB [set stocks stocks + 1    set cash cash - last exePrice]
          ask randomAgent agS [set stocks stocks - 1    set cash cash + last exePrice]
          set logB but-first logB
          set logS but-first logS
        ]
      if sell [ set logS lput tmp logS]
      set logS sort-by [item 0 ?1 < item 0 ?2] logS
      if (not empty? logB and not empty? logS) and
      item 0 (item 0 logB) >= item 0 (item 0 logS)
        [
          set exePrice lput (item 0 (item 0 logB)) exePrice
          let agB item 1 (item 0 logB)
          let agS item 1 (item 0 logS)
          ask randomAgent agB [set stocks stocks + 1    set cash cash - last exePrice]
          ask randomAgent agS [set stocks stocks - 1    set cash cash + last exePrice]
          set logB but-first logB
          set logS but-first logS
        ]
      ]
    ]
  if random-float 1 < out-of-marketLevel
    [if last exePrice > 150    [set out-of-market True]
      if last exePrice < 50    [set out-of-market False]]
  graph ]
  set dailyPrice lput last exePrice dailyPrice
end

```

The bargain procedure even if it may look complex relies on a simple confrontation between the logS list that includes all submitted ask prices by buyers, and logB, that includes all submitted bid prices by sellers for every tick.

When a buyer announces his intention to purchase a stock at his bid price and in the market there are sellers with lower ask prices he will make a deal at the lowest ask price available which is considered as the *exePrice* for this transaction. Vice versa when a seller announces his intention to sell a stock at his ask price and in the market there are buyers with higher bid prices he will make a deal at the highest bid price available which is considered as the *exePrice* for this transaction. Then of course the buyer will get one unit of *stock* purchased at the corresponding *exePrice*, reducing then his *cash* endowment. Meanwhile the seller, by giving away one unit of *stock*, benefits an increase in his *cash* endowment of the same amount. The final part recalls *out-of-marketLevel* and it forces traders to opt out the market whether the asset's price is too low or it forces them to re-enter again (in case they were out) into the market whether asset's price is too high.

Simple graph procedures allow the observer to supervise the *exePrice* on each tick thanks to the following code:

to graph

```
set-current-plot "exePrice"  
plot last exePrice
```

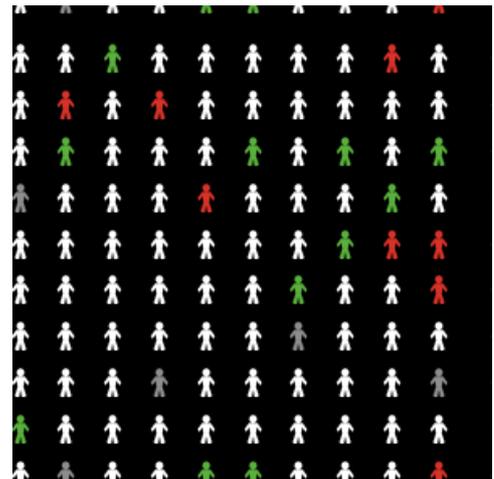
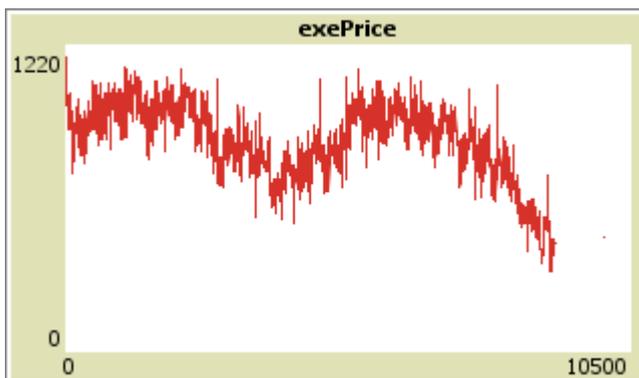
end

The output then will appear on the appropriate interface's plot, where on x-axis there are the prices and on the y-axis the ticks' progression. One single tick simulation with one hundred traders displays a numerous number of *exePrice* as the result of the same number of underlying transactions between sellers and buyers.

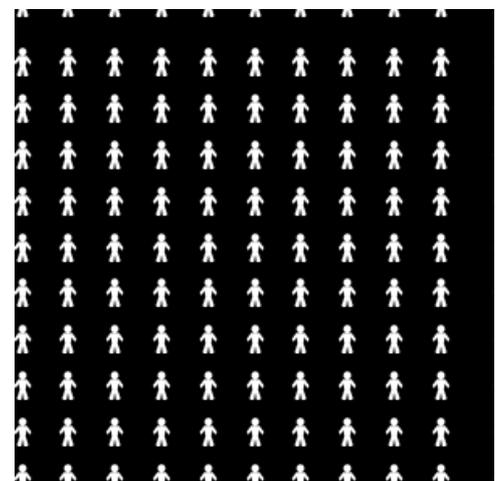
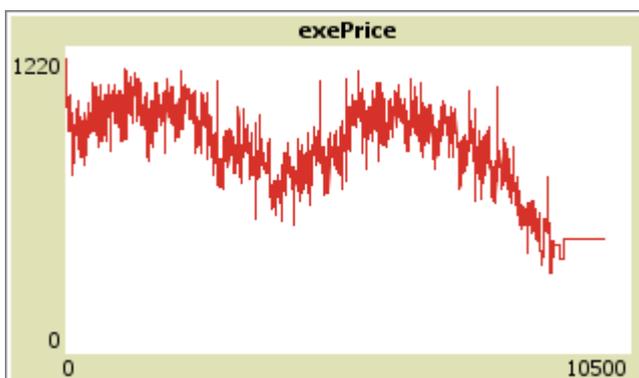


4.1.7 Conclusions and remarks on model 1.0

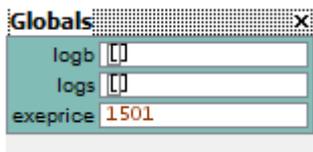
Although simple and reductive the model so far does what we wanted. It replicates coherently the CDA mechanism seen in Smith (1962) and in Gode and Sunder (1993). We are able here to simulate a financial market where agents display different impatience rates, which is set by the observer through the *passLevel* slider, or with different preferences regarding the *out-of-marketLevel*. Consider now a market with one hundred traders and let's simulate how those two variables can separately affect the output result. In the first case for instance traders have the maximum amount of *out-of-marketLevel* (0.50) and a low value for *passLevel* (0.15). After one hundred ticks we may observe the following plot and world situations:



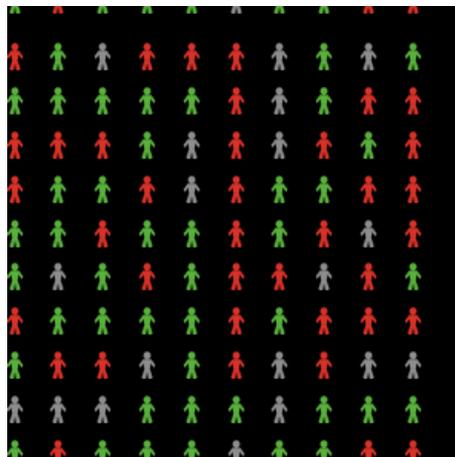
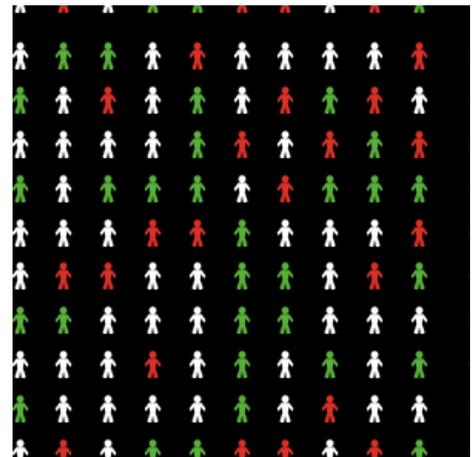
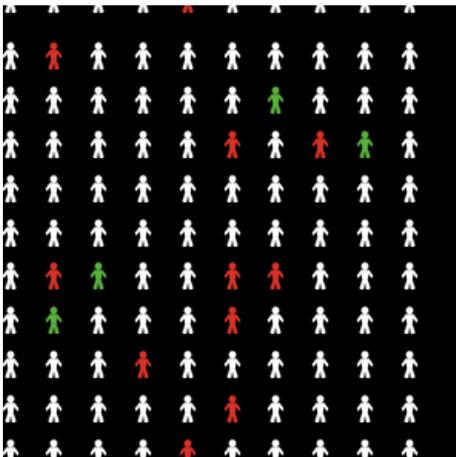
Since the price started to drop under 500 more and more traders in the market prefer to opt out waiting the price to increase over 1,500 before starting participate again in the transactions. Even if the simulation run for one hundred ticks the number of total underlying transactions is definitely higher, around 10,000. Similarly, the Netlogo world replicates these behaviors. In fact white traders are the most numerous representing the agents out of the market while there is a less numerous number of sellers (greens), buyers (reds) and of traders not involved in any transaction (grey). If we let the simulation running we would be able to see the following extreme situation where all traders are out of the market:



In order to check whether the model behaves as described let's impose on the next tick an *exePrice* greater than 1,500. We can do that by right-clicking on Netlogo world and directly modifying the *Globals*, as it follows:



We may justify this abrupt change of prices due to some shock that affected the financial market but our speculations are here unnecessary since the purpose it is only to check for model consistency.



As the price keep floating around 1,500 the traders who were out of market start to re-join the transactions almost instantly but since the *out-of-market* procedure is based on a probability then a little bit of time is needed in order to have all the traders again in the market. In the other extreme case where *passLevel* is set at its maximum (1.0), regardless the other variables values, nothing interesting happens since the *exePrice* stays put at its initial level of 1,100 and all traders are neither selling not buying

4.2 Model 2.0: ZI traders *sapiens*:

Although ZI traders in a CDA market are able to display aggregate rationality starting from totally irrational behaviors (Gode and Sunder, 1993), the purpose to this work is to simulate a financial market where human bias and rational mistakes occur according to a consistent framework.

This is why the model has to be improved in order to include more interesting features. The main ones here are a

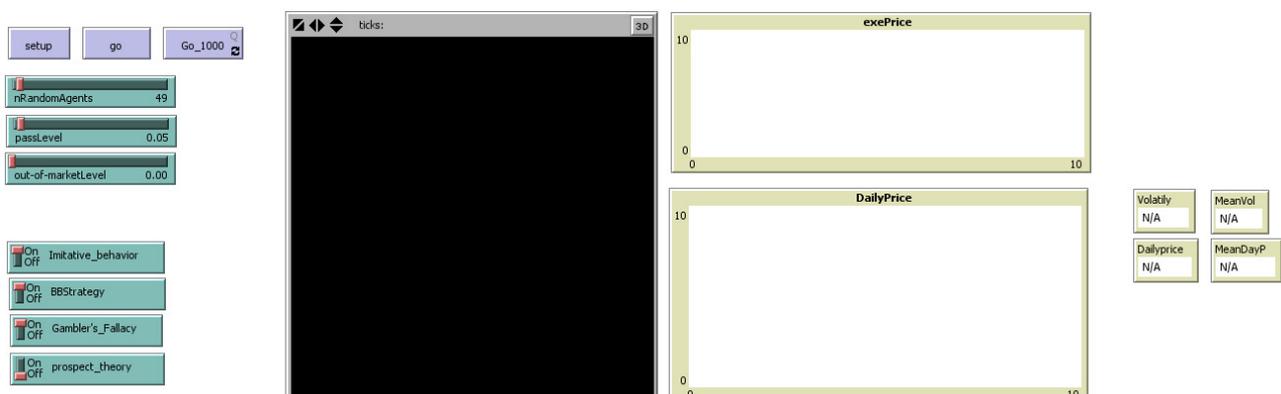
- trading technique based on Bollinger bands for those proficient investors who perform relying on a rigorous technical strategy;
- an imitative strategy more likely to occur for less skilled investors in order to account for herd behavior in financial markets
- a rational mistake identified as the Gambler's fallacy that introduces the concepts of heuristics in the decision process as theoretically emphasized in section 3.7 .

As we can notice traders are now designed as intelligent (no longer ZI) thank to their *skill* variable. This variable is used as a proxy for agent's intelligence as well as other relevant cognitive abilities such as mathematical and statistical knowledge, financial markets understanding, economical insights et cetera... . All agents are created with a specific level of skill randomly assigned when created and hold constant for the whole simulation.

What we are interested to see here it is if with these three assumptions the market remains as randomly produced while the smartest investors make profits to the detriment of less-skilled ones.

4.2.8 Interface and variables

The updated model has the following interface



We can notice how four new switches have been implemented on the left side. Those correspond to the new features of the model which can be easily activated or not at the observer's will, according to his research projects.

In particular each switch may activate or not a specific strategy or procedure as it follows:

1. *Imitative_behavior*: activates or not the possibility for traders to look for a leader to follow when deciding their next move.
2. *BBStrategy*: when activated it allows the smartest traders to use a technical financial indicator, in this case Bollinger's bands, to lead their decision in the market.
3. *Gambler's_Fallacy*: when activated it affects more easily the less skilled traders by misleading their judgments due to simple heuristics.

An additional plot has been introduced called *DailyPrice* and it represents the final price for every trading day (or tick) that basically corresponds to the last *exePrice*. On the right part of the interface instead some output boxes appear. They report the last daily price as well as its mean and standard deviation.

Additionally new global and local variables have been introduced in order to introduce the above procedures. The new global variables are:

- *UB* and *LB*. They are respectively the upper and lower Bollinger Bands which are explained in details further.
- *sdMemory* and *sdMemoryLong*. Both are lists containing information on the standard deviations of the *DailyPrice* but with a different length in term of memory and amount of data stored
- *p_fallacy* is a list with some of the previous price level. Such list is extremely useful for the Gambler's fallacy implementation.
- *BBprice_memory* contains the previous prices in order to implement Bollinger's Bands

While the new agents' variables are:

- *wealth* is a list memorizing all the trader's cash history
- *leader* is a dummy variable used to define whether or not the trader is a leader
- *Skill* is a random float variable that goes from zero to one. Every trader has its own skill value as a sort of unique measure that reflects each individual's ability, know-how, mathematical and analysis knowledge.

4.2.9 NLS extensions for Netlogo

With the purpose of keeping the original basic CDA mechanism the closest possible to Smith's work in order to implement new command I decided to use separated *nls* files that are recalled by the Netlogo itself thanks to the following lines:

```
__includes ["BBStrategy.nls" "imitation.nls" "colors.nls" "fallacy.nls" "richest.nls"  
"CDA_priceformation.nls" "CDA_bargaining.nls" "pththeory.nls" "data.nls"]
```

Each of these .nls file contains an important contribution to the model that will be analyzed on the next pages.

4.2.10 Herd behavior in Netlogo

The algorithm beyond the implementation of herd behavior has been saved as a distinct nls file named imitation and included in the **__includes** [] command written in the original Netlogo file. Then we can easily recall the include.nls procedure just by selecting the nls file we are interested into. By simply clicking on the Code > Includes a drop-down menu will appear with the list of all the nls files linked to the model. Of course to analyze or modify how the Herd behavior has been written in the model we'll select *imitation.nls* . The imitative behavior can be activated or not according to the observer's will through its own switch.

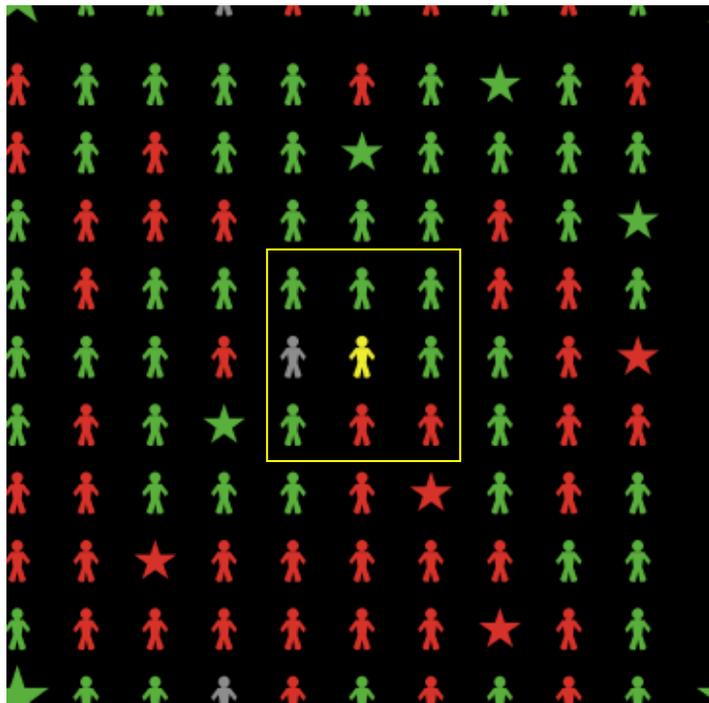
This part of the program can be broken conceptually into two different parts. The first one identifies who are and not the leaders. The second one implements the consequences of being or not a leader.

```
if Imitative_behavior [  
  let side sqrt nRandomAgents let step max-pxcor / side let n 0 let leader_price 0  
  let olead[] let center[]  
while [n < nRandomAgents]  
[ask randomAgent n  
  [set center lput cash center set center lput who center set leader_price price  
  ask randomAgents in-radius (step * 1.5)  
    [let lead[] set lead lput cash lead set lead lput who lead  
    set olead lput lead olead set olead remove center olead  
    set olead sort-by [item 0 ?1 > item 0 ?2] olead ]  
  if (item 0 center) > item 0 (item 0 olead) [set leader true set shape "star"]  
  if (item 0 center) < item 0 (item 0 olead) [set leader false set shape "person"]
```

The model exploits the grid disposition of trader in the Netlogo world. In fact every trader seeks around if someone is performing better in the market in terms of cash amount. Therefore, every trader looks his neighborhoods and seeks someone performing better. If this occurs the richer trader around would be identified as a leader by his neighborhood. Traders are not able to directly see if their neighborhoods are more skilled which would be unrealistic. In the other side, they can compare their performance with the potential leader's one in terms of cash won or lost.

This process is implemented thanks to a comparison between the centre and olead lists. The centre list contains the identification number and cash amount of the centered trader (temporarily in yellow in the picture below). While the olead list has stored the cash amount of all the traders in the square except the centered trader.

Two different scenarios might arise. In the first case the centered agent has not performed better so it won't be identified by his peers as a leader. In the second case instead he is recognized as a leader since he has been beating the market regardless if he is only lucky or really skilled. The leader trader then changes its shape into a star (to follow under the uncertainty of the market) in this way it is easily possible to recognize them in the Netlogo world.



If there is a leader which is not out of the market he will provide advises to the ones around him. These advices correspond to his own position on the market regarding if he will pass, buy or sell and at which price. The ones surrounding him decide to copy his move or not according to a probability that that depends on the traders' skill level.

In particular the ones with less ability to understand what is going on the market are more likely to follow the leader rather than decide on their own. Graphically this result is quite intuitive to be grasped. In fact, red- buyer stars (leaders) are mainly surrounded by red buying traders while green-selling stars by green sellers.

The second part of the code it is the continuation of the first one and it is written as:

```
if any? randomAgents with [leader = true]
  [ask randomAgent n
    [if leader and not out-of-market
      [let leader_advise[] set leader_advise lput price leader_advise
        set leader_advise lput who leader_advise
        ask randomAgents with [leader = true]
        [ifelse pass
          [ask randomAgents in-radius (step * 1.5)
            [if not out-of-market
              [set pass true set buy false set sell False] ] ]
          [ask randomAgents in-radius (step * 1.5)
            [set pass false] ] ]
        if buy
          [ask randomAgents in-radius (step * 1.5) with [pass = false]
            [if not out-of-market and random-float 1 > Skill
              [set price leader_price set buy true set sell false] ] ]
        if sell
          [ask randomAgents in-radius (step * 1.5) with [pass = false]
            [if not out-of-market and random-float 1 > Skill
              [set price leader_price set sell true set buy false] ] ]
      ]
    ]
  ]
set n n + 1 set olead[] set center[] ] ]
end
```

4.2.11 Bollinger Bands in Netlogo

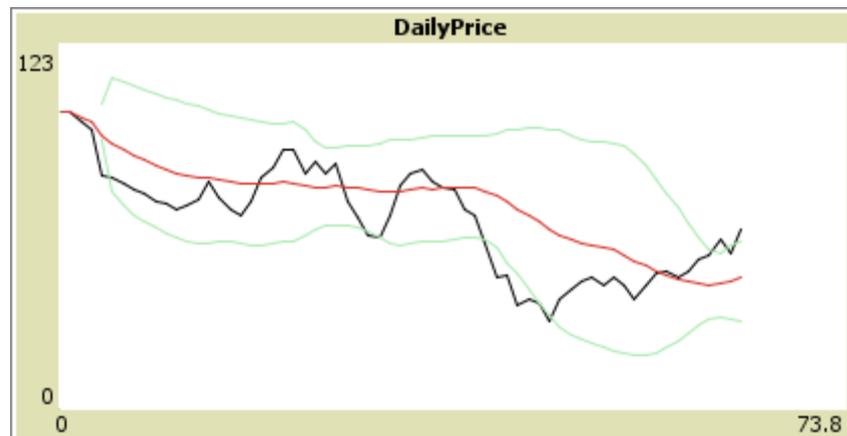
To implement successfully the Bollinger's Band a new graph command has been added. Its algorithm corresponds to:

```
to graph2
if ticks > 1
  [let ss standard-deviation dailyPrice set UB (mean dailyprice) + (2 * ss)
  set LB (mean dailyprice) - (2 * ss) set sdMemory lput ss sdMemory
  if ticks > 20
    [set sdMemory remove-item 0 sdMemory
    set dailyPrice remove-item 0 dailyPrice ]
]
end
```

This procedure adds in the DailyPrice chart the two Bollinger's bands together with a 20-day Moving average .

Both the upper band (UB) and the lower band (LB) are computed based on the standard deviation of the DailyPrice according to their original formulation described in section 3.2 and 3.2.2.1 .

The Daily prices together with Bollinger's Band and MA are graphically represented in the following chart:



It is easy to recognize the Bollinger's Bands in green while the twenty day Moving average is drawn as a red line. The DailyPrice, that corresponds to the last executed price for the trading day appears as a black line.

With a quick glance at the chart the observer can have an insight on what is happening to the market price together with some intuitive information on mean and standard deviation.

4.2.12 A simple strategy for *ZI sapiens* based on Bollinger's Bands

The strategy based on Bollinger's Bands as well as the one for imitative behavior can be implemented or not through the appropriate switch named BBStrategy in Netlogo's Interface.

This basic strategy exploits heavy selling (oversold) and heavy buying (overbought) conditions that correspond to the breaks of lower or upper bands respectively. Usually, once a lower band breaks due to oversold, the price of the stock reverts back above.

Therefore the Netlogo traders are willing to buy whenever the price is below the lower band and sell whenever the price is above the upper band.

```
to BStrategy
set BBprice_memory lput last exePrice BBprice_memory
if ticks > 19
  [set BBprice_memory but-first BBprice_memory]

if BBStrategy and ticks > 19
[ask randomAgents
  [if not pass and last dailyPrice >= (UB * ( 1 + ((random 11)) / 1000) )
    [ if 0.2 + random-float 0.8 < Skill
      [set sell True set buy False]
    ]
  ]
ask randomAgents
  [if not pass and last dailyPrice <= (LB * ( 1 - ((random 11)) / 1000) )
    [if 0.2 + random-float 0.8 < Skill
      [set sell False set buy True ]
    ]
  ]
ask randomAgents
  [let a random 6
    if not pass
      [ if last dailyPrice < ( mean BBprice_memory * ( 1 + (a / 1000) ))
        and last dailyPrice > ( mean BBprice_memory * ( 1 - (a / 1000) ))
          [if 0.2 + random-float 0.8 < Skill
            [set pass True set sell False set buy False set color grey ]
          ]
        ]
  ]
]
end
```

The traders start following this strategy on the 20th tick, this value can of course be increased but it has been chosen in order to have sufficient past data to obtain reliable twenty periods Bands and the corresponding Moving Average.

This technical strategy is based on three different market price conditions that trigger the agents' decision:

- Whenever prices are greater or equal to the Upper Band plus a random percentage value, then traders are likely to sell stocks.
- Whenever prices are lower or equal to the Lower Band minus a random percentage traders are likely to buy stocks.
- Whenever prices are instead in random percentage neighborhood of the MA, traders are likely to pass rather than buy or sell.

We can notice how traders' decisions are based on the relationship between their own Skill ability and a random float value plus a fixed value.

The basic assumption here is that not all traders know or possess the sufficient skill to perform well according to this strategy. In order to apply it profitably traders must at least have a Skill amount greater than 0.2 plus a random amount between 0 and 0.8.

4.2.13 Gambler's fallacy in Netlogo

As some researches pointed out stock markets are affected by investors biased decisions. In particular it has been shown how investors are more likely to buy rather than to sell when the reference chart displays a significant high than a low (Mussweiler and Schneller, 2003). Gambler's fallacy has been implemented in the model as an optional parameter that can be included or not in the simulation by the observer. In fact it is enough to set on or off the corresponding switch in the interface in order to implement or not this fallacy. The code behind this procedure is :

```
to Gambler_fallacy
if Gambler's_Fallacy
  [set p_fallacy lput last exePrice p_fallacy
  if ticks > 4
    [let At item 5 p_fallacy
    let At-1 item 4 p_fallacy
    let At-2 item 3 p_fallacy
    let At-3 item 2 p_fallacy
    let At-4 item 1 p_fallacy
    set p_fallacy but-first p_fallacy
    let n 0
    while [n < nRandomAgents]
      [ask randomAgent n
      [if 0.20 + random-float 0.8 > Skill
        [if At < At-1 and At-1 < At-2 and At-2 < At-3 and At-3 < At-4
          [set buy true set sell false]
        if At > At-1 and At-1 > At-2 and At-2 > At-3 and At-3 > At-4
          [ set sell true set buy false ]
        ]
      ]
    ]
    set n n + 1
  ]
end
```

The investors are tempted to follow their heuristics after at least five increasing or decreasing day prices in a row. When markets show an increasing trend (or alternatively decreasing) investors are tempted to beat against the trend and then to sell (or alternatively to buy). A new list has been introduced here named p_fallacy which contains the last five daily prices and it is of course updated each tick.

This decision process does not apply to all investors it is instead based on each own investor's abilities which are represented in the model as the Skill variable. Therefore low-skilled investors are the one more likely to display a biased decision as opposed to more skilled investors. Here we imposed that those with a Skill value lower than 0.20 fall systematically into the Gambler's fallacy.

4.2.14 Additional procedures

Two additional procedures are still to be described. The first one is a simple check-up procedure that re-edits all agents' colors according to their move: grey if the investor passes, red when buys, green when sells and white if he is out of the market.

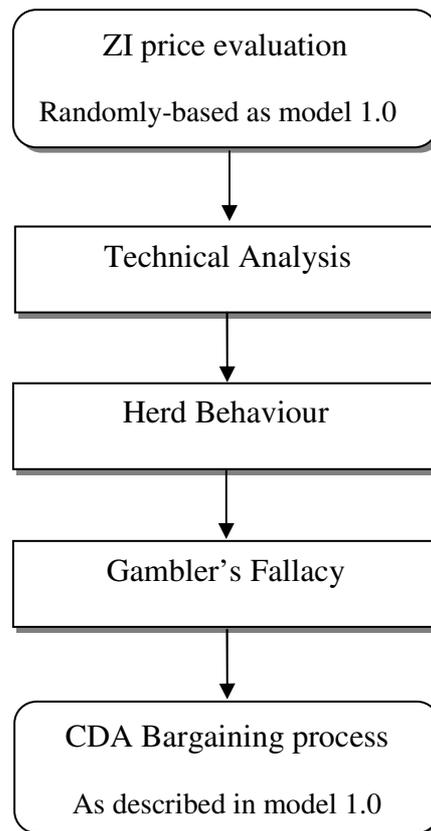
```
to Update_colors
ask randomAgents
[if pass           [set color gray]
 if buy           [set color red]
 if sell          [set color green]
if out-of-market  [set color white]
]
end
```

Finally the last procedure detects who is the richest investor at the end of an arbitrary cycle of one thousand days. Additionally it changes the shape of the richest agent by enlarging him and it reports on the Command Centre of the Interface the skill value of the richest trader.

```
to richest
let wealth_comparison[]
ask randomAgents
[if not out-of-market
  [set wealth_comparison lput cash wealth_comparison
   set wealth_comparison sort-by > wealth_comparison
  ]
]
let HC item 0 wealth_comparison
ask randomAgents
[ ifelse cash = HC
  [set size 3 ]
  [set size 2]
]
if ticks = 1000
[ ask randomAgents with [cash = HC ] [ show skill ]
  ask randomAgents with [cash = HC ] [ set richest_skill skill set richest_cash cash ]
]
end
```

4.2.15 Weaknesses of model 2.0

So far the investors' decisions are built on a sell-buy dichotomy with a small exception for passing included on the Bollinger's Band strategy. The investor's decisions follow a sequential and rigid path that still starts with a random choice as the previous CDA model 1.0. This initial decision is then modified according to the additional strategy and decision-processes included. The whole decision process can then be summarized as it follows.



As already said each step can be included or not according to the observer's will. However what is important here to stress it is that in each step the previous decision is revised and may change. It means that with all procedures included if investor A decides according to his analysis on Bollinger's Band to buy or to sell then this choice could be nullified by the further steps in which he may decide to behave conversely. The problem here is that we have a sequential tool (Netlogo software) to simulate a parallel process (human decisions). In order to develop a more realistic decision-process which can also allow adding successfully the prospect theory approach (Kahneman and Tversky, 1979) the model has to be improved. A good strategy here is to use a probability approach where instead of a binary decision between sell or buy at each stage the investors increase or decrease their probability to buy to sell or to pass on each stage. Each strategy then may contribute or not to increase or decrease those probabilities and when the investor at the end of the process will decide how to behave in the market it will be the result of the different meta-strategies.

4.3 Model 3.0

The final model comprehends and enriches all the features introduced in the previous ones. Among the already-known characteristics:

- Agents can be asked to be ZI (Gode and Sunder, 1993) or *smart* enough to decide to operate actively in the market;
- The *smart* agents' orders are based on different strategies and heuristic fallacies which apply according to their *skill* variable. Such strategies are:
 - A Imitative strategy
 - A technical analysis strategy based on Bollinger Bands
 - The gambler's fallacy based on the representativeness heuristics.

The new interesting aspects introduced are:

- The substitution of the sequential decision process with a more humanlike one.
- The introduction of subjective probabilities following de Finetti (1930).
- The introduction of the prospect Theory based on Kahneman and Tversky (1979, 1992)
- The possibility for agents to trade different number of stocks
- A more sophisticated statistical analysis is undertaken thanks to the rServe extension that links Netlogo with R.

4.3.1 Interface

As we can see, the new interface presents more sliders and switchers than the previous models according to the design of a more complex system. Moreover also additional plots have been introduced but we would prefer here to use R to produce the plots and statistical analysis thanks to the rServe connection. By doing that it is possible to overcome the physical limitations imposed by Netlogo. In the interface the RServe connection can be established by the button *Open the door to RServer* together with the *r-idle*; the latter helps to stabilize the connection between the programs avoiding crashes.

setup Go_P

On ZI Simulation_length On Var_Stock

nRandomAgents 100 Skill_threshold 0.75

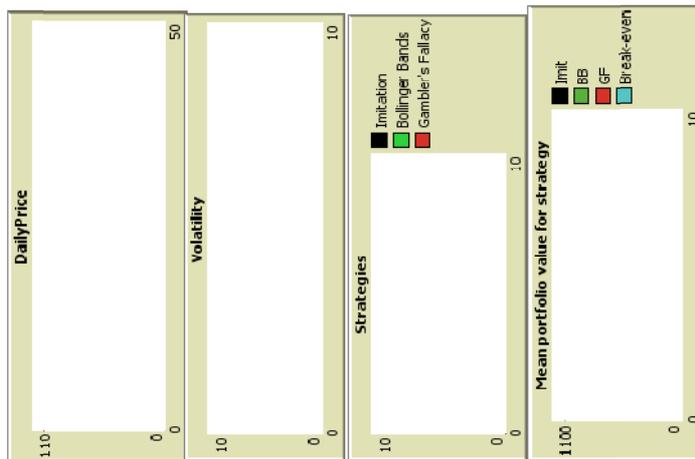
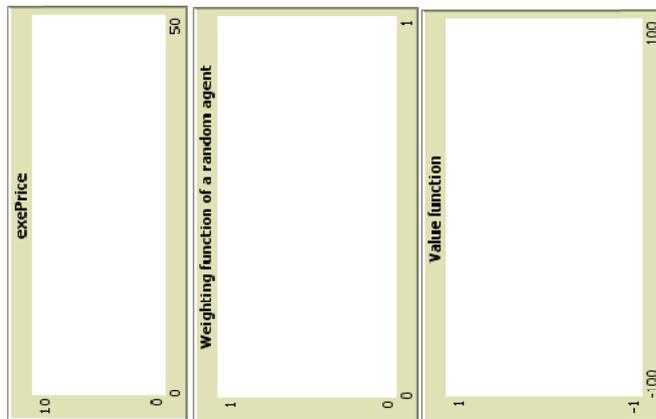
On Imitative_behavior Off

Off On Imit_Impact 1.00 On Imit_sensibility 5

On BBStrategy Off BB_Impact 50

On Gambler's_Fallacy Off GF_Impact 50 GF_sensibility 5

On prospect_theory Off



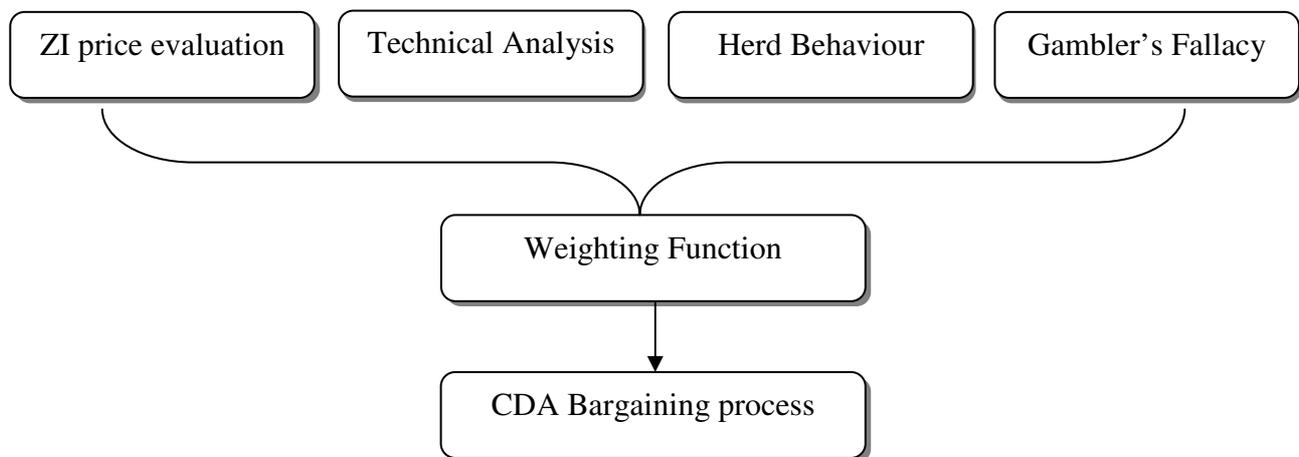
4.3.2 The code structure

As already mentioned the sequential approach was one of the main weaknesses of model 2.0 and it has been here replaced by a more humanlike one.

In fact in model 2.0 the agents' offers where either buy or sell according to a dummy variable which was shaped by the strategy adopted by the agents. In practice an agent following for instance the technical strategy would have bought or sold according to it. What if the same agent would have looked around for a leader according to the imitation strategy? In this case, the agent would completely forget its previous valuation in favor of the new one. It then becomes crucial the way the strategies are written in Netlogo since in these cases the last strategy will tend to nullify the others.

The alternative proposed in this model is based on a subjective probability assumption (De Finetti, 1930). In this way the strategies do not offset each other but they would rather contribute to the agent's decision. At the extreme the single agent's decision may be the result of all the strategies which does not require the strategy to share the same market direction. On the contrary it accounts for those situations where the first strategy may suggests buying while the second to sell and the third to pass.

Model 3.0 can then be represented by the following flow diagram



4.3.3 Agent's decision process

The strategies involved in the assessment process of agents are the evolution of the ones implemented in model 2.0. However some improvements have occurred and some new processes added and they are presented here in a new way by stressing the formulas and logic behind them.

It is important to remember that the strategies are picked by the agent according to their *skill* variable which is randomly assigned to each of them during the setup. In this new version the user can directly choose the amount of skill level (*Skill_threshold*) required for the strategies to be applied.



Let consider S_T as the value imposed by the user as *Skill_threshold*. The table below provides a quick recap of our model highlighting the connection between the strategies and the agent's skill variable, s_i . Moreover it includes all the variables implied in the decision process.

Strategies	Conditions	Driven by	Affected by	Impact on probabilities
ZI price evaluation	$\forall s_i$			$U(0, 0.10)$
Technical Strategy	$s_i \geq S_T$	Market	Agent's evaluation	$0.20 + (BB_Impact / 1000)$
Imitation Strategy	$s_i < S_T$	Environment	Imit_sensibility	$0.20 + (Imit_Impact / 1000)$
Gambler's Fallacy	$s_i < (1 - S_T)$	Market	GF_sensibility	$0.20 + (GF_Impact / 1000)$

The algorithm behind the strategies have been exhaustively described in model 2.0 in sections 4.2.10; 4.2.11; 4.2.12; 4.2.13. Additionally the ZI procedure has its roots on the model 1.0 described in section 4.1.6. Therefore the attention is here drawn on the logic of the process rather than on its computational technicality.

The imitation strategy, technical strategy and gambler's fallacy are chosen by the agents either complementary or not. The number of times they are applied is represented in the plot named "Strategies" in the Interface and here reported.



4.3.3.1 ZI Price evaluation

If we would consider only this procedure the agents involved will just be ZI as described in model 1.0 and as well as by Gode and Sunder (1993). This opportunity can be freely explored by the user simply setting off the other strategies with their relative switches.

Every agent in the model behaves according to three different subjective probabilities p_{sell} , p_{buy} , p_{pass} which correspond respectively to the probability for the agent to sell, buy or pass in the market.

At each new trading day (each tick) all agents reconsider the market situation and re-decide how to operate in it. The probability for each possible action (p) is here defined by a random procedure according to which p is any value in the interval $[0, 0.10)$.

The best way to formalize this is thought a continuous uniform distribution $U(a, b)$ where each member of the interval are equally probable with a and b as the minimum and maximum values of distribution.

$$p_{sell} \sim U(0, 0.10)$$

$$p_{buy} \sim U(0, 0.10)$$

$$p_{pass} \sim U(0, 0.10)$$

$$p_i \in Q^+ \quad \forall i = sell, buy, pass$$

At later stages this probabilities are going to be modified by the other strategies and agent evaluations as we will discuss in the next pages. However at the end of the decision processes the agent will behave according to the highest probability value. Therefore

$$\left\{ \begin{array}{l} \text{if } p_{sell} > p_{pass} \text{ and } p_{sell} > p_{buy} \Rightarrow \text{the agent's order is to sell} \\ \text{if } p_{buy} > p_{pass} \text{ and } p_{buy} > p_{sell} \Rightarrow \text{the agent's order is to buy} \\ \text{if } p_{pass} > p_{buy} \text{ and } p_{pass} > p_{sell} \Rightarrow \text{the agent does not place any order and passes} \end{array} \right.$$

Whenever the agent does not pass but instead decide to put an order either to buy or to sell he will propose a price level. Each agent proposes a different price at which he buys or sells according to the following formula where P denotes the proposed price and P_{t-1} is the market price of the last trading day.

$$P_t = P_{t-1} + X \quad \text{with } X \sim N(0, 10)$$

As we can see from the table in section 4.3.3., the ZI procedure is the one with the lowest impact on agents' probabilities. This has been done since we want to deal with a system where agents are more likely to take decisions based on the available strategies rather than having a model where total randomness dominates.

4.3.3.2 Technical analysis strategy

One crucial aspect in this strategy is the fact that it is not designed for all agents. The reason why this limitation has been introduced is to account for a part of the population to access to some technicalities and methodologies which are not available for all.

Despite the fact that the strategy adopted is really simple it can be considered as a nice proxy that introduces some discrimination among agents' abilities in the market.

This procedure is activated by its corresponding switch in the interface:



An additional variable called *BB_impact* has an impact on how the strategy is executed and it can be modified by the user according to the following slider



The strategy is based on Bollinger's Bands which are described in section 3.2.2.1. We denoted with UB the Upper Band, with LB the Lower Band and with MA the 20-days moving average of prices while we assume that the agents involved in this analysis already meet the skill requirements. The decisions are taken according to the following formula where \ddot{p}_i , with $i = \text{pass, buy or sell}$, corresponds to the probability value on i -decision heretofore from the other strategies' contributions. P as before denotes the price level.

$$\left\{ \begin{array}{l} \text{If } P_{t-1} \geq UB \left(1 + \frac{y}{1000}\right) \quad \text{then } p_{\text{sell}} = \ddot{p}_{\text{sell}} + 0.20 + \left(\frac{BB_{\text{Impact}}}{1000}\right) \\ \text{If } P_{t-1} \leq LB \left(1 - \frac{y}{1000}\right) \quad \text{then } p_{\text{buy}} = \ddot{p}_{\text{buy}} + 0.20 + \left(\frac{BB_{\text{Impact}}}{1000}\right) \\ \text{If } MA \left(1 - \frac{z}{1000}\right) \leq P_{t-1} \leq MA \left(1 + \frac{z}{1000}\right) \quad \text{then } p_{\text{pass}} = \ddot{p}_{\text{pass}} + 0.20 + \left(\frac{BB_{\text{Impact}}}{1000}\right) \end{array} \right.$$

$$\begin{array}{l} \text{where } y \in Z \text{ and } y \sim U(0,10) \\ z \in Z \text{ and } z \sim U(0,5) \end{array}$$

Briefly put this strategy leads agent to play on break out situations when prices are too high (or too low) and agents expect price to move back in the opposite direction. The interesting feature here is that even if the strategy is the same the individual evaluations might differ.

The agents' individual evaluation passes through the y and z values which are randomly chosen for each agent and each tick. Consider for instance the case where the price is rising, it is above the UB and two identical agents are undertaking the technical analyst strategy. They will face different values for y for instance it is 0 for the first and 10 for the second. Price may rise up to the corresponding value with $y < 10$ and then only the first agent is more likely to sell while the second does not change his idea on the market. The *BB_Impact* variable takes value between [10, 100] and it then increases the impact that technical analysis has on probabilities.

4.3.3.3 Imitation strategy

Imitation strategy implements in the model what theoretical has been described in section 3.6 as herd behavior. As we already might know from the table in section 4.3.3 this strategy is adopted by those with low skill values. The underlying assumption here is that agents with the complex task to guess what is going to happen in the market seek around for someone who seems more competent and then try to do the same. This strategy can be activated or not according to the simulation purpose thanks to this switch:



Moreover two additional variables influence both the way the imitation occurs and its impact on agents' decisions.



The code did not change much from its previous version adopted in model 2.0 explained in section 4.2.10. For this reason we focus here on the overall logic resuming formally how imitation works in order to highlight the new features and variables implemented in model 3.0. For those interested the full code behind Imitation strategy can be found in Appendix D.3 .

The imitation mechanism can be split in two parts. In the first one, agents seek around then in order to identify a potential leader on the basis of the cash owned. In the second phase the followers look at what the leader is doing and decide whether or not to imitate. Consider a potential leader x_L with its own cash value c_L and the set of the neighbors around him defined as X_{NL} .

$$X_{NL} = [x_1, x_2 \dots x_N] \quad \text{such that} \quad d(x_i, x_L) \leq d_z^9$$

$$x_L := \text{leader}(X_{NL}) \quad \text{iff} \quad c_L > c_j \quad \forall j \text{ such that } x_j \in X_{NL}$$

As we can see, the leader is selected by the imitators on the basis of the cash rather than on the skill variable which is unobservable to the agents.

Consider s_i as the skill variable for agent i , and S_T as the skill threshold

if $s_j < S_T$ and if $Imit_{sensibility} \geq y$:

$$\left\{ \begin{array}{l} \text{if } x_L \text{ buys} \quad \Rightarrow \quad p_{buy}^j := p_{buy}^{\dots j} + 0.20 + \left(\frac{Imit_{Impact}}{1000} \right) \text{ and } P^j := P_L \pm \varepsilon \\ \text{if } x_L \text{ sells} \quad \Rightarrow \quad p_{sell}^j := p_{sell}^{\dots j} + 0.20 + \left(\frac{Imit_{Impact}}{1000} \right) \text{ and } P^j := P_L \pm \varepsilon \\ \text{if } x_L \text{ passes} \quad \Rightarrow \quad p_{pass}^j := p_{pass}^{\dots j} + 0.20 + \left(\frac{Imit_{Impact}}{1000} \right) \end{array} \right.$$

where $y \in Z$ and $y \sim U(0,10)$

⁹ The distance d_z is chosen in Netlogo so that each leader may affect the decision of the closet agents around him. For a graphical representation of the area of influence of the leader in Netlogo see section 4.2.10 .

A leader whose decision is to buy will tempt the ones around him to do so, we say “tempt” since first of all the conditions on skill and *Imit_sensibility* have to be satisfied and secondly because since decisions are based upon probabilities imitation might also not occur

Those imitating a leader whose decision is to buy will increase the probability of buying too according to the formula written above. The same dynamic holds whenever the leader sells or passes. Remember that as for the technical strategy \ddot{p}_i^j is the probability for agent j to take the decision i before the imitation mechanism occurs. Therefore it accounts for other strategies' contributions that might or not have influenced the agent's decision.

Moreover whenever the leader does not pass and he places a buy or sell order the followers are likely to do the same with similar prices. It is worth to notice that there is not a copy and paste imitation mechanism on price but it is recognized a certain degree of free will. In fact the ε term on the price imitation formula has been added to allow both a certain degree of individual decision from the imitator side and to account for information asymmetry in the model.

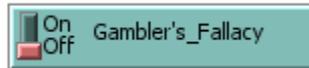
The *Imit_sensibility* variable allows different scenarios and types of imitation mechanisms it can be freely chosen by the user from the interval [1, 10]. In fact a really low sensibility leads to few imitators among the leader's neighbors while at maximum the leader decision is automatically considered by the followers and taken into account. Of course the followers still have to meet the skill criteria condition.

As for previous cases the *Imit_Impact* variable takes value between [10, 100] and it increases the imitation effect on probabilities.

4.3.3.4 Gambler's fallacy

Gambler's fallacy is a cognitive bias that arises from the representativeness heuristic. Its theoretical background has been deeply explored in the section 3.7 and its sub-sections as well as the psychological aspects behind it.

The assumption here is that some investors are not really trained to trade in the market and then they use low-profile strategies affected by rational biases as for the gambler's fallacy. This is the reason why in the model this strategy is adopted by the agents with the lowest value for skill. As the others strategies presented so far it is activated by its own switch:



Additionally two more variables influence both the way the fallacy occurs and its impact on agents' decisions.



Taking as granted the skill conditions we focus now on how Gambler's fallacy has been implemented in the model.

if $GF_{sensibility} \geq x$:

$$\begin{cases} \text{if } P_T < P_{T-1} < P_{T-2} < P_{T-3} < P_{T-4} \Rightarrow & p_{buy}^j := p_{buy}^{j\ddot{}} + 0.20 + \left(\frac{GF_{Impact}}{1000}\right) \\ \text{if } P_T > P_{T-1} > P_{T-2} > P_{T-3} > P_{T-4} \Rightarrow & p_{sell}^j := p_{sell}^{j\ddot{}} + 0.20 + \left(\frac{GF_{Impact}}{1000}\right) \end{cases}$$

where $x \in Z$ and $x \sim U(0,10)$

The idea and formula above are both pretty intuitive. When agent sees the price raising (or falling) five times in a row he will be tempted to bet on the change of direction as it occurred in the Monte Carlo casino in 1913 when gamblers rushed over the roulette to bet aggressively on the red since the black was out 15 times ignoring the fact at the end the black came up a record 26 times in succession.

The $GF_{sensibility}$ allows the simulation of different scenarios and it can be freely chosen by the user in the interval [1, 10]. In fact a really low sensibility leads to few investors, among the ones with low skill value to apply it. On the opposite the maximum value leads to the situation where all traders with sufficiently low skill apply almost automatically this strategy when the market triggers it.

Instead GF_{Impact} variable takes value between [10, 100] and it increases the gambler's effect on probabilities.

4.3.4 Prospect theory in the game

In model 3.0 both value function and the weighting function have been added in order to include the main features of Prospect Theory in the simulation. For the theoretical background behind prospect theory and its evolution we suggest to give a look to sections 1.2, 1.3 and 1.4.

These features can be activated at the will of the user from the switch:



The TPWF implemented is based on the work of Gonzalez and Wu (1999) since it stresses two underlying psychological aspects, discriminability (γ) and attractiveness (δ), and it provides a function able to account for a broader variability among the agents compared to the one parameter function of Kahneman and Tversky (1992).

Moreover we found the following equation linking the two parameters, as demonstrated on Appendix A:

$$\gamma = 0.2965(\delta) + 0.2079$$

In particular the ε term has a normal distribution with mean 0.2079 and standard deviation 0.02. Designing ε like this it provides more variability on the parameters' values without leaving away the theoretical insights.

$$\gamma = 0.2965(\delta) + \varepsilon \quad \text{with } \varepsilon \sim N(0.2079, 0.02)$$

Each single agent i is created with its unique value for attractiveness (δ^i) that is randomly drawn with equal probability from the interval [0.21, 1.52). Such interval has been chosen based on the minimum and maximum values observed in the sample of Gonzalez and Wu (1999).

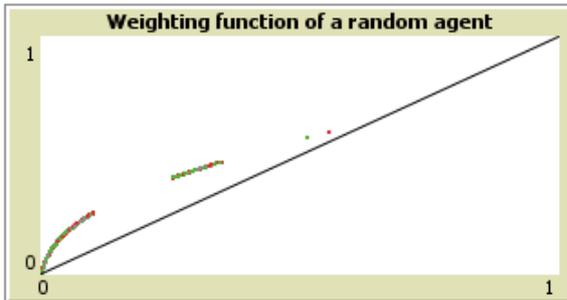
During the simulation then each agent reconsiders its probabilities according to its own weighting function:

$$w(p) = \frac{\delta^i p^\gamma}{\delta^i p^\gamma + (1-p)^\gamma}$$

Moreover the distorted and real values for probabilities of a random trader have plotted providing an intuitive idea of how the agent's weighting function is made. It is worth to remember that attractiveness can be conceived as the gambler's attraction for the game where the higher attractiveness is, the higher is the weighted probability function. While the degree of the curvature is determined by the level of individual's discriminability.

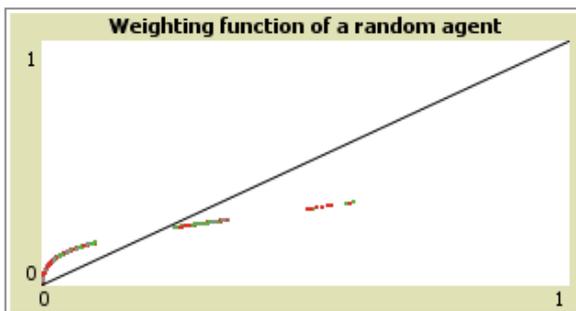
Consider now how weighting functions affects probabilities during the simulation and the corresponding “full” shape based on the same individual parameters.

In this case we have an agent with a high values both for attractiveness ($\delta = 1.33$) and discriminability ($\gamma = 0.60$). As reference point it is worth to remind that in the observed population the median values for attractiveness and discriminability were respectively 0.77 and 0.44

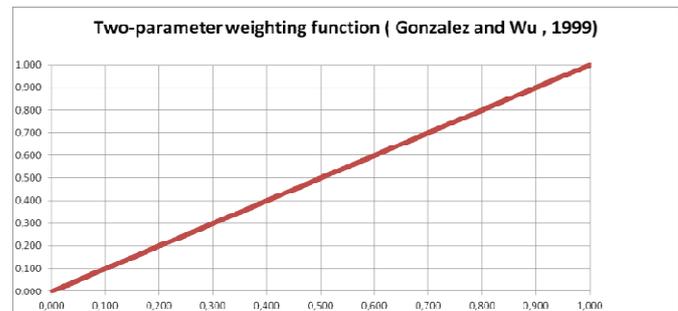
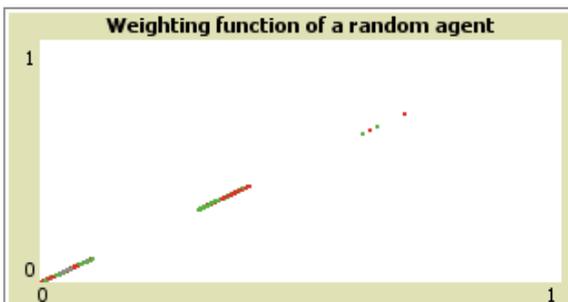


In this case it is clear how the agent high attractiveness for the financial market drives him to over-estimate most probabilities. Moreover his underlying weighting function confirms this idea and it tells us that how the all range of probabilities are conceived by that specific individual.

In the case we face of a more cautious agent with lower values for the parameters ($\delta = 0.46$ and $\gamma = 0.34$) we see an opposite trend that lead the agent to mainly underestimate probabilities and to be prudent on his own choice.



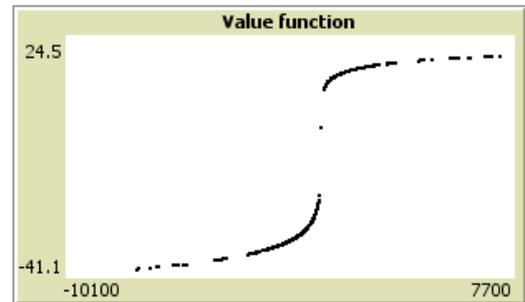
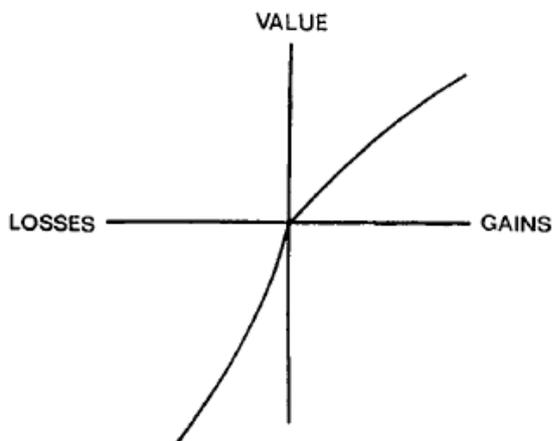
Of course it is quite intuitive that when the prospect theory is not activated in the simulation the agent’s probabilities stay put and are not weighted, the resulting function would then corresponds with the bisector. In this case the weighting function parameters take values $\delta = \gamma = 1$.



At the same time the agent's value function is drawn based on the experienced gains and losses. As the theory tells us (section 1.4.1) the crucial points here are:

- The role of the reference point of the agent that corresponds in the model to the cash value owned the day before.
- Diminishing marginal sensitivity to changes in both gain and loss directions.
- A gain and a loss in absolute terms do not have the same effect on individual, but a loss has a proportionately greater impact. (consistent with the loss aversion)

On the left we see the theoretical function according to Kahneman and Tversky (1979, 1992) while on the right the resulting function implemented in Netlogo.



An important point has to be stressed here. Despite we'll refer to the utility function of the agent as the sum of the experiences values associated with both gains and losses, the agents are not utility-maximizer but the value function is just a descriptive tool.

First of all it allow us to implement all the crucial elements of prospect theory while it tells us that agents experience both good and bad times during their investments in spite of positive or negative final outcomes.

In the model the value function is based on two parts: one accounting for losses one for gains. In particular we used two log functions with different bases in order to show both diminishing marginal sensitivity and a greater impact of losses over gains. Consider a randomly chosen agent with cash value c_T at time T and c_{T-1} at time t-1. At each tick of the simulation then:

$$\text{let } \Delta c_1 := c_T - c_{T-1}$$

$$\left\{ \begin{array}{ll} \text{if } \Delta c_1 > 0 \Rightarrow U' = (\log_{100} \Delta c_1) * 10 & \text{and } U_T = U_{T-1} + U' \\ \text{if } \Delta c_1 = 0 \Rightarrow & U' = 0 \quad \text{and } U_T = U_{T-1} + U' \\ \text{if } \Delta c_1 < 0 \Rightarrow U' = [\log_{10}(-\Delta c_1)] * (-10) & \text{and } U_T = U_{T-1} + U' \end{array} \right.$$

$$\text{with } U_0 = 1'000 \text{ and } c_0 = 1'000$$

4.3.5 Stock variability

An interesting additional feature of model 3.0 is given by the possibility for agents to exchange more than a unitary stock. This opportunity can be explored whether the following switch is activated:



The stock variability is implemented in two different ways according whether or not the prospect theory is on for the running simulation. In fact when prospect theory is on, the agent's attractiveness value is used as a proxy measure for risk-seeking. On the other hand when the prospect theory is not activated, agents' decisions on stock change are randomly assigned at the beginning of the simulation.

Whenever the prospect theory is off in the simulation, the following procedure defines for each agent i the traded number of stocks (n_{stock}^i)

$$n_{stock}^i := n + 1 \quad \text{where } n \in Z \text{ and } n \sim U(0,4)$$

It then means that each agent trades a different number of stocks from a minimum unitary amount up to five which is held constant over the simulation.

When prospect theory is in game, the number of stocks traded depends on the agent's attractiveness variable. For agent i with a value of attractiveness δ^i its choice on the number of stocks is based on the following formula:

$$\begin{cases} \text{if } \delta^i \in [0.210, 0.472) \Rightarrow n_{stock}^i := 1 + s_0 \\ \text{if } \delta^i \in [0.472, 0.743) \Rightarrow n_{stock}^i := 1 + s_1 \\ \text{if } \delta^i \in [0.743, 0.996) \Rightarrow n_{stock}^i := 1 + s_2 \\ \text{if } \delta^i \in [0.996, 1.258) \Rightarrow n_{stock}^i := 1 + s_3 \\ \text{if } \delta^i \in [1.258, 1.520) \Rightarrow n_{stock}^i := 1 + s_4 \end{cases}$$

$$\text{where } s_j \in Z \text{ and } s_j \sim U(0, j) \quad \forall j = 0,1,2,3,4$$

Again agents here trade from a minimum unitary stock up to five; the higher the attractiveness the greater is the number of traded stocks.

Not only the formulas are different but also the way agents' choices occur. Whenever prospect theory is off agent's decision on n_{stock} it is made the first day and then followed for the rest of the simulation. On the other hand, when prospect theory is on, agent's n_{stock} changes over time but it is strictly related to the value of attractiveness, independent on time. Therefore the riskiest investors are the one who are likely to trade up to five stocks even if they might randomly decide to trade a lower amount. In turn the most risk adverse traders do not take the chance to trade more than the unitary stock.

4.3.6 Netlogo and R

One major enhancement of model 3.0 is its ability to fruitfully communicate with R. This has been extremely useful because it allowed us to overcome the graphical limitations imposed by Netlogo and provided the statistical power of R for a deeper quantitative analysis.

This has been done thanks to a Netlogo's extension named rServe that provides an interface which can be used by applications to perform computations in R. Thanks to a client/server design rServe provides a flexible, fast and separated tool to call R commands through Netlogo. (Urbanek, 2003).

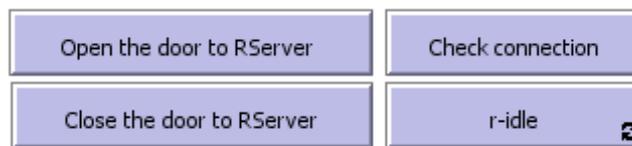
Assuming the packages are already installed both in R and Netlogo what has to be done is to allow the communication between the two programs. In R it is essential to load and run the Rserve package thanks to the following command:

```
>library(Rserve)
```

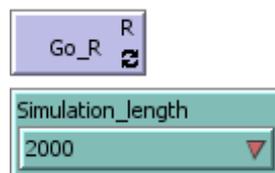
```
>run.Rserve
```

```
>Rserve()
```

Once rServe is launched, R can be freely closed since Netlogo can independently open and manage a door to communicate with it. In particular the communication process in Netlogo is performed thanks to the following buttons included in the interface.



It is essential to open the door so that R and Netlogo can communicate as well as allow the r-idle so that the program does not crash while producing the desired outputs. The last thing to do is to select the desired number of ticks the simulation has to run and then simply click the Go_R button.



At the end of the simulation the commands requested on Netlogo are performed through R. The list of commands allowed by rServe can be found on Appendix C while the whole communication code is in section D.8. The code is designed to call R primitives so a basic knowledge of R is needed in order to understand it. The graphical and statistical outputs from R can be found in the experiments chapter.

5. EXPERIMENTS AND RESULTS

In this chapter are presented the most significant experiments and results based on the final model 3.0. Almost all variables are going to be changed in order to provide a deep analysis of the model and its simulation. The default or standard variables setting requires that:

- The initial asset's price is 100
- All agents own an amount of cash equals to 1'000
- Whenever not specified differently the total number of agents is 100.
- Whenever not specified differently the Skill_threshold value is 0.75.
- The simulation runs for 2'000 ticks¹⁰ where each tick represents a trading day

We will investigate both the micro and macro sides of the model. With Terna's words (2013) we are interested both in the ants and their anthill. We want to analyze how ants' decisions affect their performances in the financial markets as well as what is the aggregate product of all these individual choices. Since we deal with large datasets many charts and graphs have been produced by R thanks to Rserve in order to overcome the physical limitations imposed by Netlogo. Further details on these software packages and their use in this work can be found in Appendix C.

The research agenda proposed here starts by a ZI model (Gode and Sunder, 1993) simulation, then each strategy has been implemented one by one in order to depict the single strategy effect both in micro and macro terms. The final model instead comprehends all the three strategies in order to check for consistency of previous single results when the model is complete.

The logical description behind the procedures can be finding in section 4.3 where the main formulas and logic are provided whereas the whole code can be found in Appendix D.

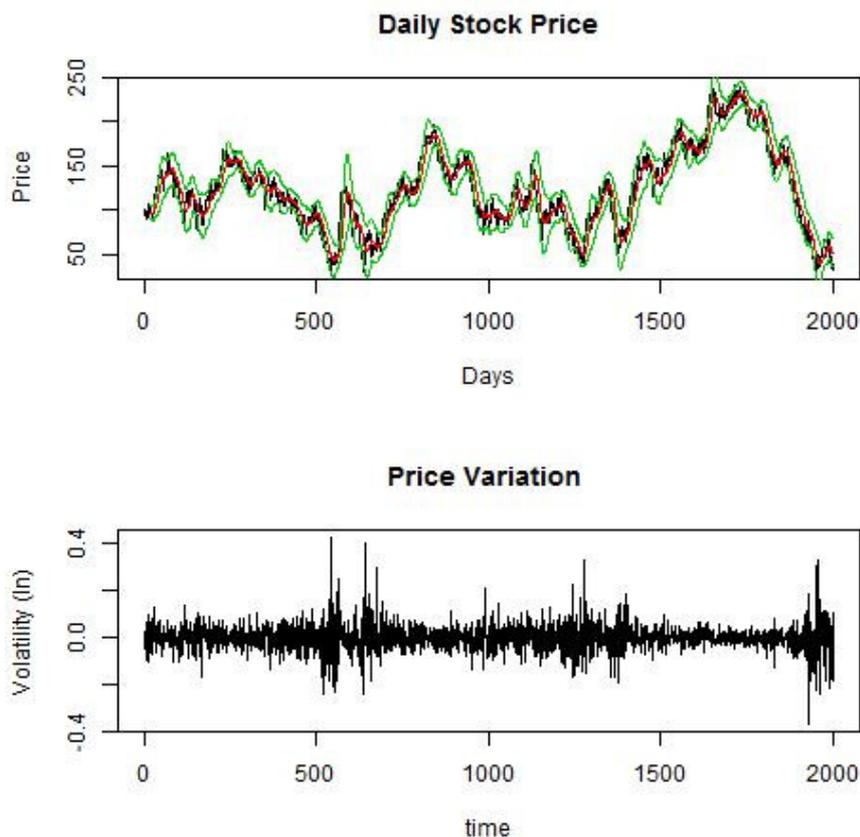
¹⁰ This value has been chosen since the Shapiro Wilk test can be successfully undertaken with a sample size up to 2000 data

5.1 ZI

5.1.1 First scenario: Gode and Sunder

Despite model 3.0 took a significant distance from the ZI model (Gode and Sunder, 1993) it is worth to perform here a simulation of a financial market based on ZI traders who should produce a market driven completely by randomness. Therefore from the “anthill standpoint” we expect to deal with a random walk process.

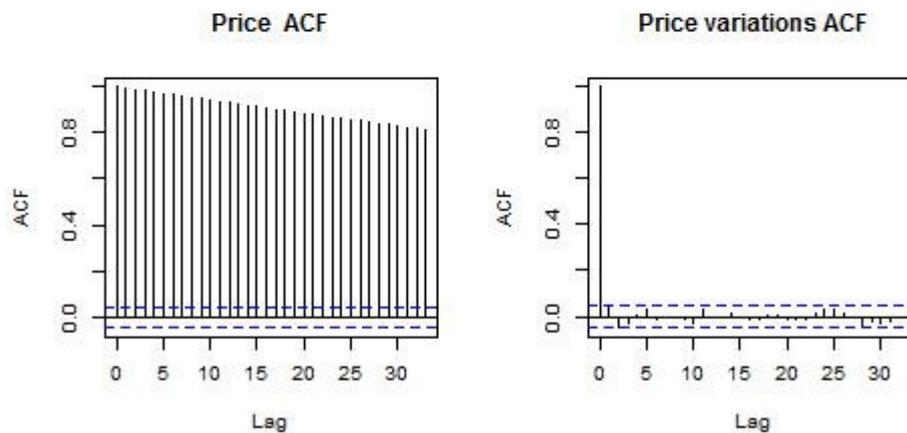
In this simulation the technical trading strategy, the imitative behavior, the gambler’s fallacy as well as prospect theory are not implemented. Additionally the agents can only trade a unitary stock. All agents have a skill value that is by default randomly assigned at their creation, which is completely useless. Therefore we expect that those able to make profits in the market are simply lucky. In this simulation the final stock price of 31 arises from the following past data with a minimum of 30 and a peak of 241.



Volatility is based on daily price variations that are computed with the following formula:

$$\Delta P_{T,T-1} = \ln(P_T) - \ln(P_{T-1}) \quad \text{where } P_0 = 100$$

Consider now the autocorrelation correlogram in order to understand which process is behind the price movements:



On the left it is plotted the ACF of the daily prices while on the right the same chart but for the daily price variations. As we can see, daily prices follow a random walk process.

On the other side, the ACF for price variations displays a white noise behavior without any correlation for lags greater than zero.

From the ACF correlogram we can see how the stochastic component dominates the price variations residuals in favour of a random walk process for price and of a white noise process for price variations.

In order to provide more information we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = -13.64200850531399

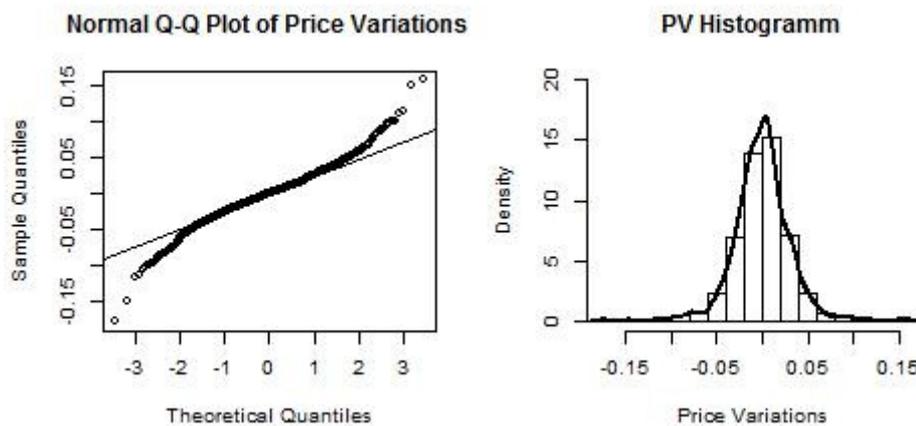
Lag order = 12

P-Value for the test = 0.01

Alternative HP: stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the following Q-Q plot. The Q-Q plot compares the data on variations generated in the model on the vertical axis to a standard normal population on the horizontal axis. The more the points display linearity more likely are the data to be normally distributed. Graphically normality seems far away here as confirmed by the histogram where the thicker black line represents the fitted density curve of price variations.



In order to have a more precise idea on the normality of price variations we performed both a Shapiro-Wilk test together with a Jarque-Bera one

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.9515751638490885
 Approximate p-value for the test = 3.435477479566121E-25

Jarque-Bera Test on price variations

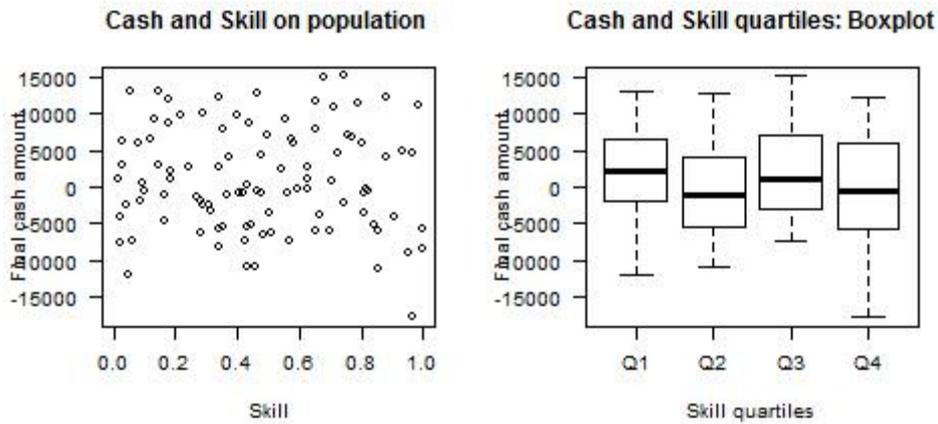
Value of the Jarque Bera test statistic = 1647.8101467402691
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05. The Shapiro-Wilk statistic value here is equal to 0.95 displaying a slightly non normality of price variation distributions since it is really close to the maximum value of 1, corresponding to a Gaussian distribution of the data. However the p-value here is less than alpha according to which the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

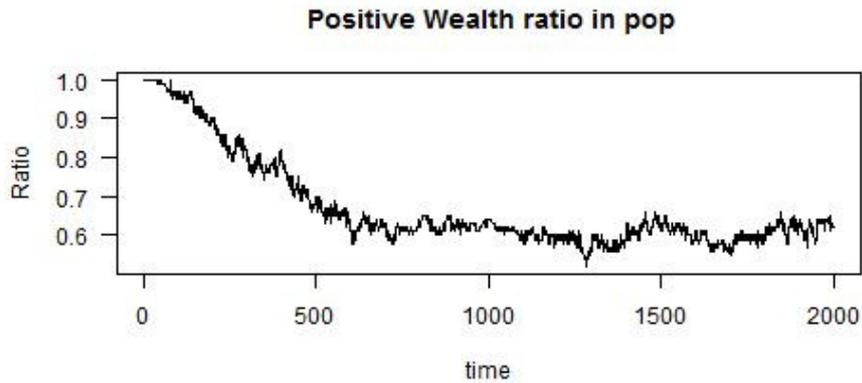
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both test suggest us that the price variations do not replicate a Normal distribution

According to what we saw on price and its variations the underlying process here is a random walk whose residuals despite seem to be draw from a Gaussian distribution they do not.

On the agent side instead we can check how skill is ineffective as there are no effect on the final amount of cash owned by agents. In statistical terms we face an almost zero Pearson correlation between the skill and cash vectors on the population (0.060). Graphically we can investigate the cash-skill relationship by looking at the following scatter plot and box plot



As we can see, the median final cash amount is similar among the different skill quartiles while in the scatter plot total randomness rules. Every agent started with a positive amount of cash but the ratio of population with positive cash started soon to drop as the simulation runs. In particular it drops significantly in the first 500 days, it touches a minimum value of 0.52, its final value at the end of the simulation is 0.62.

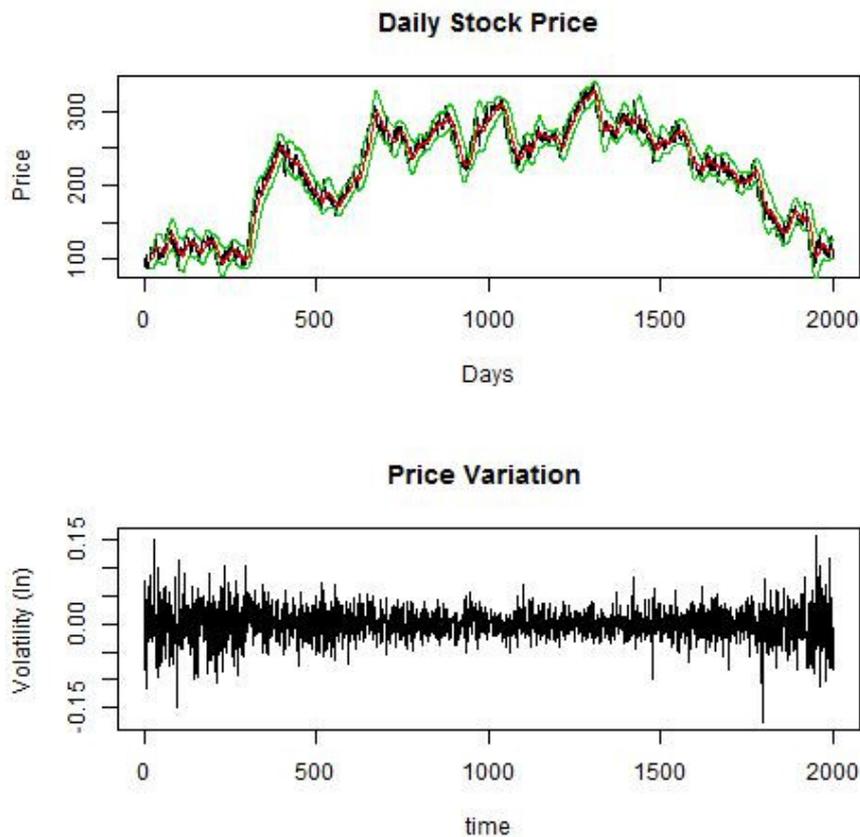


5.1.2 Second scenario: Stock variability

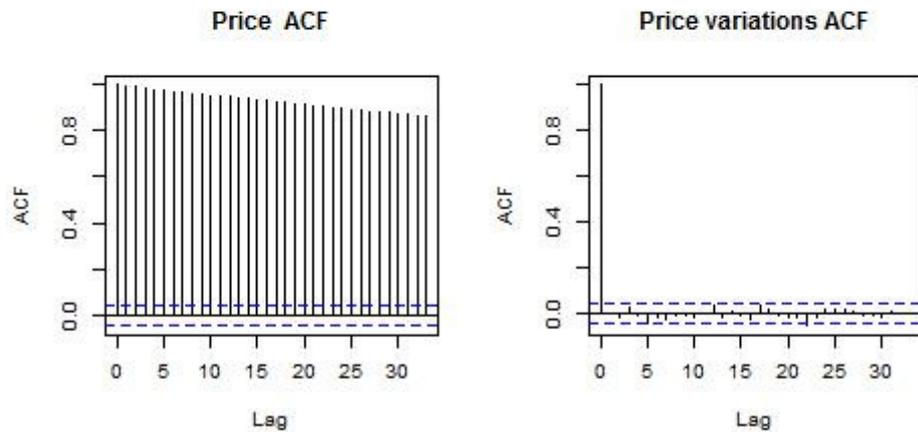
In the following simulation we use the previous settings but we allow agents to trade more than the unitary stock of the assets.

Each agent has a certain risk attitude that leads him to trade a fixed amount of stocks plus a random component in order to not deal with a deterministic choice. For more details in section 4.3.5 there are explained the formulas behind the process.

In this simulation the final stock price of 104 arises from the following past data with a minimum of 85 and a peak of 338. The volatility increases especially whenever the price falls abruptly around the minimum level of 85.



Consider now the correlograms in order to understand which kind of process is behind the price movements:



On the left it is plotted the ACF of the daily prices while on the right the same chart but for the daily price variations. As we can see, daily prices follow a random walk process.

On the other side, the ACF for price variations displays white noise behavior without any correlation for lags greater than zero except for a slight negative correlation at lag 22.

From the ACF correlogram we can see how the stochastic component dominates the price variations residuals in favour of a random walk process for price and of a white noise process for price variations.

In order to provide more information we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = - 13.08255380947571

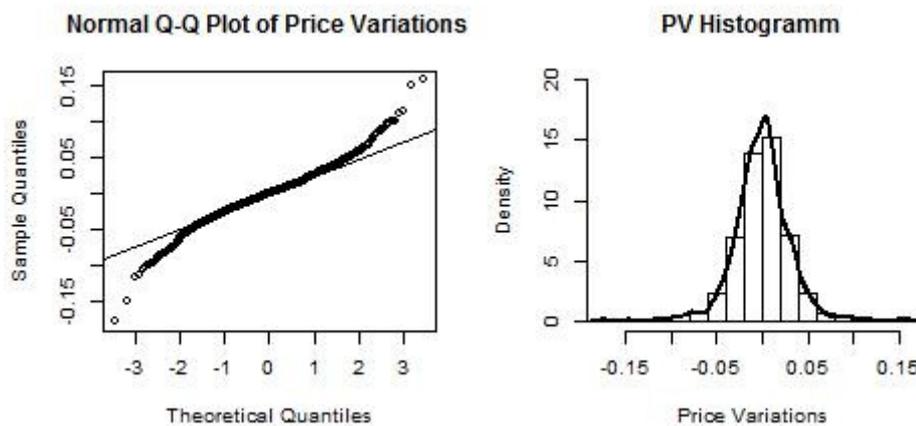
Lag order = 12

P-Value for the test = 0.01

Alternative HP: stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the following Q-Q plot. The Q-Q plot compares the data on variations generated in the model on the vertical axis to a standard normal population on the horizontal axis. The more the points display linearity more likely are the data to be normally distributed. Graphically normality seems far away here as confirmed by the histogram where the thicker black line represents the fitted density curve of price variations.



In order to have a more precise idea on the normality of price variations we performed both a Shapiro-Wilk Test together with a Jarque-Bera

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.974737724428588
 Approximate p-value for the test = 2.6384436943948403E-18

Jarque-Bera Test on price variations

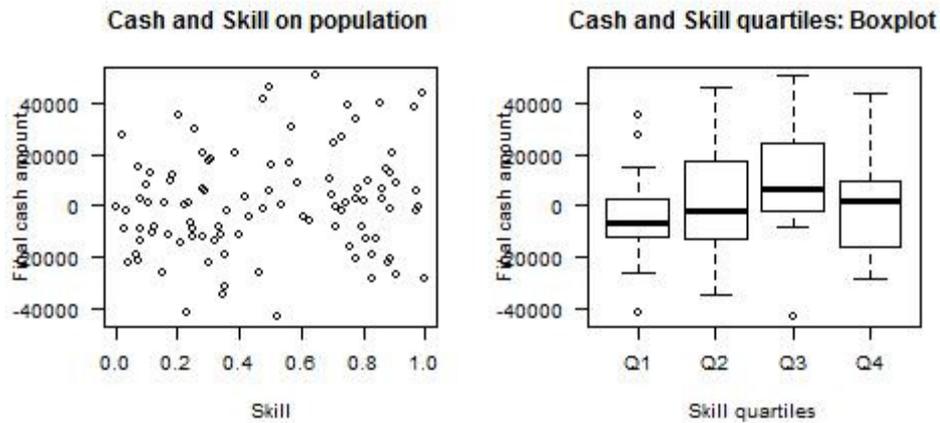
Value of the Jarque Bera test statistic = 607.2307265488956
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05. The Shapiro-Wilk statistic value here is equal to 0.97 displaying a slightly non normality of price variation distributions since it is really close to the maximum value of 1 corresponding to a Gaussian distribution of the data. It is interesting to notice that price variations are here more close to a normality distribution compared when agents trade a unitary stock. The p-value is less than alpha according to which the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

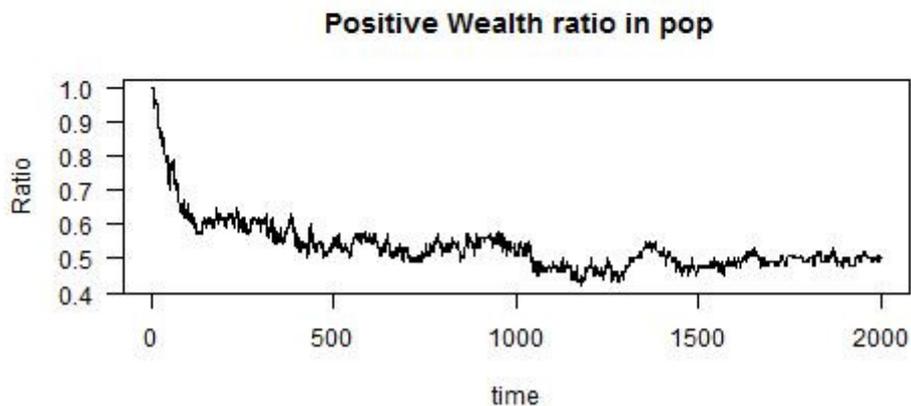
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both test suggest us that the price variations do not replicate a Normal distribution.

According to what we saw on price and its variations the underlying process here is a random walk whose residuals, despite seem to be draw from a Gaussian distribution, they do not.

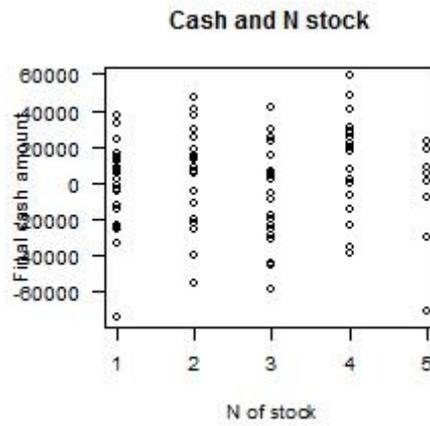
On the agent side instead we can check how skill is ineffective as there is no effect on the final amount of cash owned by agents. In statistical terms we face an almost zero Pearson correlation between the skill and cash vectors on the population (0.01). Graphically we can investigate the cash-skill relationship by looking at the following scatter plot and box plot



As we can see, the median final cash amount is similar among the different skill quartiles while in the scatter plot total randomness rules. Every agent started with a positive amount of cash but the ratio of population with positive cash started soon to drop as the simulation runs. In particular it drops significantly in the first 100 days faster than standard ZI case. It touches a minimum value of 0.42 and its final value at the end of the simulation is 0.5.



In order to investigate the effect of trading a greater number of stocks in the final cash amount the following chart comes at handy. There is not a specific trend related to the greater number of stocks. In fact we face similar losses both among the unitary stock traders and the five stocks ones. Conversely there are similar profitable agents among the five groups with the best performing one on the fourth group.



5.2 Imitative behavior

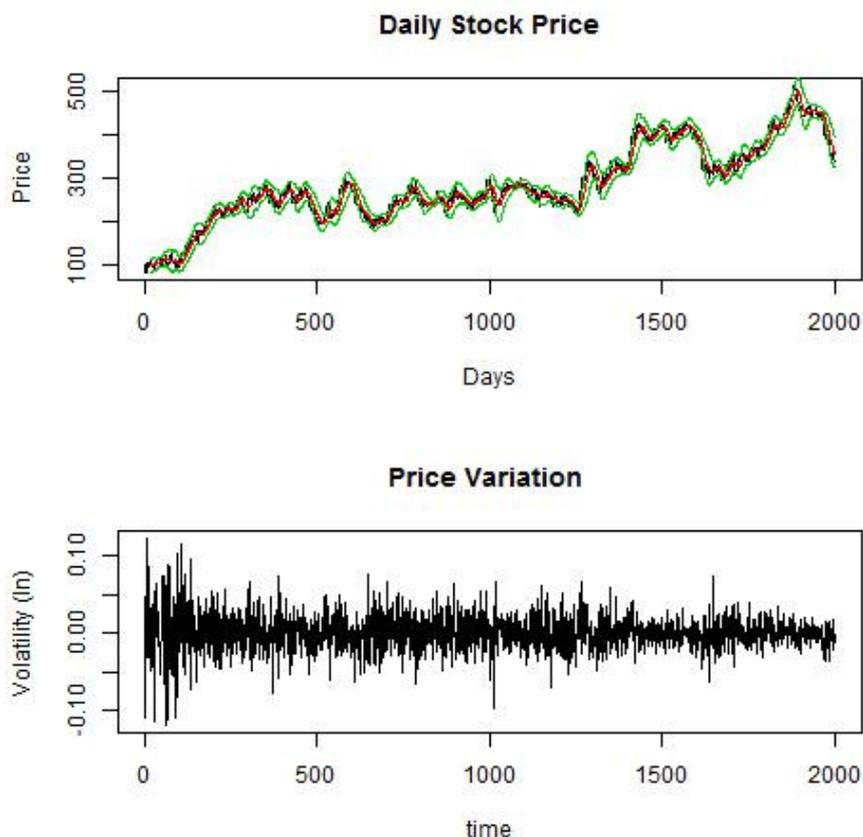
5.2.1 First scenario: standard case

In this simulation it has been added the opportunity for the agents to apply the imitative strategy in order to imitate the most successful traders around them as explained in the methodological part in section 4.3.3.3. Starting from the previous ZI simulation we switch on the Imitative_behavior and impose the following values on the Imitative's parameters:

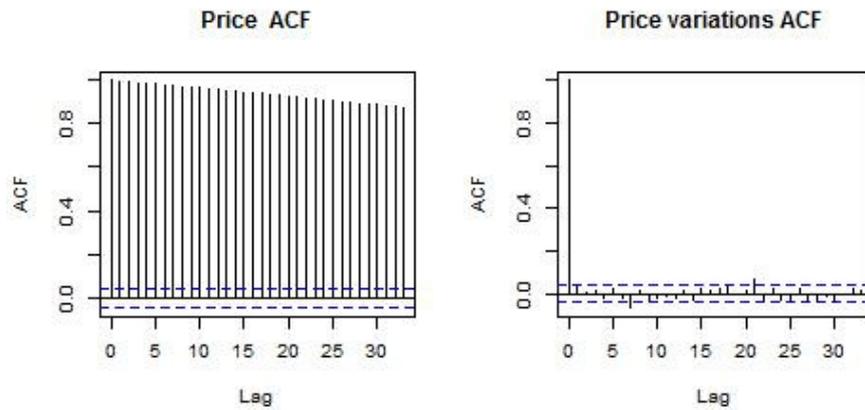
- Imit_impact : 30
- Imit_sensibility: 3

It is worth to notice that since Imitation is the only strategy available to agents modifying Imit_impact does not have a real effect on agents' decisions here. Conversely the higher the Imit_sensibility the greater is the chance that a leader is going to be followed by those surrounding him. It is worth to remember that agent's skill has here an impact on whether or not following this strategy, according to the table in section 4.3.3. We present now the case where skill threshold is imposed on the default value (0.75) while different scenarios involving changes on Imit_sensibility, Skill_threshold and the number of agents are going to be proposed later on.

The stock price has a peak at 516 and a trough at 81 and it is 340 when simulation finished while facing high volatility especially during the first part of the simulation and some peaks when price abruptly changes.



Consider the autocorrelation correlogram in order to understand which kind of process is behind the price movements as well as price variations.



On the left it is plotted the ACF of the daily prices while on the right the same chart but for the daily price variations. As we can see, daily prices follow a random walk process.

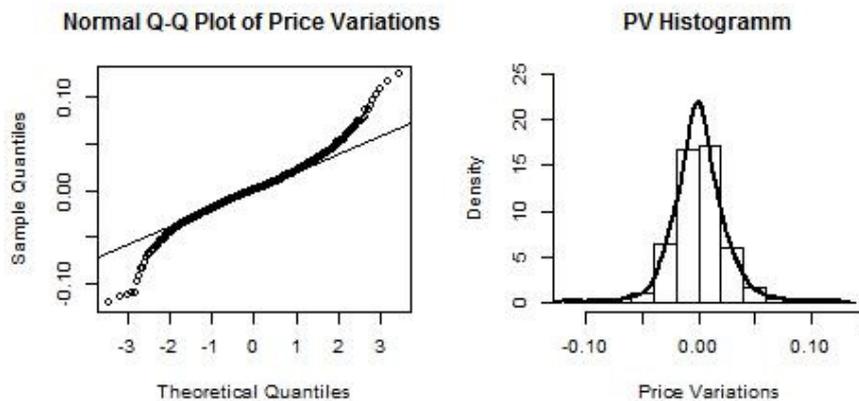
On the other side, the ACF for price variations displays white noise behavior with a slightly positive autocorrelation for the lag equal to 21 and a slightly negative one for lag 7 while for all the other k greater than zero there is not autocorrelation. To check if this graphical intuition is correct we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test:

data: Price Variations
 Dickey-Fuller = -13.270257941517396
 Lag order = 12
 P-Value for the test = 0.01
 Alternative HP: stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the following Q-Q plot and histogram.



Similar to previous cases the Q-Q plot shows many data outside of the q-q line. At the same time the fitted probability function seems sharper than a Gaussian. Before to state any conclusion we need to have more reliable information through both Shapiro-Wilk and Jarque-Bera tests.

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.9666524734184302
 Approximate p-value for the test = 4.238141709194077E-21

Jarque-Bera Test on price variations

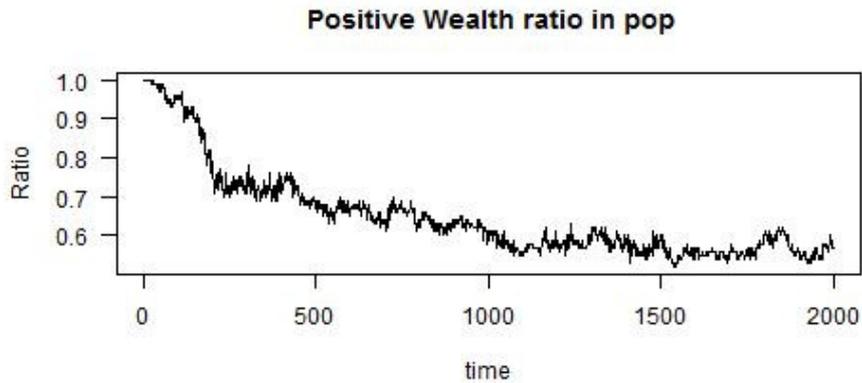
Value of the Jarque-Bera test statistic = 821.6013156358187
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05. The Shapiro-Wilk statistic value here is equal to 0.96 displaying a slightly higher normal behaviour compared to the ZI case (0.95). Similar to the ZI case the p-value here is less than alpha according to which the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

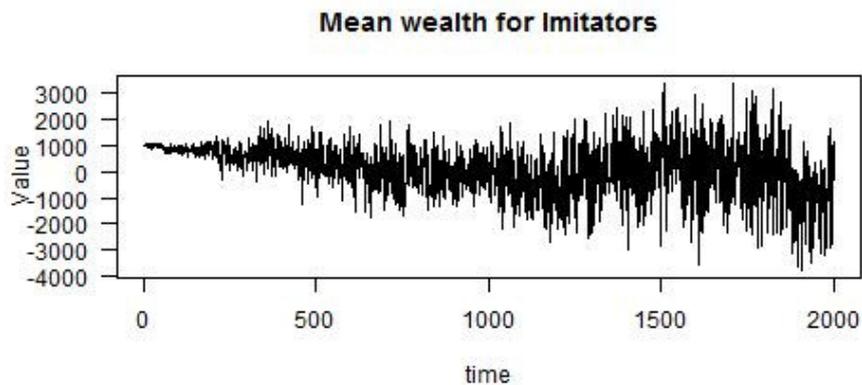
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both test suggest us that the price variations do not replicate a Normal distribution

This result is really close to what has been found for the ZI simulation.

Here we focus instead on the agent's side and how they performed in the market with the Imitative strategy at their disposal. The positive wealth ratio among agents has a similar trend to ZI case. More specifically here it has a minimum value of 0.52 before ending at 0.57.

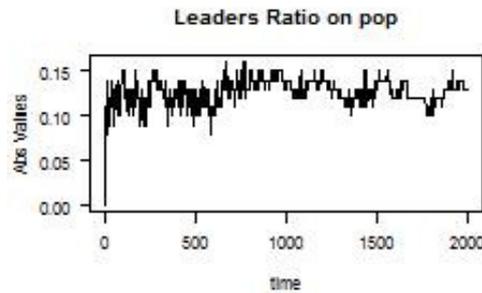


To answer how the imitative agents performed in the market we provided the following chart where are plotted the mean cash values for each day of those applying the Imitative strategy.

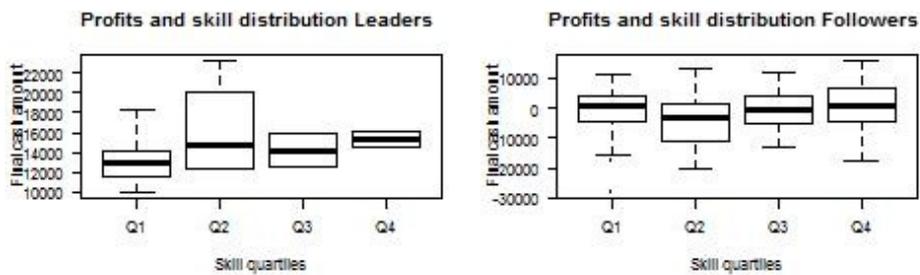


As we can see, this strategy is highly volatile with uncertain returns over time. Moreover its volatility increases over time. Its range is between the minimum value of -3'773 and the maximum of 3'388. At the end of the simulation on average the imitative agents were losing 318 each.

In the population the number of leaders is not constant over time but the leader ratio fluctuates between the 10% and the 15% with a peak of 16 leaders out of 100 agents.

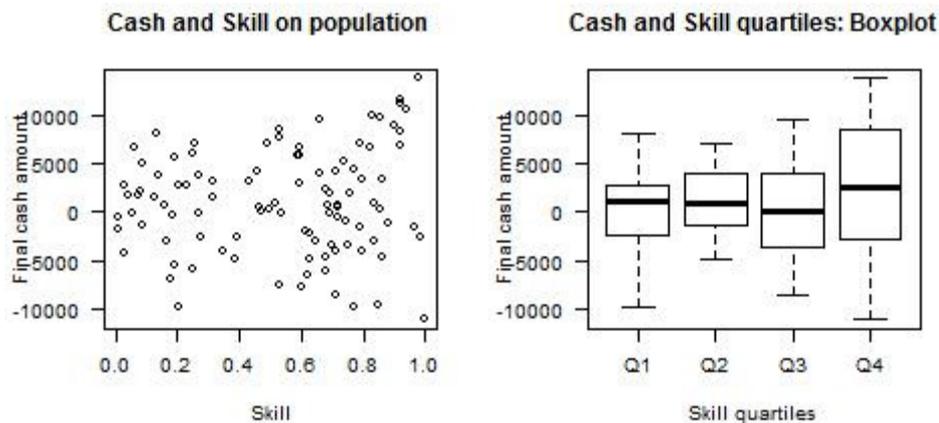


In order to have an intuitive idea on the differences in skill and final cash among leaders and their followers, the following box plots come at handy.



First of all we may notice that leaders' cash is always positive. This does not surprise us that much since as we already said the leaders are selected on cash basis. Instead it is interesting to underline that while followers' median values are almost stable among the quartiles then for leaders higher skill leads to higher returns in median terms. This effect is interesting since Q4 leaders are the ones who do not imitate and this choice seems to be rewarded with higher median cash value.

Performances might be influenced by skill values in the overall population of traders. The following charts provide an intuition whether this is true or not.



The part of population with a higher skill value than the threshold corresponds to the fourth quartile and displays a slightly higher median cash amount compared to others. This is an interesting fact since the first second and third (Q1, Q2 and Q3) quartiles were the ones who during the simulation are likely to follow the imitation strategy. Moreover a similar pattern has been found between skill and cash in the leaders' subset.

Cash and skill in the population do not seem correlated with a light positive Pearson correlation factor of 0.090. However the correlation is slightly greater for the Q4 agents, 0.150, that represent the subset of non-imitators. So far it seems that avoiding imitative behaviors is sufficient to reduce losses compared to imitative agents.

To check these intuitions more precisely we undertook a t-test on the mean cash endowment among two subsets: the lowest skilled agents (Q1) and the highest skilled agents (Q4). The null H_0 is that the mean cash for Q4 is statistically greater than that for Q1.

Welch Two Sample t-test

data: cash Q4 Skill and Q1 Skill

T test statistic value = 1.6183014059318308

Degree of freedom = 41.57471153901485

P-Value = 0.05658022709498377

Mean estimates for set2 and set1 [3118.7703393338447 481.5206732620963]

Alternative H_P greater

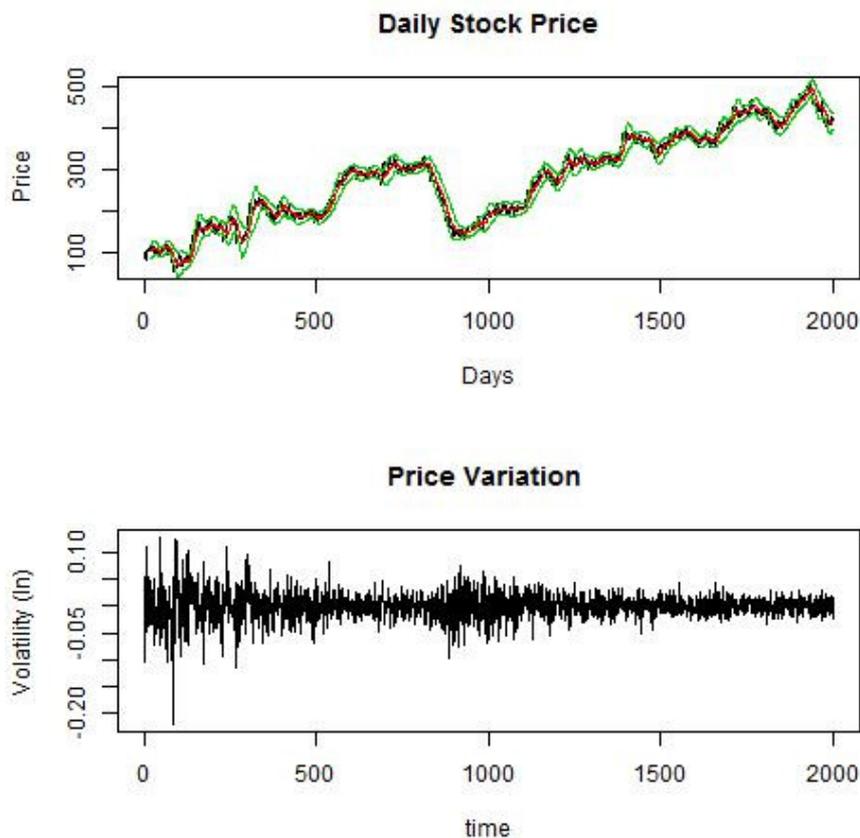
Considering an alpha significance level equal to 0.10 we can here reject the null H_0 and accept that the most skilled agents with a mean cash endowment of 3118 display a significant statistically greater amount than the lowest part of the population (481). This result does not hold for lower values of alpha such as 0.05 or 0.01.

5.2.2 Second scenario: the agent – fish schooling

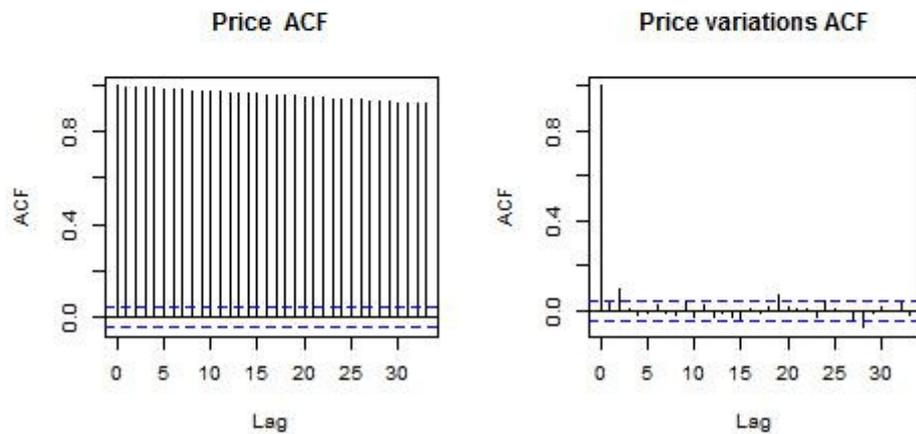
In this simulation we still investigate how Imitative behavior affects the agents' performances as well as the stock market under different assumptions. Here we want to drive the imitative behavior up to its extreme when agents are almost certainly following their leaders. At the same time there are much more investors in the markets. Therefore compared to the previous case we set here:

- Imit_sensibility to its maximum value of 10;
- Total number of agents equals to 400, fourfold the default number.

The price movement has a minimum of 55, a maximum of 507 reaching the final level of 425 while displaying high volatility in the first 100 days and when price fall and re-started increasing around the 1000th day.



Consider now the information ACF correlograms and the Augmented Dickey-Fuller Test provide about the price series.



The price seems to follow a random walk process while the price variations ACF suggests that residuals are a white noise. In particular the price ACF displays a decreasing positive autocorrelation for price as the lags increase. On the other hand the daily price differences display no relevant autocorrelation but for lags zero and slightly positive for lags two and nineteen. Additional information can be obtained from the Augmented Dickey fuller Test.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = -13.386865449961066

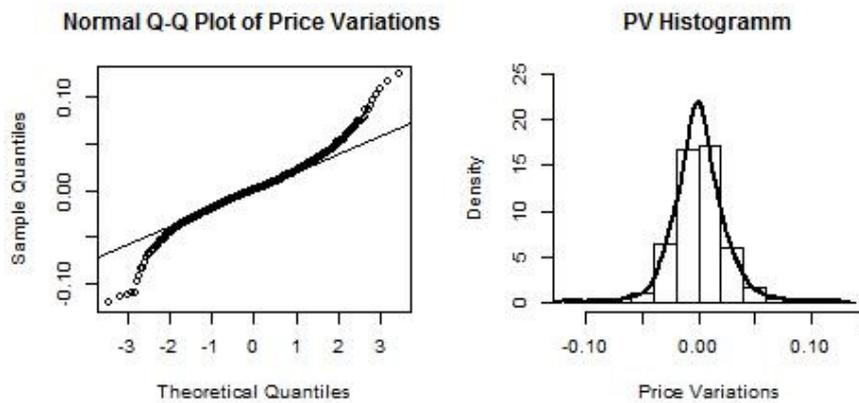
Lag order = 12

P-Value for the test = 0.01

Alternative HP stationary

The ADF test tells us that since p-value is less than the significant value alpha 0.05 the alternative H_p on stationarity is here accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the relative Q-Q plot and histogram.



Even if it seems that Price variations are far from a normality distribution before to state any conclusion further investigation is needed:

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.8699033591671964
 Approximate p-value for the test = 6.645356012671341E-38

Jarque-Bera Test on price variations

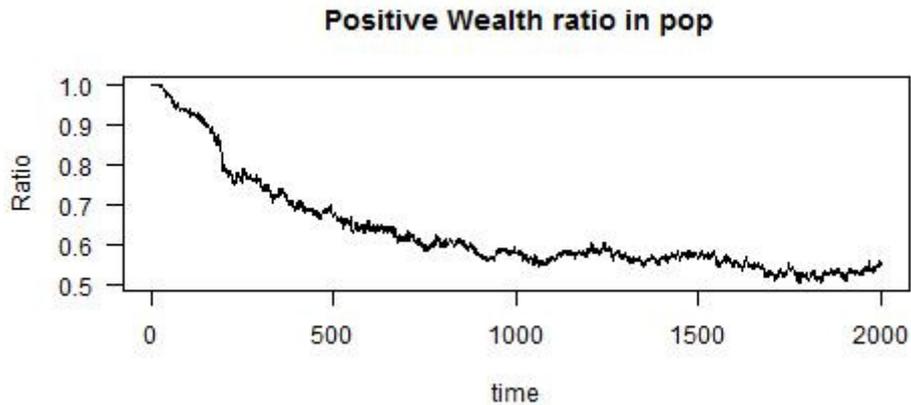
Value of the Jarque-Bera test statistic = 19615.316847764276
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05. The Shapiro-Wilk statistic value here is equal to 0.86 displaying a less normality behaviour than both standard Imitative scenario and ZI one. The p-value here is less than alpha according to which the null hypothesis is rejected and then the data tested are not from a normally distributed population.

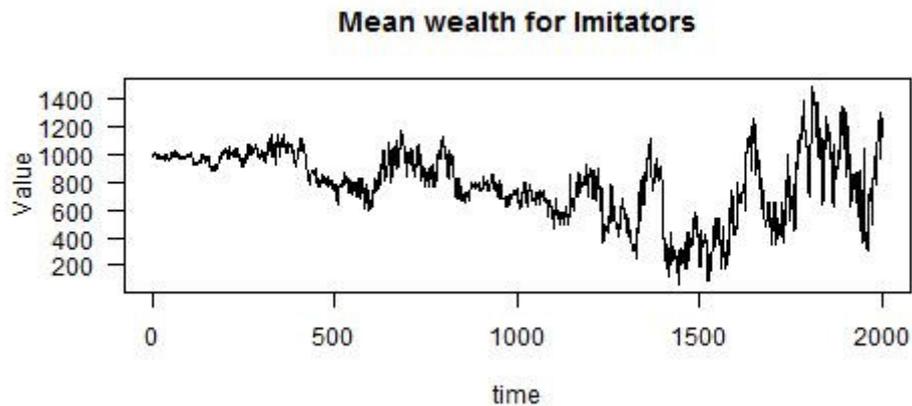
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both test suggest that price variations do not replicate a Normal distribution.

According to what we saw on price and its variations the underlying process for prices here is a random walk whose residuals do not follow a Normal distribution.

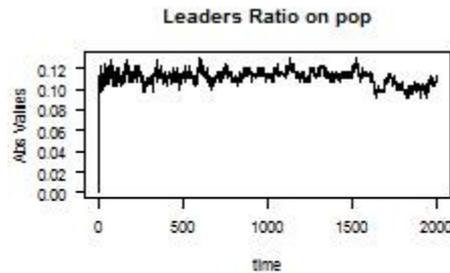
At micro-agent level, similarly the amount of positive cash in the population has the same trend as previous cases. In fact we face a final amount of 0.548 with a minimum of 0.502.



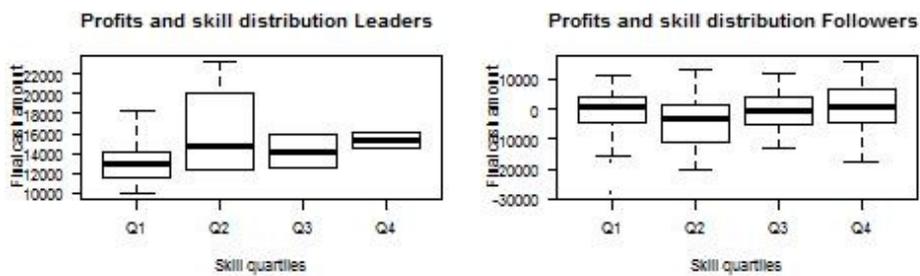
The performance of this strategy is still highly volatile. In particular it seems to be more and more volatile as the time goes. Its range is between a minimum value of 61 and a maximum of 1494. At the end of the simulation on average the imitative agents owned 1228.



Despite both number of agents and the Imitation sensibility are higher in the simulation the Leader ratio on the population decreased slightly. It has a peak of 0.13 before settling at 0.11 at the end of the simulation:

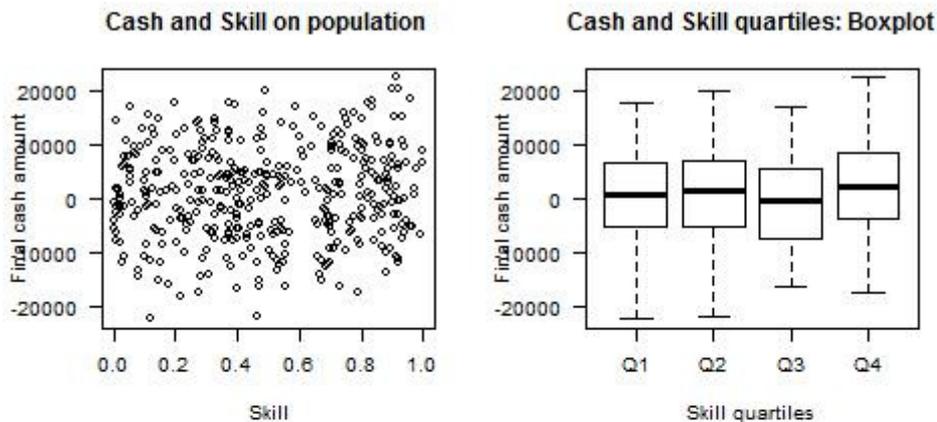


To compare in detail the differences in skill and final cash amount between leaders and their followers, the following box plot comes at handy



First of all we may notice that Leaders' median cash is always positive. This does not surprise us that much since as we already said the leaders are selected on cash basis. Instead it is interesting to see that while followers median values are almost stable among the quartiles, for the leaders the higher the skill the higher the final cash. This trend has also been noticed in the previous classical scenario for Imitation and it suggests that avoiding imitation improve agent's returns.

Skill and final cash distributions among the population can be graphically described as:



At first glance the greater number of agents is easily recognizable by the greater number of dots in the scatter plot. Higher skill values seem to have a slightly positive effect on the final median cash amount. However this effect is almost null since the correlation between cash and skill is 0.0648.

From the box plot instead it is worth to notice that the Q4 median is higher than the other ones. This is an interesting fact since Q4 corresponds to the population not joining the Imitation behavior actively. Additionally the same pattern has been found for cash and skill in the leader's subset in both the two scenarios for imitation.

To check whether or not being imitative is detrimental to the final cash position we performed a two side t-test on the sample Q1 and Q4 and their means.

T test statistic value = 1.6387821190297385

Degree of freedom = 197.6488253621186

P-Value = 0.05142473166002558

Mean estimates for set2 and set1 [2536.059080254729 586.3988796513754]

Alternative HP: two.sided

Considering an alpha significance level equal to 0.10 it is possible to reject the null H_0 and accept the alternative H_1 according to which the final cash mean for Q4 (2536) is statistically greater than the Q1 ones (586).

It is worth to notice that this result has been also found in the standard Imitative simulation.

5.3 Gambler's Fallacy

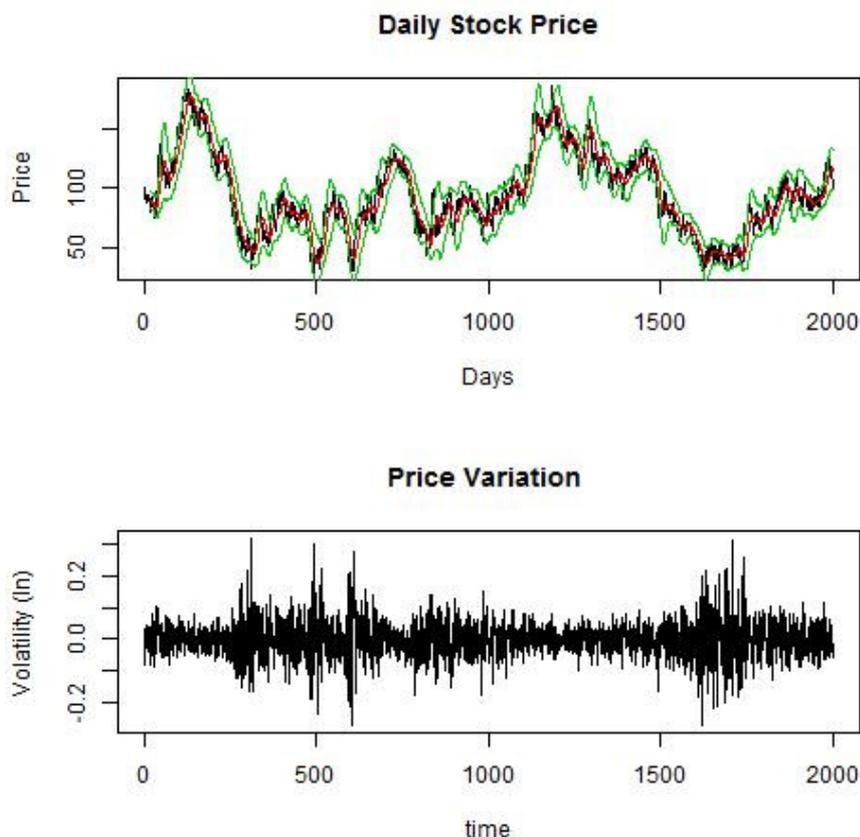
5.3.1 First scenario: standard case

This simulation moves its step from the ZI one by simply adding the gambler's fallacy (or Montecarlo's fallacy) at agents disposal. In section 4.3.3.3 it has been explained the rules that regulate this procedure while the whole code behind it can be found in Appendix D. 4.

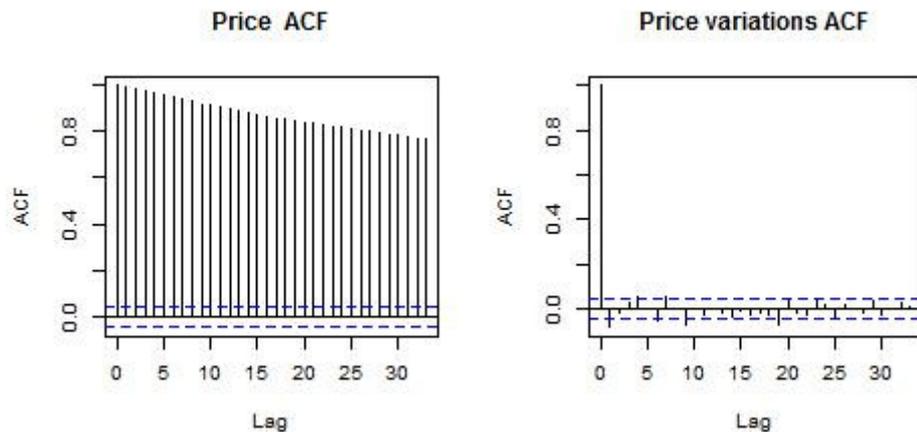
Starting from the previous ZI simulation the Gambler's_fallacy (GF) option is switched on and the following values are imposed on its relative parameters:

- GF_impact : 30
- GF_sensibility: 3

It is worth to notice that since GF is the only strategy available, modifying GF_impact does not have a real effect on agents' decisions here. Conversely the higher the GF_sensibility the greater is the chance, once the market conditions are satisfied, that this strategy will be performed. It is worth to remember that agent's skill has an impact on whether or not following this strategy according to the table in section 4.3.3. We present now the case where skill threshold is imposed on the default value (0.75) while different scenarios are going to be proposed later on. With the selected value of skill's threshold only the lowest skilled part of the population will be tempted by the gambler's fallacy. The history of price displays a maximum value at 188 and a minimum of 28 before reaching the final value of 99. Additionally the volatility is higher whenever the price falls too low.



Consider now the information ACF correlograms and the Augmented Dickey-Fuller Test provide about the price series.



On the left it is plotted the ACF for daily prices while on the right the ACF for daily price variations. As we can see, daily prices follow a random walk process.

On the other side, the ACF for price variations displays white noise behavior with some slight negative correlations for lags one, nine e nineteen.

From the ACF correlograms we can notice how the stochastic component dominates the price variations residuals in favour of a random walk process for price and of a white noise process for price variations.

In order to provide more information we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = -12.99386590757982

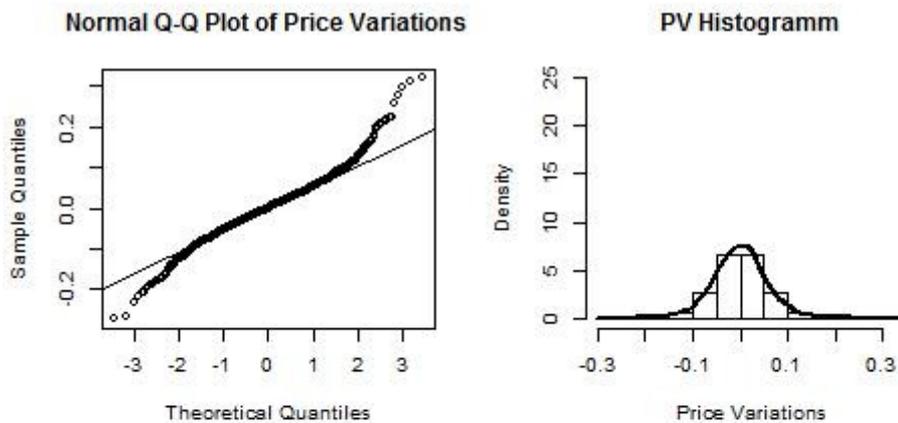
Lag order = 12

P-Value for the test = 0.01

Alternative HP stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the relative Q-Q plot and histogram.



The q-q plot seems pretty similar to previous cases while the fitted density function is far shorter than before. This is due because central values for variations were less present while extreme ones have occurred more often.

Before to state any conclusion we need to have more reliable information through both Shapiro-Wilk and Jarque-Bera tests.

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.9754652584989958
 Approximate p-value for the test = 5.046087660778307E-18

Jarque-Bera Test on price variations

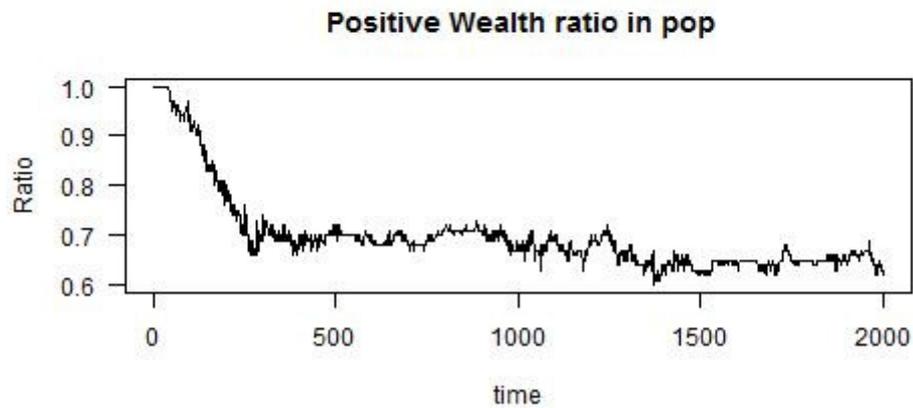
Value of the Jarque-Bera test statistic = 502.0923549969363
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05. The Shapiro-Wilk statistic value here is equal to 0.97 being closer to a Normal distribution than both standard Imitative scenario and ZI one. The p-value here is less than alpha according to which the null hypothesis is rejected and then the data tested are not from a normally distributed population.

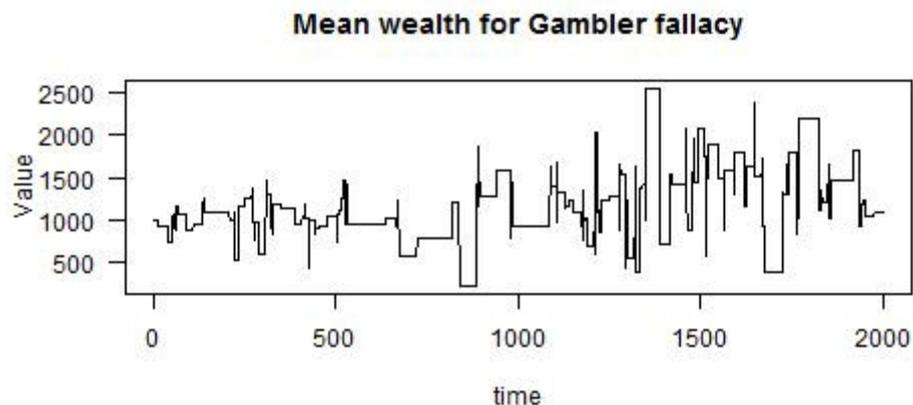
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both tests suggest us that the price variations do not replicate a Normal distribution

According to what we saw on price and its variations the underlying process here is a random walk whose residuals do not follow a Normal distribution.

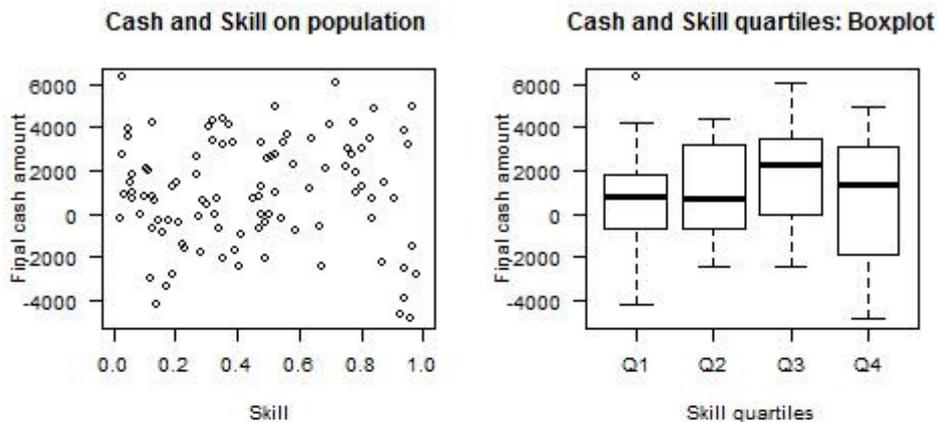
Similarly to previous cases the amount of positive cash in the population has the same trend. Compared to the ZI case the ratio of positive cash in the population is slightly higher. In fact we face a final amount of 0.62 with a minimum of 0.6 compared to the minimum value of 0.48 and the final value of 0.49 of ZI simulation.



The performance of this strategy is highly volatile as the imitative one. Its volatility seems also to increase over time as for Imitation. Its range is between a minimum value of 227 and a maximum of 2'505. At the end of the simulation on average the imitative agents owned 1'101 not that far from the initial endowment of 1'000.



Skill and final cash distributions among the population can be graphically described as:



The skill and cash relation does not seem strong in this case confirmed also by the almost null positive correlation value of 0.0327. Q1 corresponds to the subset of agents likely to follow the Gambler's Fallacy and despite the fact that the richest agent belongs to Q1 from the chart it does not seem that this strategy leads agents to outperform compared to the others. On the other hand Q2, Q3 and Q4, by simply choosing randomly how to trade in the market display similar or better returns.

To check whether it exists or not a statistically difference in the final cash amount among Q1 and Q4 we performed the following two side t-test.

T test statistic value = 0.23335089665644212

Degree of freedom = 44.55874688929165

P-Value =0.8165576421119787

Mean estimates for set2 and set1 [1056.248841023445 873.8330538744511]

Alternative HP: two.sided

Although Q1 has a mean final cash of 873 and Q4 of 1056 we cannot accept the alternative H_p of a statistical meaningful difference among the two samples. For any alpha value the null hypothesis cannot be rejected.

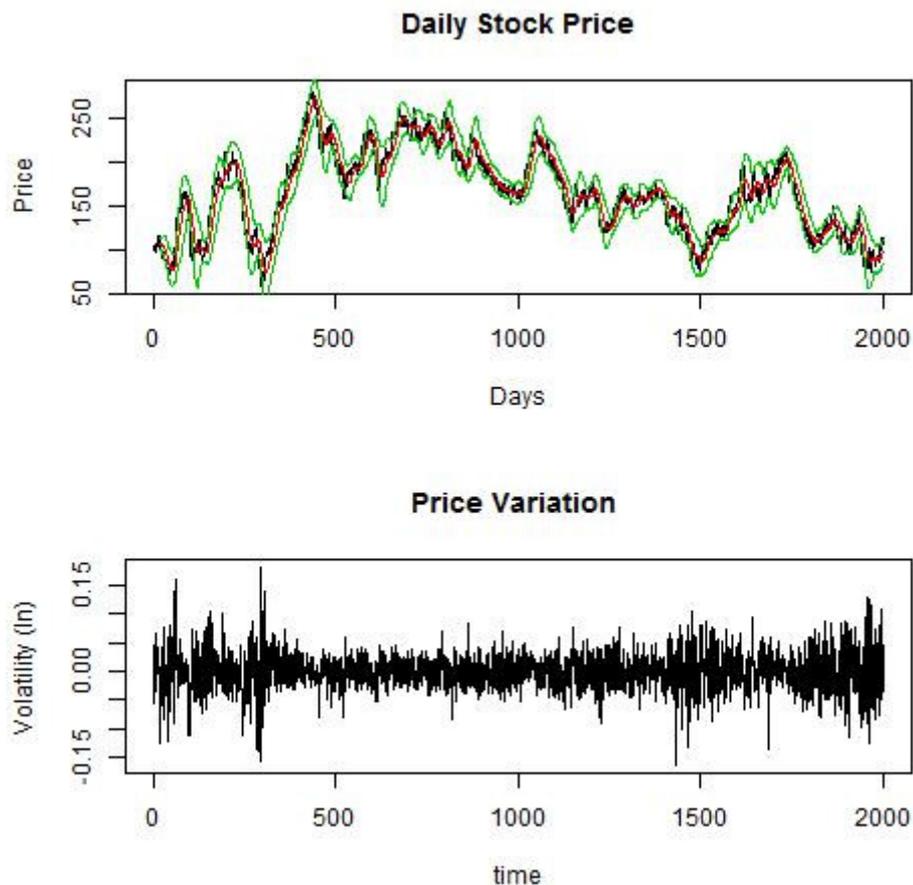
5.3.2 Second scenario: high gambling temptation

In the following simulation we still investigate how Gambler's fallacy affects the agents' performances and the stock market under different assumptions. The Gambler's fallacy behavior is implemented at its extreme conditions leading agents to follow such strategy whenever the market conditions on price are met. Meanwhile a greater number of traders are now present in the market compared to the previous scenario.

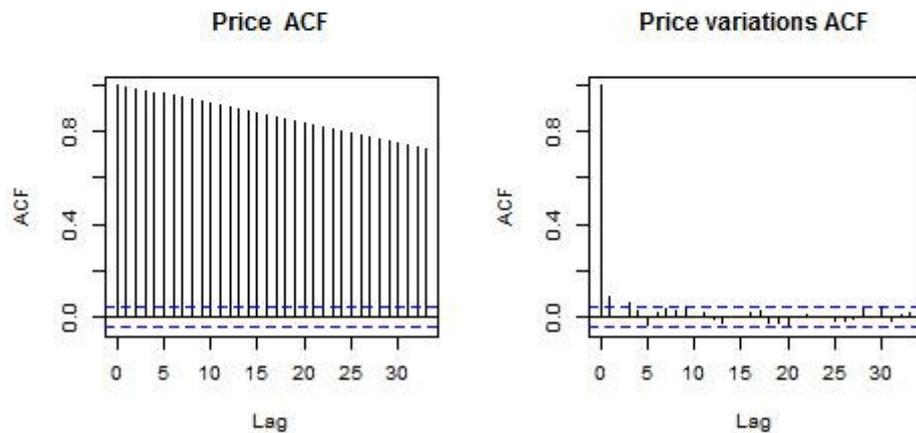
This different scenario has the same variables values as the previous one but the following one:

- GF_sensibility is at maximum value of 10;
- The total number of agents is 400, fourfold the default number.

The price movement has a minimum of 58, a maximum of 285 reaching the final level of 112 while displaying similar volatility levels as the previous cases. In particular when prices fall and raise abruptly volatility raises.



Consider now the information ACF correlograms and the Augmented Dickey-Fuller Test provide about the price series.



On the left it is plotted the ACF of the daily prices while on the right the same function is plotted for the daily price variations. As we can see, daily prices follow a random walk process.

Differently the ACF for price variations displays a white noise process without any correlation for lags greater than zero, except a slight positive correlation for lag one and three.

From the ACF correlogram we can see how the stochastic component dominates the price variations residuals in favour of a random walk process for price and of a white noise process for price variations.

In order to provide more information we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = -11.980190742357083

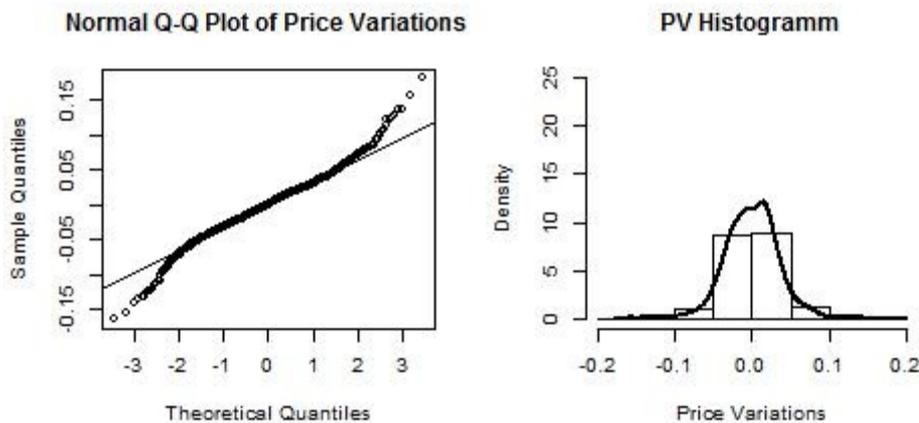
Lag order = 12

P-Value for the test = 0.01

Alternative HP stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the relative Q-Q plot and histogram.



The left Q-Q plot suggests us that price variations are not generated from a normal distribution process. On the other hand the fitted density function could be at first glance confused with a Gaussian function.

Therefore before to state any conclusion further investigation is needed:

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.9826218005902557
 Approximate p-value for the test = 7.040177358569722E-15

Jarque-Bera Test on price variations

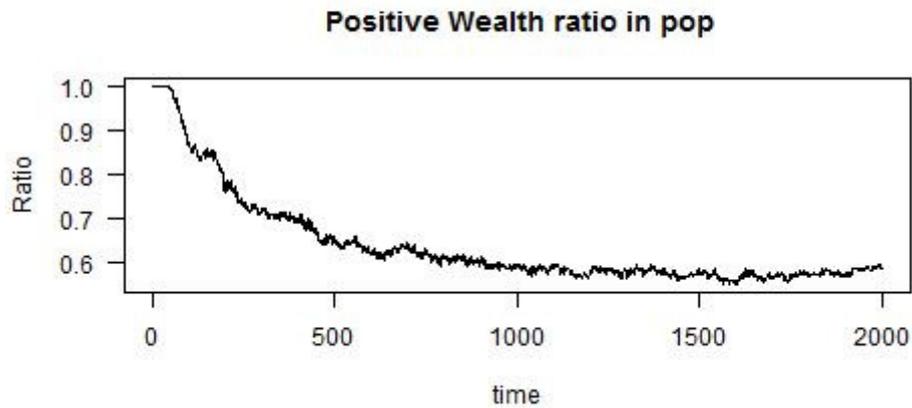
Value of the Jarque-Bera test statistic = 300.2831010432701
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05 The Shapiro-Wilk statistic value here is equal to 0.98 being closer to a Normal distribution than all previous cases. The p-value here is less than alpha according to which the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

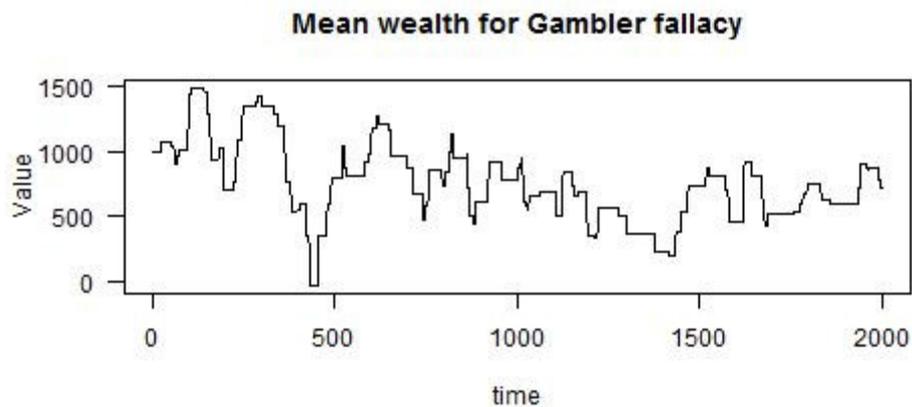
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both tests suggest us that the price variations do not replicate a Normal distribution according to our graphical intuition.

According to what we saw on price and its variations the underlying process here is a random walk whose residuals do not follow a Normal distribution.

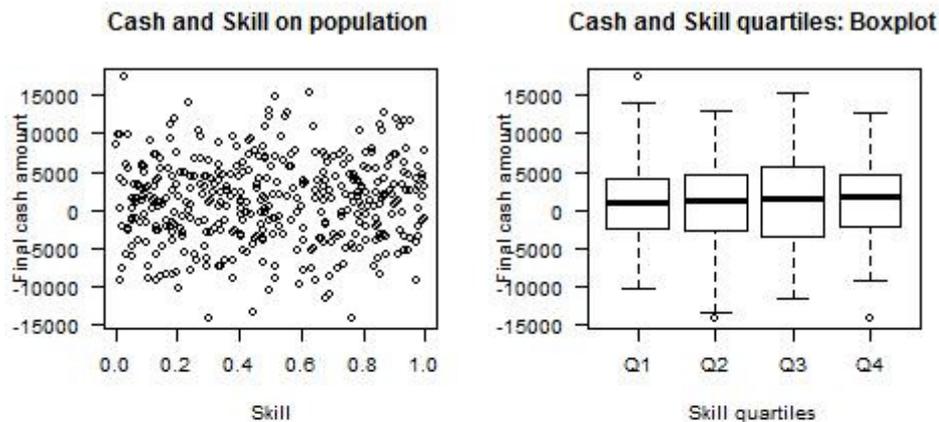
Similarly to previous cases the amount of positive cash in the population has the same trend. Every agent started with a positive amount of cash but the ratio of population with positive cash starts to drop as the simulation runs. In particular it drops significantly in the first 500 days touching a minimum value of 0.54 and its final value at the end of the simulation is 0.58.



The performance of this strategy is highly volatile with volatility increasing over time. Its range is between a minimum value of -29 and a maximum of 1'490. At the end of the simulation on average the imitative agents owned 703 that is less than the initial endowment of 1'000. Higher values of GF_sensibility led to a more repeated use of this strategy from the agents. In fact the mean wealth in this second scenario experiences more changes than previous case since agents are more willing to apply the fallacy in their decisions.



Skill and final cash distributions among the population can be graphically described as:



At first glance the greater number of agents is easily recognizable by the greater number of dots in the scatter plot. The skill and cash relation does not seem strong in this case as confirmed by the slightly positive correlation value of 0.046. The Q1 corresponds to the subset of agents likely to follow the Gambler’s Fallacy and despite the fact that the richest agent belong to Q1, from the chart it does not seem that this strategy lead agents to outperform compared to the others. In this case Q2, Q3 and Q4 by simply choosing randomly how to perform in the market display similar median values.

To check whether it exists or not a statistically difference in the final cash amount among Q1 and Q4 we performed the following two side t-test.

T test statistic value = 1.0821597074108191
 Degree of freedom = 197.5613008278342
 P-Value = 0.28050065363487475
 Mean estimates for set2 and set1 [1436.778264765435 626.2219709310474]
 Alternative HP: two.sided

Although Q1 has a mean final cash of 626 and Q4 of 1436, we cannot accept the alternative Hp of a statistical meaningful difference among the two samples then accepting the null hypothesis. We found the same result on the previous Gambler’s Fallacy simulation scenario.

5.4 Technical strategy

5.4.3 First scenario: standard case

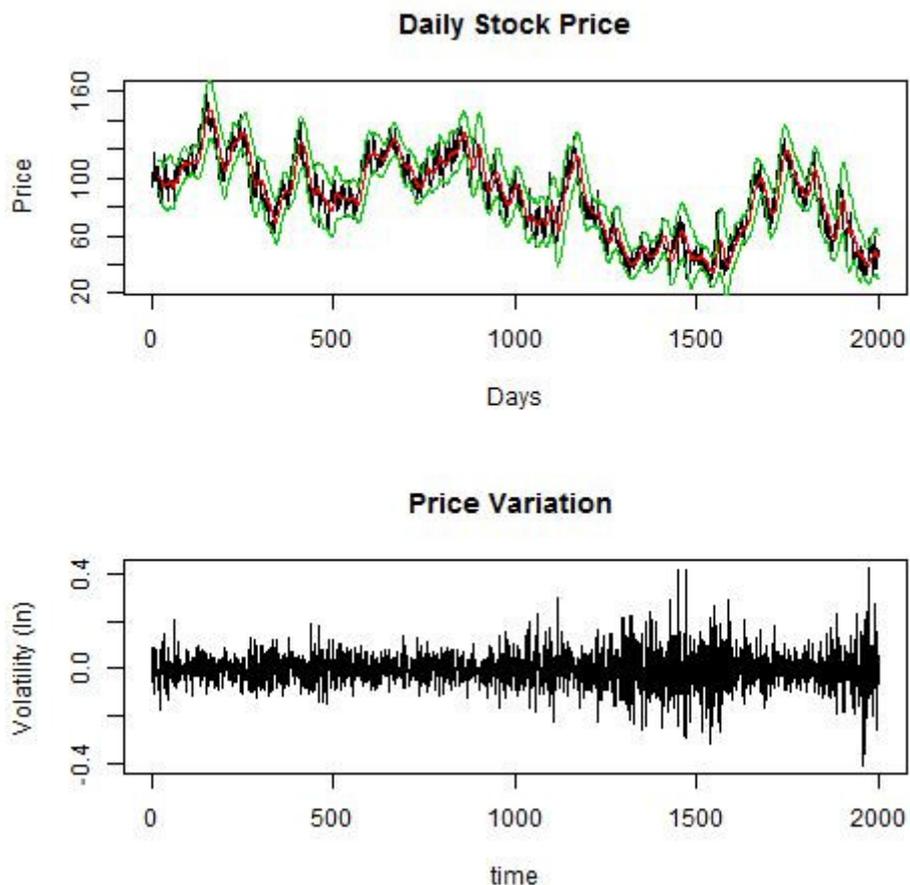
This simulation moves its step from the ZI version by simply adding the Bollinger's Band strategy at agents disposal. The way this is performed by agents has been described in section 4.3.3.2. while the code behind it can be found in Appendix D. 5.

Starting from the ZI simulation variable values BBStrategy option is switched on and the following value is imposed on its relative parameter

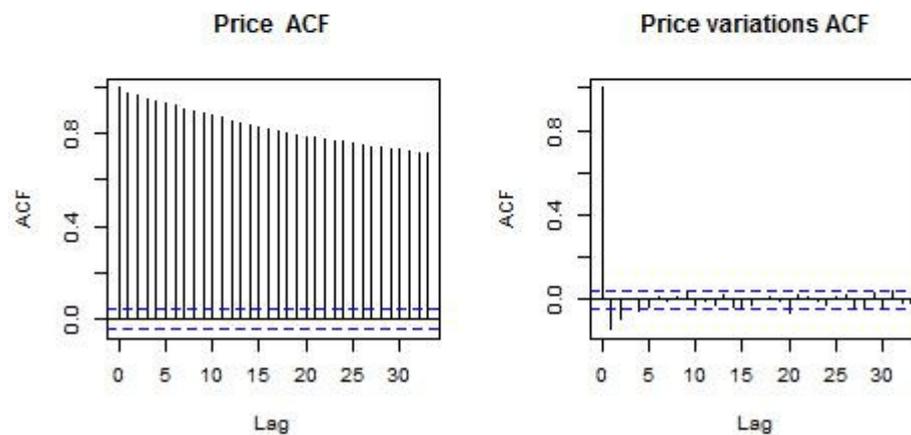
- BB_impact: 30

It is worth to notice that since this is the only strategy available to agents modifying BB_impact does not have a real effect on agents' decisions here. Instead agent's skill has an impact by determining which agents apply this strategy according to the table in section 4.3.3. It is important to remember that since skill threshold is imposed on its default value (0.75) then only the ones with greater skill than 0.75 can rely on the technical strategy.

The price movement has a minimum at 24, a maximum of 162 reaching the final level of 46 while displaying similar volatility levels as the previous cases. In particular when prices fall and then raise abruptly volatility raises.



Consider now the information ACF correlograms and the Augmented Dickey-Fuller Test provide about the price series.



On the left it is plotted the ACF of the daily prices while on the right the same function but for the daily price variations. As we can see, daily prices follow a random walk process.

On the other side, the ACF for price variations is a white noise process without any correlation for lags greater than zero displaying a slight negative correlation at lags one and two.

From the ACF correlogram the stochastic component dominates the price variations residuals in favour of a random walk process for price and of a white noise process for price variations.

In order to provide more information we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = -17.384539218397258

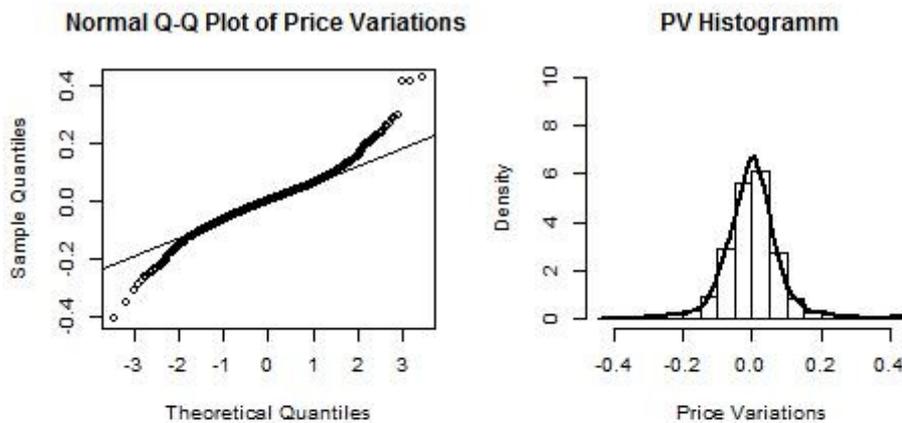
Lag order = 12

P-Value for the test = 0.01

Alternative HP stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the relative Q-Q plot and histogram.



The left Q-Q plot suggests us that price variations are not generated from a normal distribution process. Similarly the fitted density function seems at first glance too sharp to be a Gaussian. However before to state any conclusion further investigation is needed as provided by following tests.

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.9824576401164332
 Approximate p-value for the test = 5.837436986509132E-15

Jarque-Bera Test on price variations

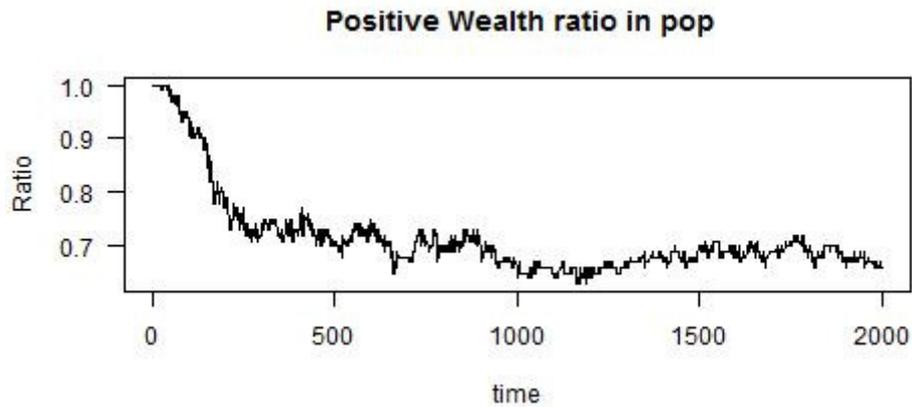
Value of the Jarque-Bera test statistic = 367.92303499268854
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05 The Shapiro-Wilk statistic value here is equal to 0.98 being really close to a Normal distribution. The p-value here is less than alpha according to which the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

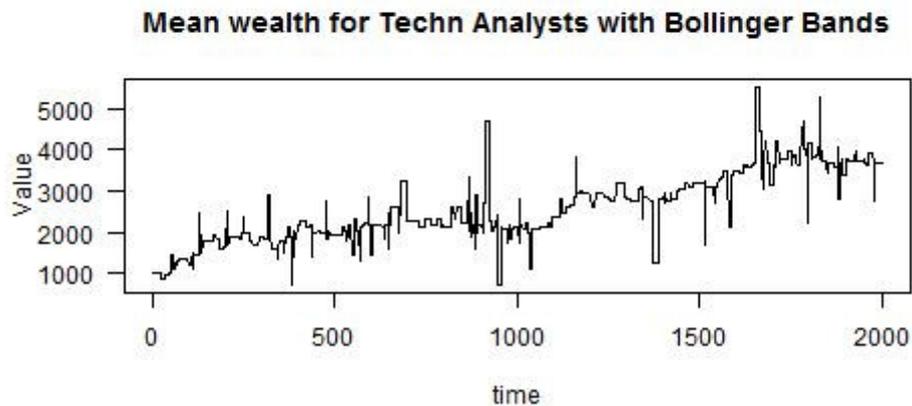
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both tests suggest us that the price variations do not replicate a Normal distribution according to our graphical intuition.

According to what we saw on price and its variations the underlying process here is a random walk whose residuals do not follow a Normal distribution.

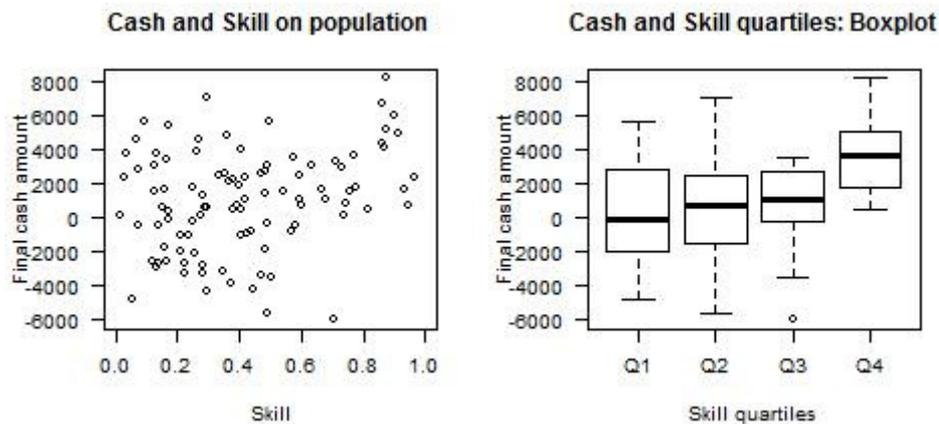
Similarly to previous cases the amount of positive cash in the population has the same trend. Every agent started with a positive amount of cash but the ratio of population with positive wealth starts to drop as the simulation runs. In particular it drops significantly in the first 500 days touching a minimum value of 0.63 and the final value at the end of the simulation is 0.66.



The performance of this strategy in terms of returns display a positive sloped trend over time. It has a minimum value of 719 and a maximum of 5534. At the end of the simulation on average the imitative agents owned 3681 almost three times the initial endowment of 1000.



On the agent side instead we can check how skill affects the final amount of cash owned by agents. In statistical terms we face a positive Pearson correlation between the skill and cash vectors on the population (0.235), this correlation increases for the fourth quartile of skill population (0.391). Graphically we can investigate the cash-skill relationship by looking at the following scatter plot and box plot:



These two plots show some interesting features. The Q4 subset includes those agents with skill higher than 0.75 and who are likely to perform the technical strategy. In the scatter plot these agents have positive or null cash amounts at the end of the simulation. Additionally in the box plot the Q4 median is considerably higher than the others.

To check whether it exists or not a statistically difference in the final cash amount among Q1 and Q4 we performed the following two side t-test.

T test statistic value = 2.2286944592823565

Degree of freedom = 47.92536589232318

P-Value = 0.015279694148251765

Mean estimates for set2 and set1 [2538.7407519835724 754.1702573282358]

Alternative HP: two.sided

Considering an alpha significance level (α) equal to 0.05, it is possible to reject the null H_0 and accept the alternative H_1 according to which the final cash mean for Q4 (2538) is statistically greater than the Q1 ones (754).

It is worth to notice that this result has been also found in both the Imitative simulations but with a less significance alpha level.

5.5 Complete system

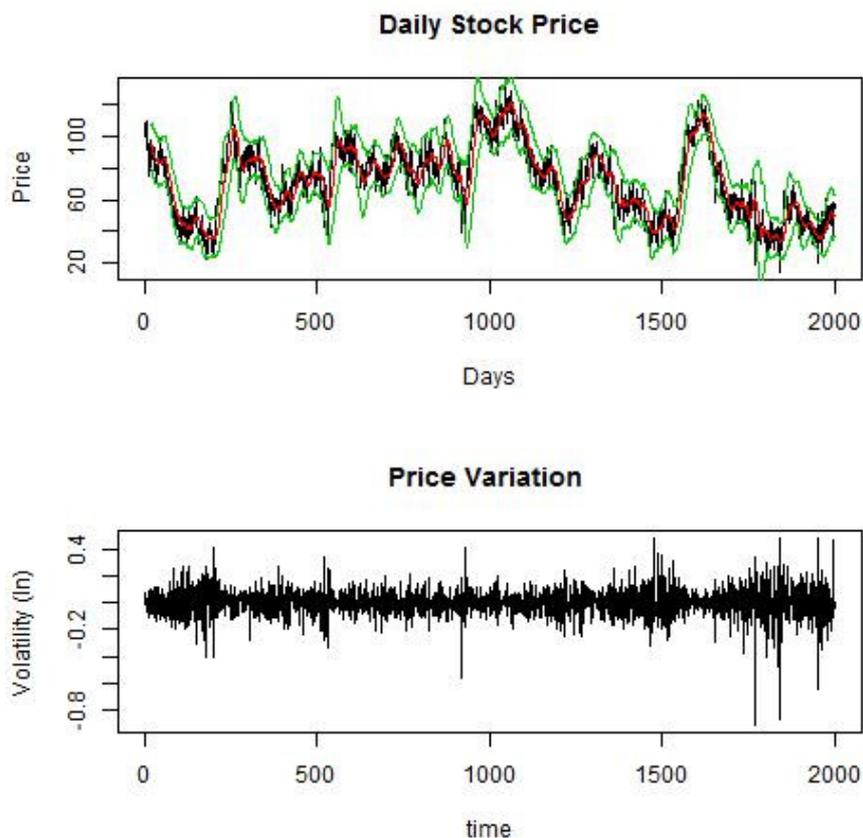
5.5.1 First scenario: standard case

In this simulation we allow the agents to apply all the strategies explained so far. They then can place their orders according to an imitative choice, to a technical strategy or to a fallacy. For a quick review on the conditions that trigger each strategy and on how the skill value affects the choices the table on section 4.3.3 summarizes them all.

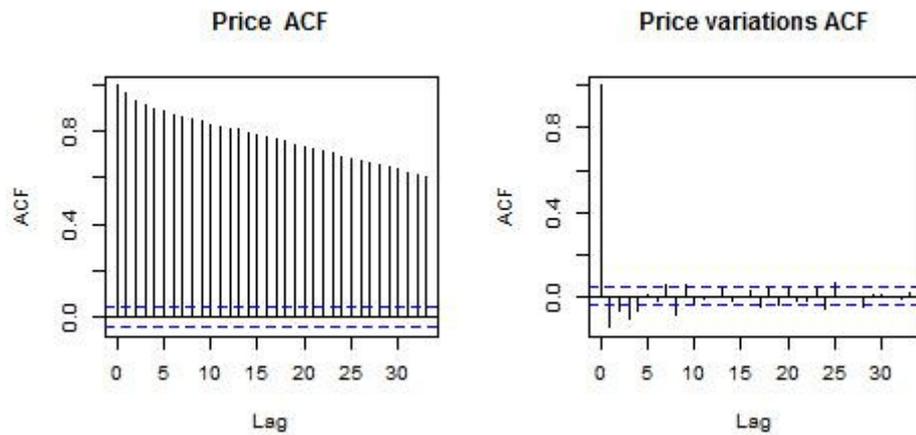
Additionally in this simulation the prospect theory has been switched on. Its theoretical foundations have been broadly discussed in chapter 1 while the rules behind it and the code can be respectively found on section 4.3.4 and Appendix D.6 .The settings under which this simulation is performed are :

- Imitative strategy : on
 - Imit_impact 30
 - Imit_sensibility 3
- BB_strategy: on
 - BB_impact 30
- Gambler's_fallacy: on
 - GF_impact 30
 - GF_sensibility 3
- Prospect_theory: on

The stock price has a peak at 133 and a trough at 14 and it is 55 at the end of simulation, while facing high volatility especially during the first part of the simulation and some peaks when price abruptly falls.



Consider now the information ACF correlograms and the Augmented Dickey-Fuller Test provide about the price series.



On the left it is plotted the ACF of the daily prices while on the right the same function is plotted for daily price variations. As we can see, daily prices follow a random walk process.

On the other side, the ACF for price variations is a white noise process without any correlation for lags greater than zero but with some slight negative correlations at lags one, two, three, four and eight.

ACF correlograms suggest that the stochastic component dominates price variations in favour of a random walk process for price and of a white noise process for price variations.

In order to provide more information we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = -15.07596802839543

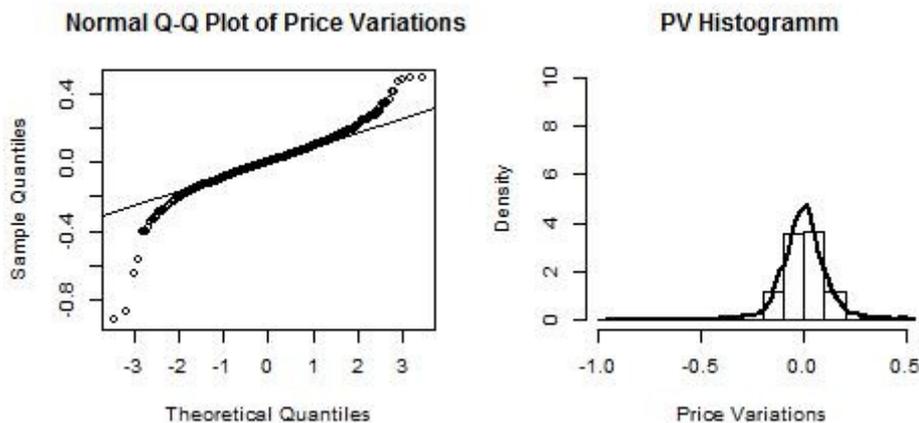
Lag order = 12

P-Value for the test = 0.01

Alternative HP stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the relative Q-Q plot and histogram.



Graphically both charts would suggest that price variations do not follow a Gaussian distribution. In order to be thorough further investigation is needed and provided thanks to the following tests:

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.9452879582370467
 Approximate p-value for the test = 1.297325207382745E-26

Jarque-Bera Test on price variations

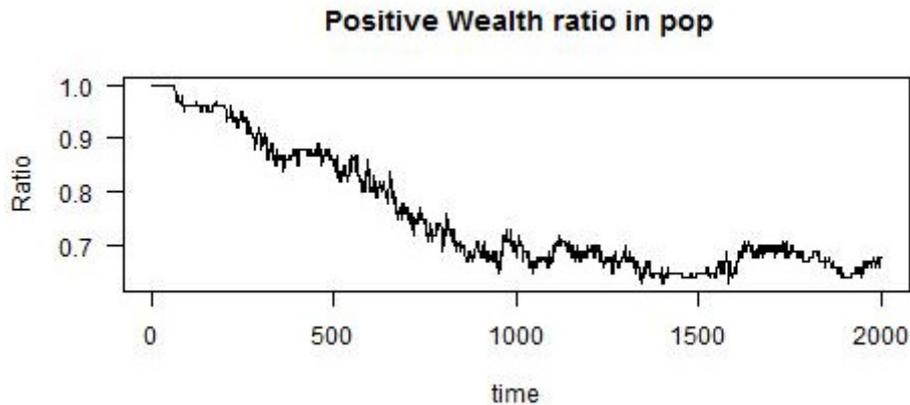
Value of the Jarque-Bera test statistic = 4515.526123730078
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05. The Shapiro-Wilk statistic value here is equal to 0.94 being sufficiently close to a Normal distribution. The p-value here is less than alpha according to which the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

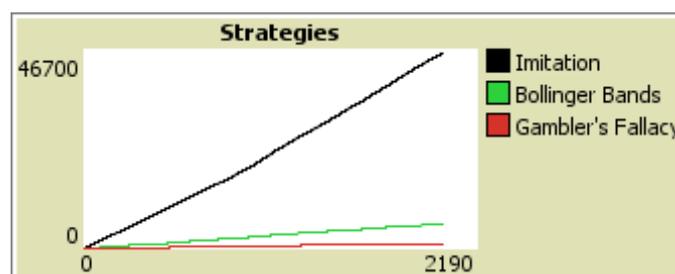
Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both tests suggest us that the price variations do not replicate a Normal distribution according to our graphical intuition.

According to what we saw on price and its variations the underlying process for price is a random walk whose residuals do not follow a Normal distribution.

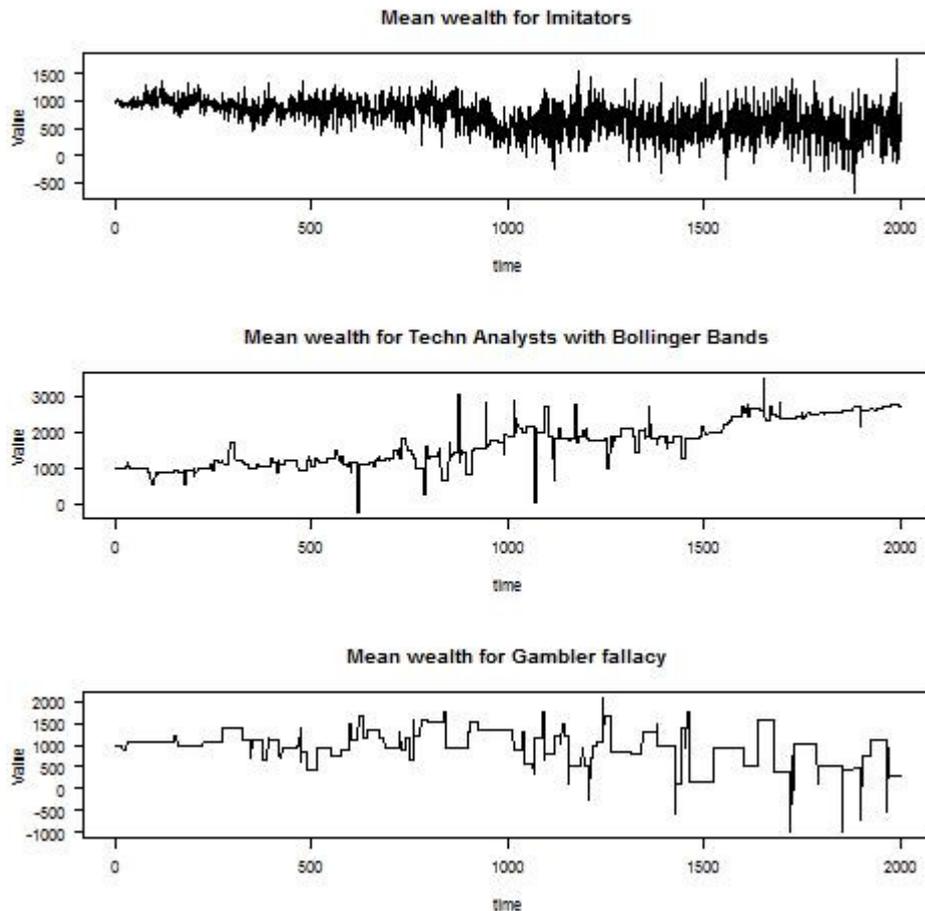
Similarly to previous cases the amount of positive cash in the population has the same trend. Every agent started with a positive amount of cash but the ratio of population with positive cash falls as the simulation runs. An interesting feature is that with more strategies available for agents the ratio of positive wealth does not drop abruptly within the 500 days as previously found. The wealth ratio has a minimum value of 0.63 and its final value at the end of the simulation is 0.68



In the following chart are illustrated the cumulative number of times each strategy is performed by all agents from the first trading day onward. The imitation strategy is the most applied and the reasons behind this are essentially two. First of all, this strategy does not depend on the market conditions to trigger it. It just needs agents to seek for the most profitable ones around and then they will imitate their leaders. Secondly under the above noted settings it is the strategy with the greatest number of agents willing to apply it.



Each strategies performance has its own trend and behaviour as illustrated in the following chart.

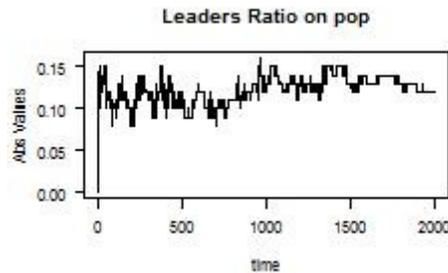


The main characteristics of each strategy’s performance are reported in the following table.

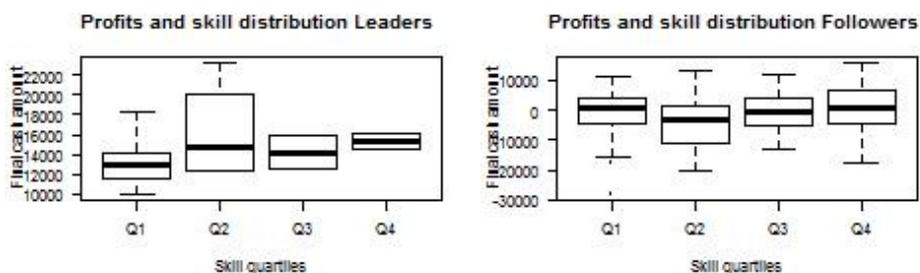
Strategy	Initial C	Min	Max	Final C
Imit	1.000	-703	1.761	783
BB	1.000	-269	3.481	2.687
GF	1.000	-1.001	2.068	274

Each of the three strategies displays consistent results with the previous cases. The imitative strategy keeps increasing its volatility over time with uncertain returns. The Gambler’s fallacy is the one that seems to be well-performing in the short run while leading to negative returns in the long-run. In fact it is the strategy with the worst values both for the minimum point and the final amount of cash. Conversely the technical strategy is the one with the lowest minimum and the highest maximum and it is the only one that leads to some gains for agents (+1687).

Does imitation change when it is not anymore the unique strategy available? Similarly to when it is the only strategy available, the number of leaders is not constant but the leader ratio fluctuates between the 10% and the 15% with a peak of 16 leaders out of 100 before falling at 12 at the end of the simulation.

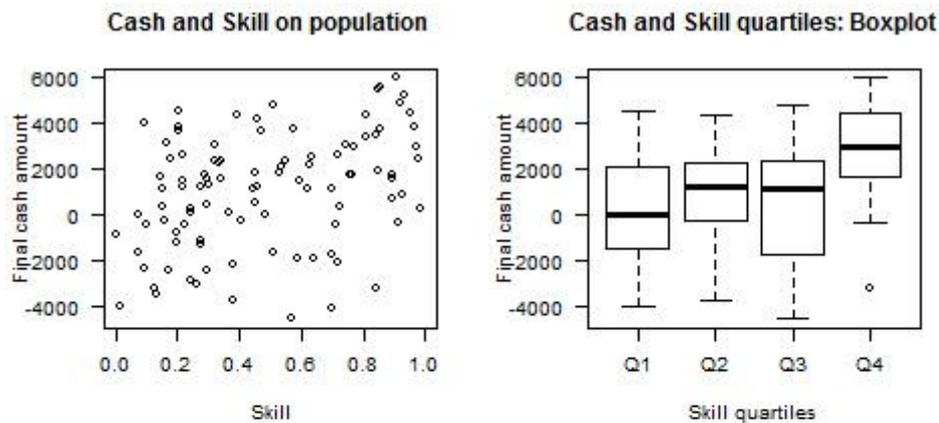


In order to have an intuitive idea on the differences in skill and final cash among leaders and their followers, the following box plots come at handy.



First of all we may notice that Leaders' cash is always positive. This does not surprise us that much since as we already said the leaders are selected on cash basis. Similarly to previous Imitative simulations for leaders the higher is the skill the higher is the final cash. This is particularly true for the Q4 subset that includes the leader whose skill value belongs to the first quartile of the population. The followers instead do not display the same trend for cash as the median values among the quartiles are similar.

On the agent side we'll investigate here how skill affects the final amount of cash owned by agents. In statistical terms we face a positive Pearson correlation between the skill and cash vectors on the population (0.356) and it is the highest positive correlation value obtained so far. Graphically the cash-skill relationship can be analyzed intuitively from the following scatter plot and box plot



Higher skill values have a positive effect on the final median cash amount consistent with the correlation factor. It is important to notice that the Q4 median is the highest one. At the same time Q1 agents display the lowest median cash. It is worth to remember that Q4 agents are the ones likely to apply the technical strategy, Q1 the ones applying the Gambler's fallacy while Q3, Q2 and Q1 can also decide to imitate. To check whether or not most skilled agents are statistically performing better than lowest skilled the following Welch's t-test has been undertaken:

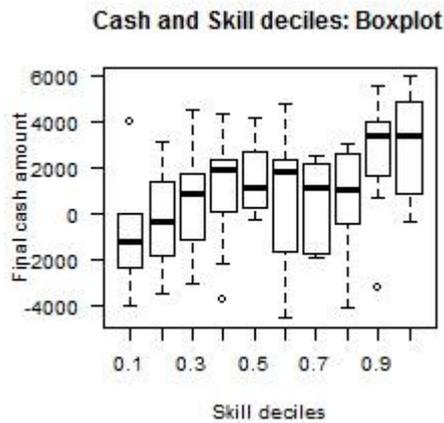
T test statistic value = 3.636714475557875
Degree of freedom = 46.93127260441102
P-Value = 3.422509196543893E-4
Mean estimates for set2 and set1 [2701.8698107923274 317.72773742946396]
Alternative HP: greater

For any significant level of alpha the null H_0 is rejected while accepting the alternative H_a according to which the final cash mean for Q4 (2702) is statistically greater than the Q1 ones (318).

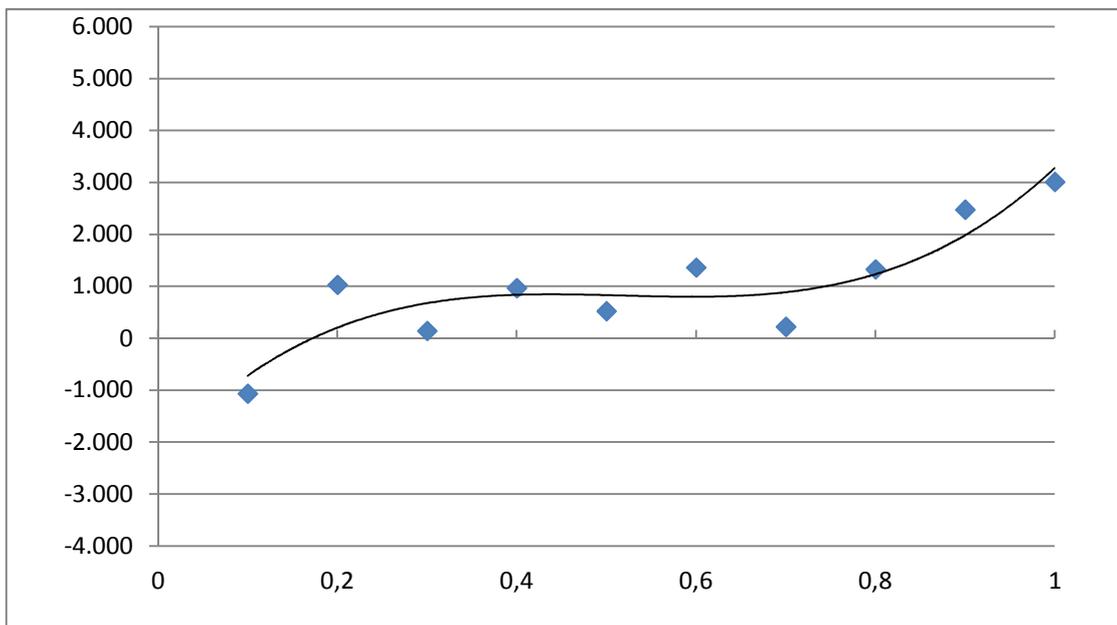
It is worth to notice that this result is the effect of jointed forces. In particular referring to the single cases previously analyzed:

- Imitative strategy has been seen to lead to a statistically higher mean value for Q4 versus Q1, as illustrated in section 5.2 .
- Gambler fallacy leads to low cash performances for those who apply it as illustrated in section 5.3.
- Technical strategy has been seen to lead to a statistically higher mean value for Q4 versus Q1 as illustrated in section 5.4.

To see more specifically how the final cash is influenced by skill the quartile box plot analysis has been extended thanks to the following decile box plot



The cash-skill relation do not have a linear shape. Using the mean data the skill cash relation based on deciles can be approximated by the following third order function.

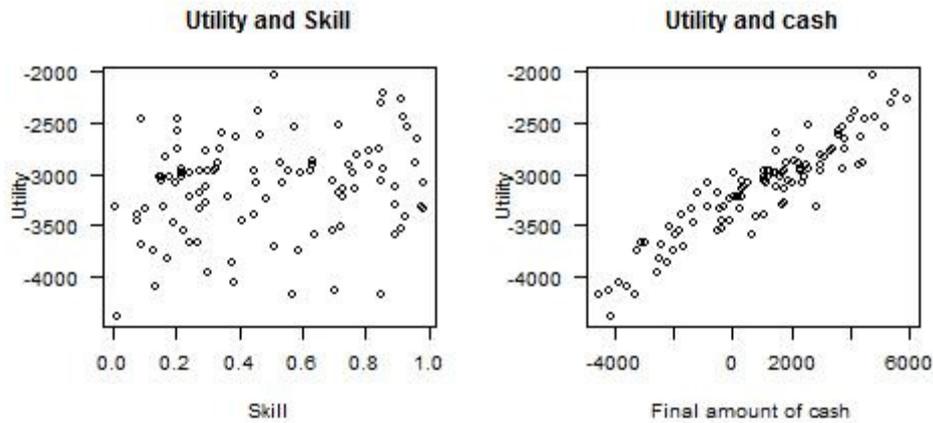


In particular the function has the following form and R^2 values. The latter suggests that the equation well fits the data.

$$y = 23784 x^3 - 36933 x^2 + 18671 x - 2244,5$$

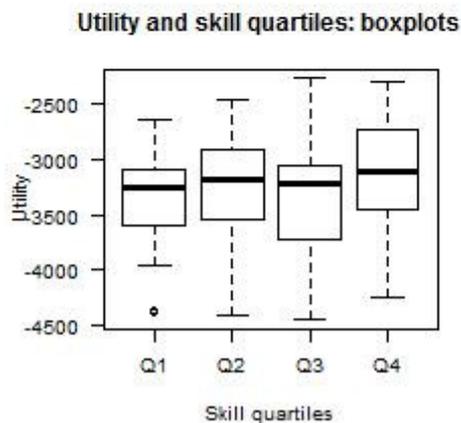
$$R^2 = 0.8147$$

The introduction of prospect theory adds to the model a descriptive utility function as described more properly in section 4.3.4. As we know the utility function is shaped according to the value function described by Kahneman and Tversky (1979, 1992).



As we can see, there is a strong positive correlation between utility and cash. This does not surprise us since the utility function is based on gains and losses of agents. The final utility values are all negatives because the losses have in absolute terms a greater effect on utility compared to gains. This is one of the main characteristic of the value function described in section 1.4.1.

Since skill has been demonstrated to have an impact on cash and knowing that utility depends on cash we would like to check whether or not utility is influenced by skill.



T test statistic value = 1.4246836057375516
 Degree of freedom = 41.785751368004085
 P-Value = 0.16167503434032546
 Mean estimates for set6 and set5
 [3238.4580747109535 -3393.121466195919]
 Alternative HP: two.sided

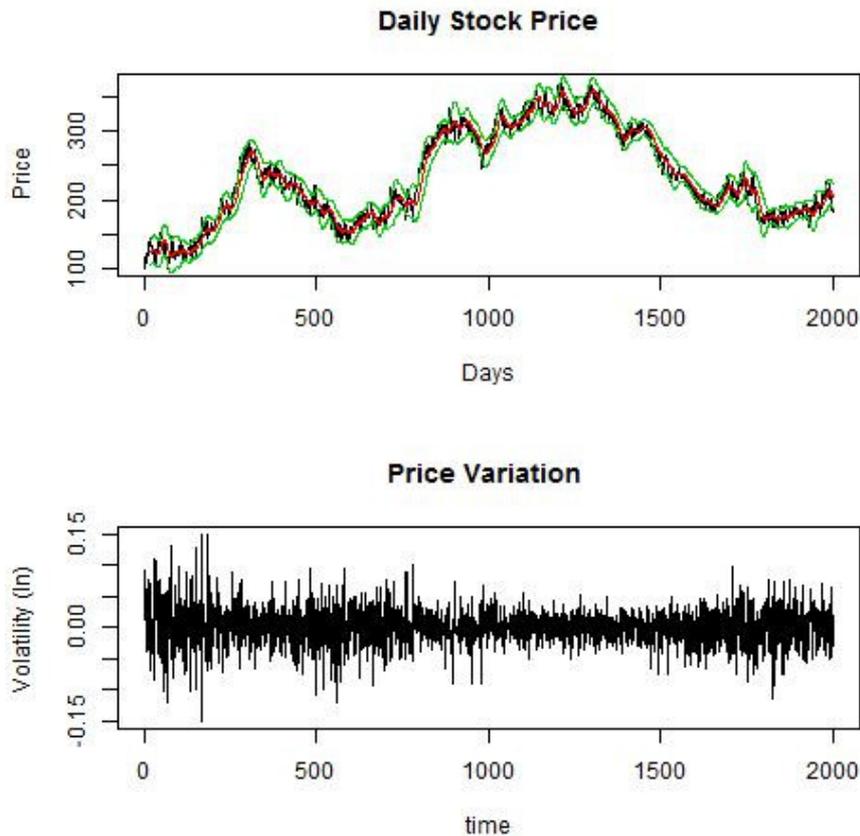
Both chart and the test suggest that there is not a statistically relevant difference between the Q1 mean utility and the Q4 one.

5.5.2 Second scenario: Stock variability and Risk-attractiveness

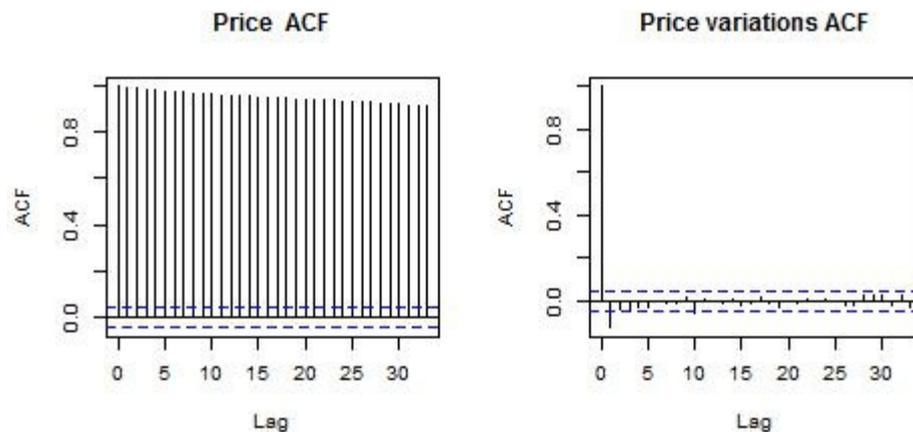
Similar to the second scenario for ZI simulation we here allow agents to trade more stocks than the unitary amount. However there is an important difference since each agent has a specific risk-attractiveness based on the two-weighting function that influences the number of traded stocks.

As for the second ZI scenario the amount of stocks traded vary between one up to five as said according to the risk-profile of each agent. Apart from stock variability the remaining strategies and procedures are set as in the previous scenario.

The stock price has a peak at 374 and a trough at 100 and it is 185 when simulation finished while facing high volatility especially during the first part of the simulation and some peaks when price abruptly falls around level 150.



Consider now the information ACF correlograms and the Augmented Dickey-Fuller Test provide about the price series.



On the left it is plotted the ACF of the daily prices while on the right the same function is plotted for the daily price variations. As we can see, daily prices follow a random walk process.

On the other side, the ACF for price variations displays white noise behavior without any correlation for lags greater than zero but with a slight negative correlation at lag one.

From the ACF correlogram we can see how the stochastic component dominates the price variations residuals in favour of a random walk process for price and of a white noise process for price variations.

In order to provide more information we performed an Augmented Dickey-Fuller Test on the price variations.

Augmented Dickey-Fuller Test

data: Price Variations

Dickey-Fuller = -14.976463490586942

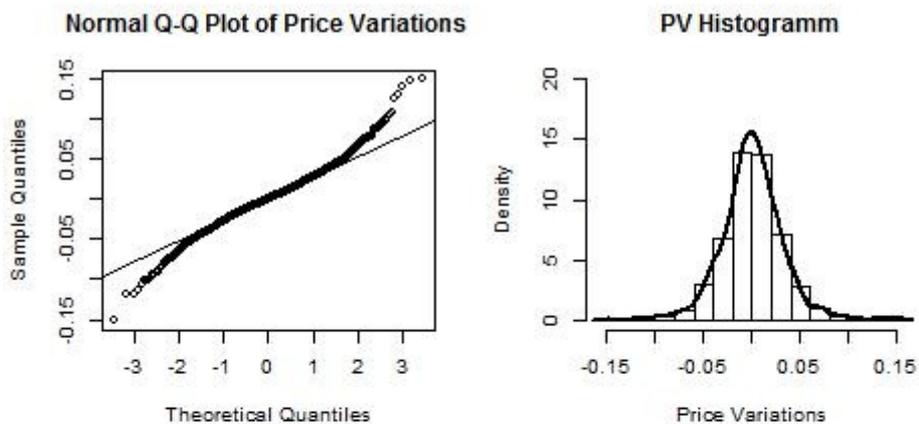
Lag order = 12

P-Value for the test = 0.01

Alternative HP stationary

With a p-value less than the significant value alpha 0.05 the alternative H_p on stationarity has here to be accepted.

In order to have an intuitive idea whether or not price movements are normally distributed consider the relative Q-Q plot and histogram.



Graphically both charts would suggest that price variations do not follow a Gaussian distribution. Normal Q-Q plot displays many values out of the linear line while the fitted density curve seems too sharp to be a Gaussian. In order to be thorough further investigation is needed and here provided:

Shapiro-Wilk Test on price variations

Value of the Shapiro-Wilk test statistic = 0.9818105735764826
 Approximate p-value for the test = 2.821530077987072E-15

Jarque-Bera Test on price variations

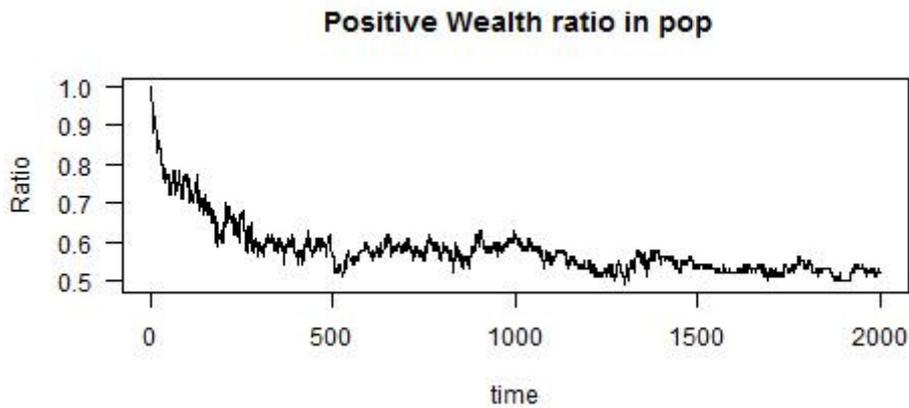
Value of the Jarque-Bera test statistic = 311.3771059524002
 Approximate p-value for the test = 0

Consider an alpha significance level (α) equals to 0.05 The Shapiro-Wilk statistic value here is equal to 0.98 being really close to a Normal distribution. The p-value is less than alpha according to which the null hypothesis is rejected and there is evidence that the data tested are not from a normally distributed population.

Similarly the Jarque-Bera test shows a p-value less than the alpha significance level. Therefore the null hypothesis of normality is again rejected. Both tests suggest us that the price variations do not replicate a Normal distribution according to our graphical intuition.

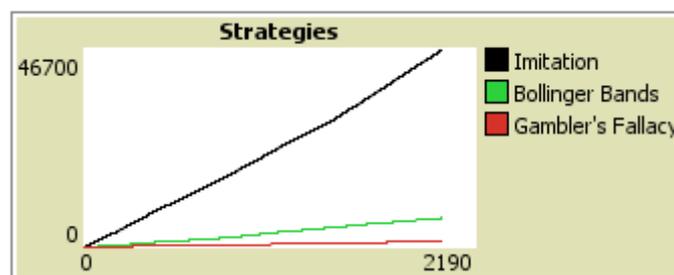
According to what we saw on price and its variations the underlying process here is a random walk whose residuals do not follow a Normal distribution.

In the standard complete scenario with more strategies available the ratio of positive wealth did not drop abruptly within the 500. Allowing stock variability instead provided a quick fall of the positive wealth ratio as also occurred for the ZI simulation with stock variability. The ratio displays a minimum value of 0.49 and its final value at the end of the simulation is 0.52

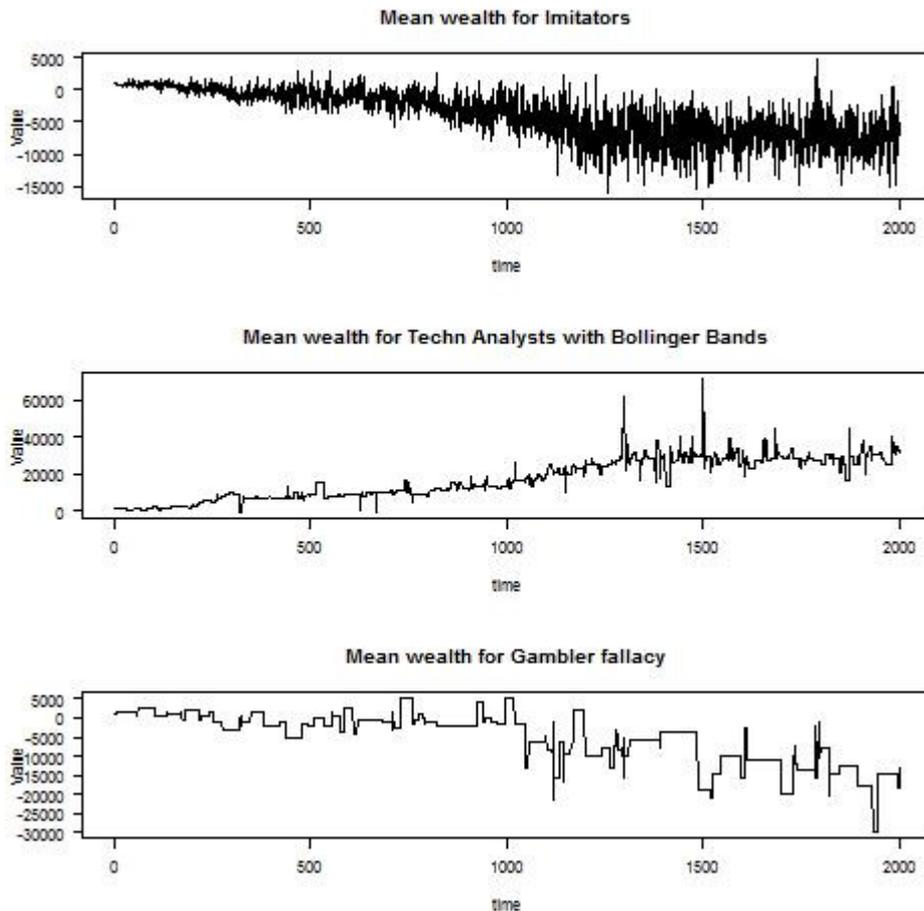


In the following chart are illustrated the cumulative number of times each strategy is performed by all agents from the first trading day onward. The imitation strategy is the most applied and the reasons behind this are essentially two. First of all, this strategy does not depend on the market conditions to trigger it. It just needs agents to seek for the most profitable ones around and then they will imitate their leaders. Secondly under the above noted settings it is the strategy with the greatest number of agents willing to apply it.

These results are completely similar to the ones found on the first scenario for the complete system.



Each strategies performance has its own trend and behaviour as illustrated in the following chart.



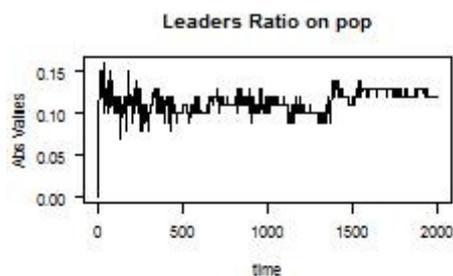
The main characteristics of each strategy's performance are reported in the following table.

Strategy	Initial C	Min	Max	Final C
Imit	1.000	-16.186	4.769	-5.119
BB	1.000	-1.009	71.288	31.892
GF	1.000	-29.946	5.088	-13.037

A first glance to the table, it tells us that dealing with more stocks increased the entire minimum, maximum and final cash amount. Despite the bigger numbers the overall picture does not change. The results in fact are similar to the previous case in terms of general performances of each strategy and relative results compared to each other.

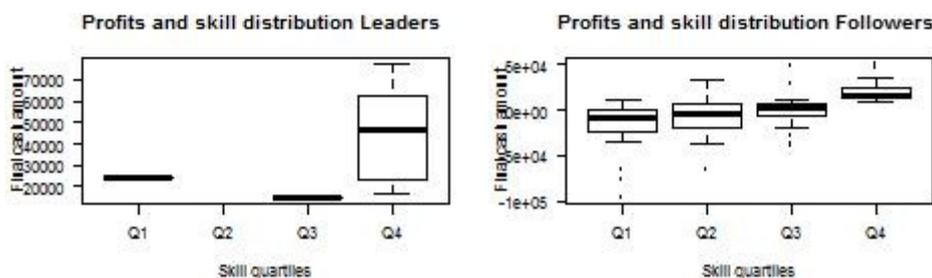
Similar to the previous simulation, the imitative strategy keeps increasing its volatility over time with uncertain returns; the Gambler's fallacy is the one that seem to be well-performing in the short run while leading to negative returns in the long-run and it is the worst for the minimum and the final cash values. Conversely the technical strategy is the one with the lowest minimum and the highest maximum and it is the only strategy leading to positive returns for agents (31892).

Consider now the Imitative strategy in order to check whether or not the previous results still hold when all the strategies are available.

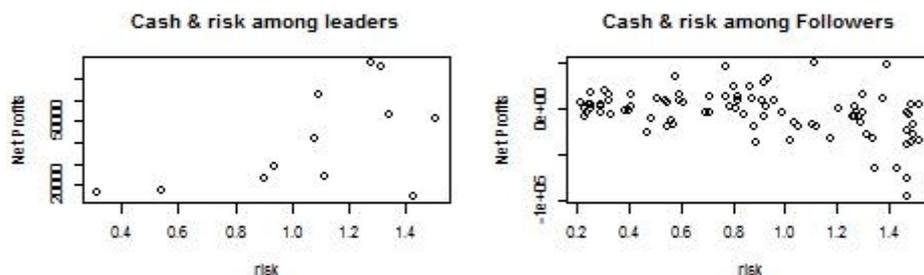


In the population the number of leaders is not constant but the leader ratio fluctuates between the 10% and the 15% with a peak of 17 leaders out of 100 before falling at 12 at the end of the simulation.

In order to have an intuitive idea on the differences in skill and final cash among leaders and their followers, the following box plots come at handy.

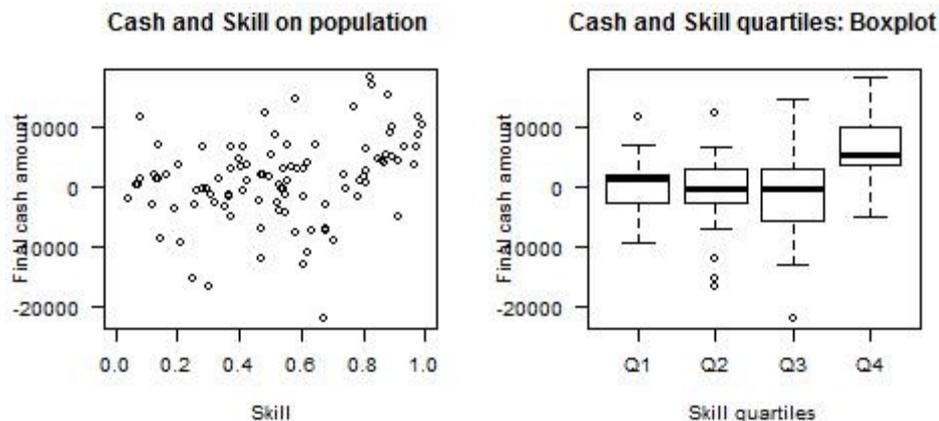


First of all we may notice that Leaders' cash is always positive. This does not surprise us that much since as we already said the leaders are selected on cash basis. On leader's side at the end most of them are highly skilled one so that selecting them on cash basis has been a nice way to detect the most skilled ones. On follower's side it is interesting to notice how cash increases as skill is higher. In this simulation risk-attractiveness determines the amount of stocks traded daily and the way risk affects performances for leaders and their followers is described by the following scatter plots:



It is outstanding to notice that cash and risk have a different relation according to the fact of being a leader or a follower. Risky leaders are in fact more likely to perform better than risk-adverse ones. Conversely for follower the greater the risk the more likely are them to lose money in the market. This is quite interesting because risk only affects the number of stocks traded and each agent's risk is completely independent from its skill value.

On the agent side we'll investigate here how skill affects the final amount of cash owned by agents. In statistical terms we face a positive Pearson correlation between the skill and cash vectors on the population (0.331) slightly less than the previous case (0.356). Graphically we can investigate the cash-skill relationship by looking at the following scatter plot and box plot



Higher skill has a positive effect on the final median cash amount consistent with the correlation factor. It is important to notice that the Q4 median is higher than the other ones. Differently from previous case Q1 agents do not display the lowest median cash. It is worth to remember that Q4 agents are the ones likely to apply the technical strategy, Q1 the ones applying the Gambler's fallacy while Q3, Q2 and Q1 can also imitate. To check whether or not most skilled agents are statistically performing better than lowest skilled the following Welch's t-test has been undertaken.

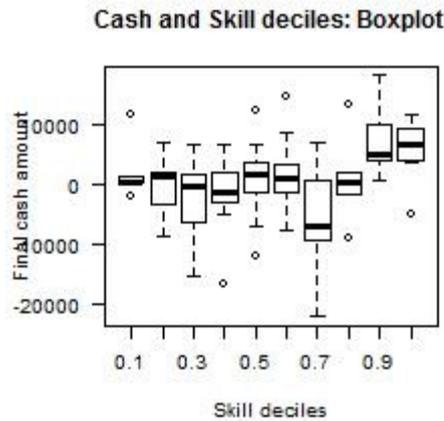
T test statistic value = 4.535803808563898
Degree of freedom =47.52350846468463
P-Value = 1.9482832211337404E-5
Mean estimates for set2 and set1 [6459.734701755508 -1232.3755358282103]
Alternative HP: greater

For any significant level of alpha the null H_0 is rejected while accepting the alternative H_1 according to which the final cash mean for Q4 (6460) is statistically greater than the Q1 ones (1232).

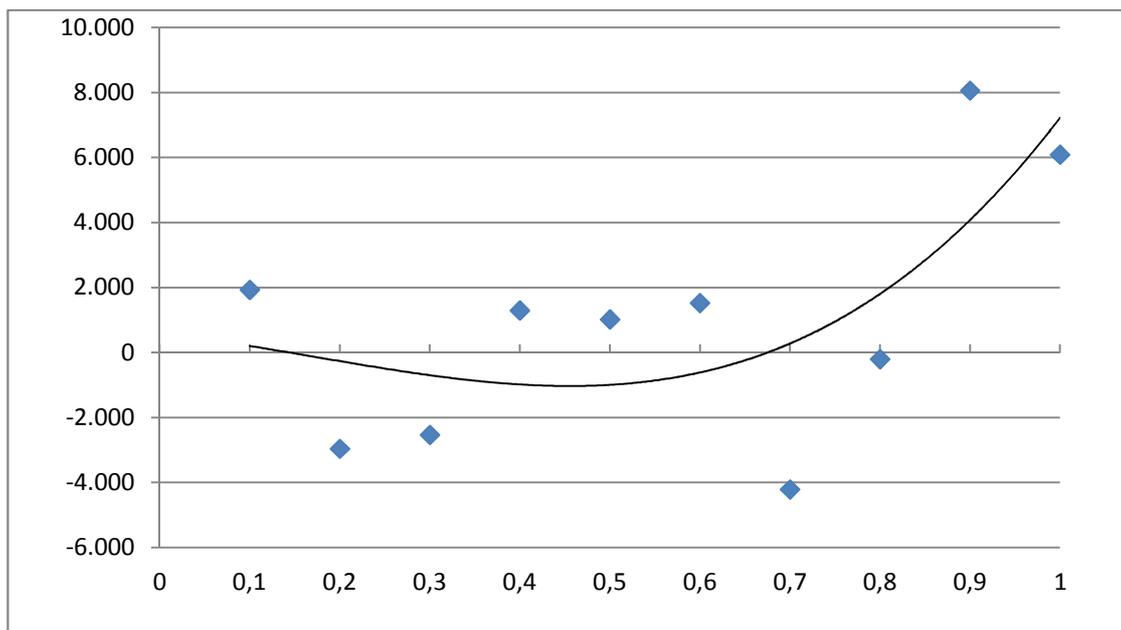
It is worth to notice that this result is the effect of jointed actions. In particular referring to the single cases previously analyzed:

- Imitative strategy has been seen to lead to a statistically higher mean value for Q4 versus Q1 as illustrated in section 5.2.
- Gambler fallacy leads to low cash performances for those who apply it as illustrated in section 5.3.
- Technical strategy has been seen to lead to a statistically higher mean value for Q4 versus Q1 as illustrated in section 5.4.

To see more specifically how the final cash is influenced by skill the quartile box plot analysis has been extended thanks to the following decile box plot



The skill cash relation seems far from a linear representation. Using the mean data the skill cash relation based on deciles can be approximated by the following third order function.

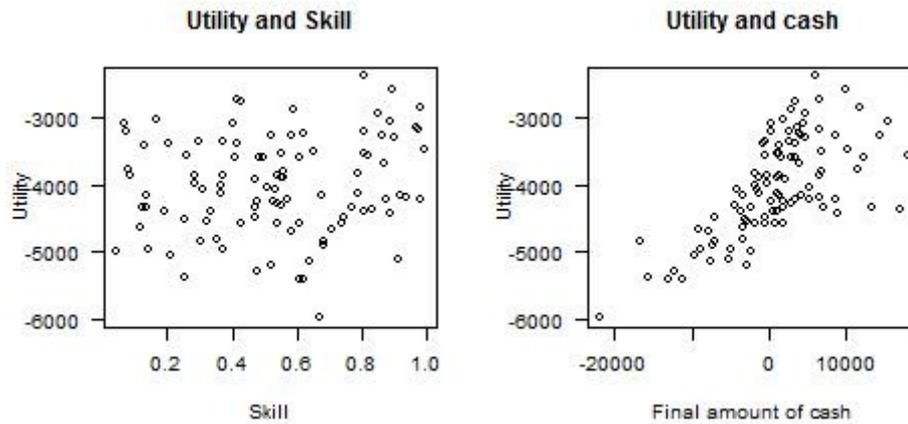


In particular the function has the following form and R^2 values:

$$y = 20067 x^3 - 10524 x^2 - 2893,5x + 576 \qquad R^2 = 0.485$$

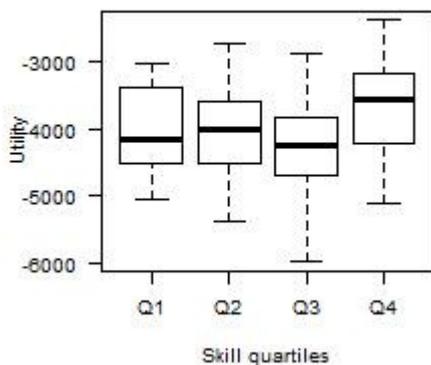
As the picture suggests us the previous relationship hold here only partially while the found third order function fits less with the real data than in the previous case. The volatility of stocks makes impossible to find totally consistent results with the previous ones. However in both cases there is a positive shaped curve for skill values greater than the skill threshold while a flat relation for the central deciles.

Prospect theory adds to the model a descriptive utility function as described more properly in section 4.3.4. As we know the utility function is shaped according to the value function described by Kahneman and Tversky (1979, 1992).



As we can see, there is a strong positive correlation between utility and cash. This does not surprise us since the utility function here is based on gains and losses of agents. The final utility values are all negatives because the losses have in absolute terms a greater effect on utility compared to gains. This is one of the main characteristics of the value function theoretically described in section 1.4.1. Differently from standard complete scenario utility here varies more for positive cash value. Since we demonstrated that skill has an impact on cash and we know that utility depends on cash we would check whether or not utility is influenced by skill.

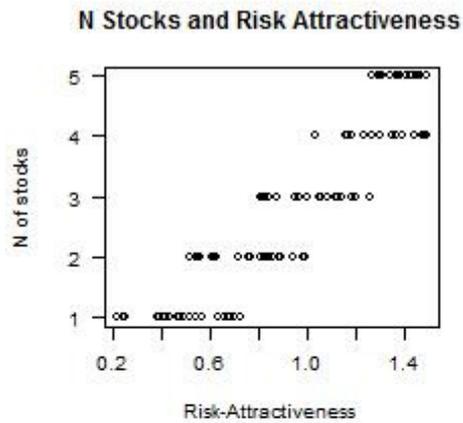
Utility and skill quartiles: boxplots



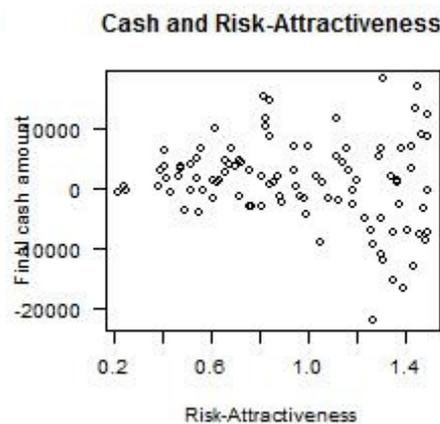
T test statistic value = 0.8816252423735672
 Degree of freedom = 45.87722107054801
 P-Value = 0.3825753463275441
 Mean estimates for set6 and set5
 [-3977.9292646629165 -4118.433389773557]
 Alternative HP: two.sided

Both the chart and the test suggest that there is not a statistically relevant difference between the Q1 mean utility and the Q4 one.

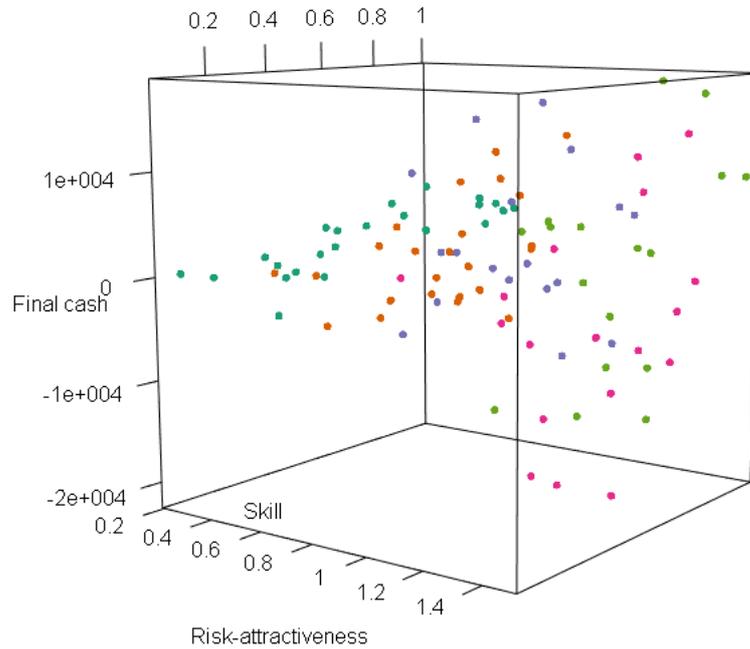
The number of stocks traded, as already said, depends on the risk-attractiveness of individuals. This is easier to be grasped from the following chart. Since the behaviour is not strictly deterministic, the number of stocks depends of course on the risk level but the same level of risk-attractiveness may lead to different number of traded stocks since a random component is embedded in the model.



Final amount of cash is highly influenced by risk-attractiveness. From the chart below it is possible to notice how higher risk-attractiveness leads to a wider gains – losses range. Really low risk levels are associated with zero final amounts while at maximum values the richest and poorest agents can be found.



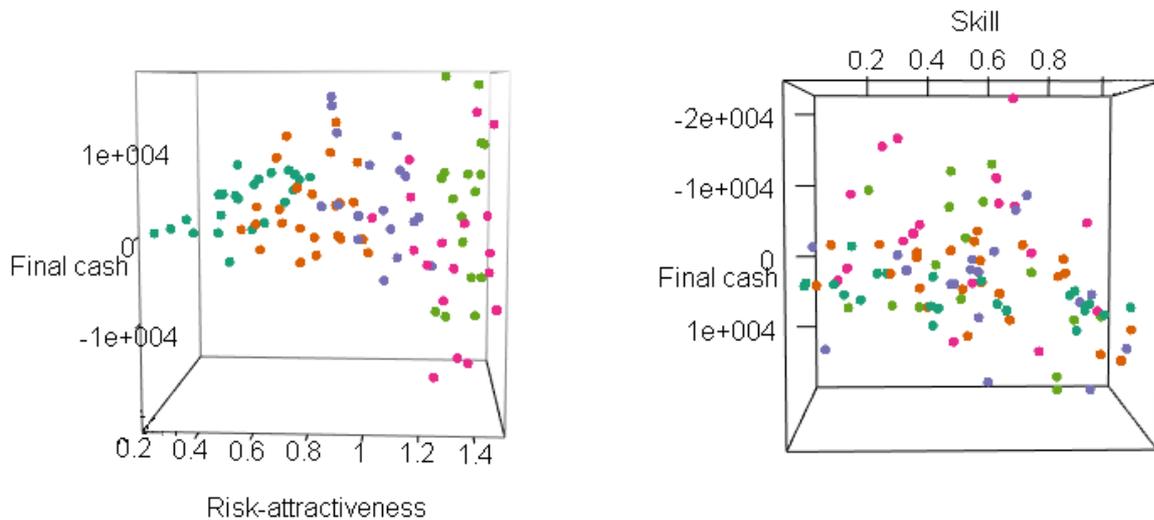
In order to summarize how skill, risk and the number of stocks traded affect the final cash the following interactive 3d chart has been created thanks to a specific R library.



This plot conveys many data seen so far. Just to describe it briefly on the x-axis is represented the level of risk for agents; y-axis the final cash amount while z-axis the skill value. Dots have different colours according to the number of stocks traded.

- One stock traded
- Two stocks traded
- Three stocks traded
- Four stocks traded
- Five stocks traded

We can see that while risk increases the final cash range widening whereas the higher the skill the more likely are higher returns. From the 3D-plot it is also possible to derive the 2D-plot for the risk-cash and for the skill-cash relations simply by moving the chart with a mouse click as it follows:



CONCLUSIONS

The main purpose of this work is to provide insights on how single agent's decisions under risk occur and affect financial markets.

The work has been presented in four parts:

1. The first part investigated how people decide when they are put in a context characterized by risk. In this domain the well-known expected utility theory has been presented as a starting point for the further development proposed by the Economic Nobel prize Kahneman and Tversky called prospect theory (1979, 1992).
2. The second part investigated the features of real stock markets by highlighting its main technical aspects and human components. From the human side the attention has been drawn on strategies, heuristics and fallacies. While from the market standpoint the CDA mechanism has been widely discuss to provide a realistic model to match agent's offers.
3. The third part presents all the models that lead to the final version 3.0. In its original form the model 1.0 is simply an adaptation in Netlogo of Gode and Sunder (1993) work. The model 2.0 instead is the first one to introduce strategies at disposal of the agents. The strategies in model 2.0 are:
 - i. An imitative strategy that let agents seek for a leader on cash basis
 - ii. A technical analysis strategy based on Bollinger's Bands
 - iii. A gambler's fallacy strategy that represents a cognitive bias from the psychological heuristic based on representativeness.

In model 2.0 these strategies are adopted according to agent's skill value but they strictly determine the agent's position on the market since the buy or sell decisions were based on dummy variables. Model 3.0 instead adds subjective probabilities behind each agent decision including also the opportunity for agents to not participate in the buy-sell process preferring to pass. Therefore the same strategies are at agents' disposal but they have been revised in order to be more individually personalized for each agent. For instance the Bollinger's bands strategy suggests agents to sell once the price is above the upper band. However the right "above price" is determined by each agent randomly according to a personal evaluation that affects the strategy. Additionally the prospect theory has been added to the framework and each agent has a personal way to underestimate or overestimate probabilities according to its own two-parameters weighting function.

The introduction of probabilities helped to develop a more dynamic decision system for agents whose positions in the market are not determined by the last strategy occurred in the agent's mind but from an overall evaluation that might involve none or all strategies. On the "background" of agent's mind a ZI component is still present in the model that influences decisions' probabilities but at a lower degree than the real strategies.

4. The last part is all about the experiments based on model 3.0. The experiments have been conducted starting from the ZI agent paradigm in order to have both a theoretical comparison with the existing literature and a benchmark to compare the further results. From here each strategy has been singularly put at disposal of agents in order to focus on the single strategy contribution before to have all of them at agent's disposal. During the experiments the attention has been drawn on both micro and macro layers of the model analyzing how decisions affect agents' performances in the financial markets as well as what is the aggregate product of all these individual choices.

The experiments' results display a price time series process following a random walk process under all the different experimental settings. Additionally the price variations do not follow a normal distribution while displaying different features case by case. These results have been found thanks to a rigorous econometric analysis that involved some graphical intuition provided by correlograms, and q-q plots as well as more technical tests on unit root as the Augmented-Dickey Fuller.

Agents' performances are different according to the strategy selected and the latter display a consistent behavior over the different simulation settings. Of course ZI trader agents' performances are totally random and driven by arbitrariness as demonstrated in the really first experiment. The strategies performances on the other more meaningful experimental settings are summarized in the following table:

Strategy	Initial C	Min	Max	Final C
Imitative : Standard case				
Imit	1.000	-3.773	3.388	318
Imitative : Agent – fish schooling				
Imit	1.000	61	1.494	1.228
Gambler's fallacy: Standard case				
GF	1.000	227	2.505	1.101
Gambler's fallacy: High gambling temptation				
GF	1.000	-29	1.490	703
Technical strategy : Standard case				
BB	1.000	719	5.534	3.681
Complete system : Standard case				
Imit	1.000	-703	1.761	783
BB	1.000	-269	3.481	2.687
GF	1.000	-1.001	2.068	274
Complete system : Stock variability and Risk-attractiveness				
Imit	1.000	-16.186	4.769	-5.119
BB	1.000	-1.009	71.288	31.892
GF	1.000	-29.946	5.088	-13.037

Imitative strategy displays a high volatility that affects the extreme minimum and maximum values while at the end provides uncertain returns that are usually lower than the initial endowment. The minimum – maximum range is greatest when agents trade different amounts of stocks that are not unitary anymore. This setting leads also to the lowest return for Imitation. Imitate the richest is not the best idea here probably for two reasons. Firstly the richest agents are not the most skilled in the market. In particular in the first imitative simulation leaders are simply the luckiest one who are so lucky to be richer than others and then get selected as leaders by their neighbors. The second is that even when imitating the smartest agents, which are among the richest on that day, it may leads to negative results. A winning strategy accounts also for losses and bad days which are compensated by the strategy itself on the long-run. In fact, imitative agents simply copy small pieces of the overall strategy adopted by the leaders and at the end they may miss the whole mosaic.

Similar negative results have been found for those adopting the Gambler's Fallacy. Once again the strategy is based on the price time series and on the heuristic biases. After five days of price increasing (or decreasing) agents expect a reversal in the other direction. When taken alone the strategy leads to final values close to the initial ones. The volatility of the performances is extremely high for this strategy too. In particular in the complete system simulations is the one with the lowest minimum value and the greatest max-min range while providing the worst returns, in both cases with and without stock variability.

The technical strategy based on Bollinger Bands is the only one with positive returns in terms of final cash minus initial cash endowment. When it is the only strategy available it displays both higher minimum and maximum values than the previous strategies under similar settings. From this perspective this strategy seems more reliable and less risky than the others. Moreover it has also strong positive returns as mentioned before. In particular in both standard case and in the first complete system it leads to similar returns: 3681 in the former, 2687 in the latter. These features are also in the complete system with stock variability but the numerical values here are greater in absolute terms. Of course trading more stocks with the right strategy leads to higher returns as shown.

It is important to remember that strategies are not available for all as reported in the table in section 4.3. While technical analysis is performed by the ones with higher skill vales the gambler's fallacy is performed by those with the lowest. The key point here is the skill's threshold as it appears in the mentioned table. For each simulation we compare the most skilled and the less one thanks to box plots and the Welch's t-test together with deeper analysis on the risk attitude in the last experiment and on leaders versus followers' profiles when imitation was on.

It is fair to notice that some simplifications have been adopted here. In fact agents do not face any transaction costs, since no spread ¹¹has been introduced, while they can keep investing on the market also when their wealth endowment is negative. Therefore agents do not face any physical constraint and at the same time they are not affected by their initial wealth. These aspects should and can be added in the model in order to improve it. Additionally it may be interesting to include the possibility for agents to learn from past based on genetic algorithms while adding a fundamental strategy as well.

¹¹ Spread here refers to the difference between the bid and the ask price of a security or an asset.

APPENDIX

A. Parameters' relation of the TPWF

Discriminability (γ) and attractiveness (δ) are the two parameters proposed in the Gonzales and Wu (1999) version of the weighting function. They estimated the individual values for both parameters according to the subjects' performances on an experimental study. In particular subjects had to choose among several gambles and their associated probability as we have seen in Kahneman and Tversky (1992). The estimated values for each subject have been reported in the following table:

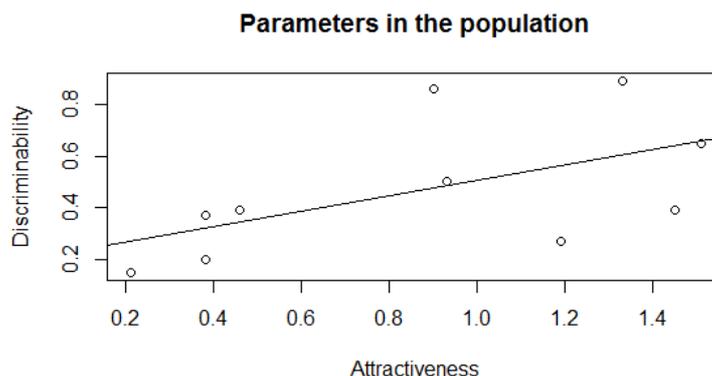
Subjects	δ	Dev St	γ	Dev St
1	0,46	0,11	0,39	0,03
2	1,51	0,46	0,65	0,04
3	1,45	0,35	0,39	0,02
4	0,21	0,04	0,15	0,02
5	1,19	0,32	0,27	0,02
6	1,33	0,15	0,89	0,03
7	0,38	0,07	0,20	0,02
8	0,38	0,11	0,37	0,04
9	0,90	0,18	0,86	0,04
10	0,93	0,26	0,50	0,03
Median Data	0,77	0,10	0,44	0,01

The purpose here is to implement successfully the TPWF in Netlogo. Since the TPWF's formula is known we need simply to generate the parameters' values among the agents. However if we would just generate both randomly we will miss the typical S-shape for the weighting function. Moreover the two parameters display a positive correlation equal to 0.5611.

The strategy adopted here it is to regress discriminability (γ) on attractiveness (δ) in order to obtain the regression equation and use it to produce consistent parameters. The estimated equation is:

$$\gamma = 0.2965(\delta) + 0.2079$$

With a R^2 value equals to 0.315. Graphically:



B. Statistical and econometric tool: Theory and R commands

B.1 Correlation coefficient

The Pearson correlation coefficient (ρ), simply known as the correlation coefficient is one of the most widely used statistics. It summarizes the relationship between two variables that have a linear relationship. Given two variables X and Y, each one having n values $x_1, x_2, x_3 \dots x_n$ and $y_1, y_2, y_3 \dots y_n$ respectively, the population Pearson correlation coefficient ρ is found as:

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

Where:

$$Cov(X,Y) = E [(X - \mu_X)(Y - \mu_Y)]$$

μ_X is the mean of X

σ_X is the standard deviation of X

The statistics takes value in the interval [-1, 1] for positive value of ρ the relationship is positive so that increases in one variable are associated with increases in the other variable. Alternatively for negative value a negative relationship is found so that increases in one variable are associated with decreases in the other. If instead ρ is close to zero it means that little or no relationship is found between the two variables.

In R the correlation between two variable X and Y is performed thanks to:

```
cor(x, y = NULL, use = "everything", method = c("pearson"))
```

B.2 Welch's t-test

The two-sample t-test is a statistical hypothesis where the test statistic follows a Student's t-distribution if the null hypothesis is accepted. It is used to check whether or not two population means are significantly different from each other. It is an adaptation of Student's t-test and it is more reliable when the two samples have unequal variances (Welch, 1947). Welch's t-statistics is defined by:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

Where

\bar{X}_1, s_1^2 and N_1 are respectively the first sample mean, sample variance and sample size. The degrees of freedom associated with this statistics are:

$$v \approx \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\frac{s_1^4}{N_1^2 v_1} + \frac{s_2^4}{N_2^2 v_2}}$$

Where

$$v_1 = N_1 - 1 \text{ and } v_2 = N_2 - 1$$

Using a two sided test, once t and v have been calculated they are used with the t-distribution to test:

$$H_{p_0}: \mu_1 = \mu_2$$

$$H_{p_1}: \mu_1 \neq \mu_2$$

If instead we are interested in a one sided test the hypothesis are:

$$H_{p_0}: \mu_1 = \mu_2$$

$$H_{p_1}: \mu_1 > \mu_2 \text{ or conversely } H_{p'_1}: \mu_1 < \mu_2$$

In R the Welch test requires the package *tstats* installed and launched and it is performed by the following code:

```
t.test(x, y, alternative = c("two.sided", "less", "greater"), mu =  
0, paired = FALSE, var.equal = FALSE, conf.level = 0.95, ...)
```

By imposing different conditions on the c vector it is possible to have either the one sided or the two sided version of the test.

B.3 Shapiro-Wilk Test

The Shapiro-Wilk Test is considered one of most valid test to determine normality distribution in a sample x_1, x_2, \dots, x_n especially for small samples (Shapiro e Wilk, 1965).

The test statistics is computed as:

$$W = \frac{(\sum_{i=1}^n a_i x_i)^2}{\sum_{i=1}^n (x_{(i)} - \bar{x})^2}$$

Where

$x_{(i)}$ is the i^{th} order statistic the i^{th} – smallest number in the sample;

\bar{x} is the sample mean

$$(a_1, \dots, a_n) = \frac{m^T V^{-1}}{(m^T V^{-1} V^{-1} m)^{\frac{1}{2}}} \quad \text{with } m = (m_1, \dots, m_n)^T$$

m_1, \dots, m_n are the expected values of the order statistics of *i.i.d.* random variables sampled from normal distribution and V is the covariance matrix of those order statistics.

The Shapiro-Wilk tests the following null hypothesis

$$H_{p_0} \text{ } X \text{ is normally distributed}$$

This is rejected whenever the p-value is less than the chosen alpha level telling us that the data are not normal. Conversely whether p-value is greater than the chosen alpha it is not possible to reject the null hypothesis that suggests that data are from a normally distributed population.

Meanwhile the W statistics takes values between zero and one, the closer the values are to one the more likely are the data to derive from a normally distributed population.

In R the Shapiro-Wilk Test requires the package *tstats* installed and launched and it is performed on data sample x by the following code:

```
shapiro.test(x)
```

B.4 Jarque-Bera Test

As the Shapiro-Wilk test the Jarque-Bera test determine normality distribution in a sample x_1, x_2, \dots, x_n . However while Shapiro-Wilk is based on two estimates for the variance the Jarque-Bera tests whether or not sample data have the skewness and kurtosis matching a normal distribution (Jarque and Bera, 1987).

The JB statistics derives from:

$$JB = \frac{n - k + 1}{6} \left(S^2 + \frac{1}{4} (C - 3)^2 \right)$$

Where n is the degree of freedom;

$$S \text{ is the sample skewness: } S = \frac{\widehat{\mu}_3}{\widehat{\sigma^3}} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^{\frac{3}{2}}};$$

$$C \text{ is the sample kurtosis : } C = \frac{\widehat{\mu}_4}{\widehat{\sigma^4}} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^2};$$

As we can see, from the formula above, $\widehat{\mu}_3$ e $\widehat{\mu}_4$ are respectively the third and fourth central moments, \bar{x} is the sample mean and $\widehat{\sigma^2}$ is the variance.

The JB statistics is asymptotically distributed with two degrees of freedom and it is used to determine normality on sample data. The null H_0 here corresponds to a joint hypothesis requesting both skewness and the excess kurtosis (kurtosis of 3) being zero.

The JB statistic has a chi-squared distribution with two degrees of freedom, so the statistic can be used to test the hypothesis that the data are from a normal distribution. The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being zero. The null hypothesis is based on the fact that normally distributed data show have an expected skewness of 0 and an expected excess kurtosis of 0.

In R the Shapiro-Wilk Test requires the package *tseries* installed and launched and it is performed on data sample x by the following code:

```
jarque.bera.test(x)
```

B.5 Autocorrelation function and plot

The autocorrelation of a random process describes the correlation between values of the process at different times, as a function of the two times or of the time lag.

The autocorrelation of a process Y_T can be defined as:

$$\rho_K = \frac{Cov(Y_T, Y_{T-k})}{\sigma^2}$$

Where the numerator $\gamma_k = cov(Y_t, Y_{t-k}) = cov(Y_{t-k}, Y_t)$ corresponds to the auto-covariance of the process while the denominator corresponds to the sample variance (Verbeek, 2004).

The autocorrelation function (ACF) or the correlogram of the series corresponds to the autocorrelation of the process as a function of k lags. ACF allows inferring the extent to which one value of the process is correlated with previous values and thus the length and strength of the memory of the process.

In R it is possible to obtain the ACF through the package *tseries* installed and launched; given a time series vector x:

```
acf(x)
```

B.5.1 Augmented Dickey–Fuller test

In order to determine whether or not a time series is stationary using an autoregressive model it is useful to perform a unit root test. One of most popular and more reliable especially for large sample is the augmented Dickey–Fuller test (ADF).

ADF set of hypothesis is

$$\begin{aligned} H_{p_0}: \theta &= 1 \\ H_{p_1}: |\theta| &< 1 \text{ (stationarity)} \end{aligned}$$

The test statistic is computed as:

$$DF = \frac{\hat{\theta} - 1}{se(\hat{\theta})}$$

Where $se(\hat{\theta})$ is the OLS standard error. Once obtained the statistics it is important to compare the relevant critical value for the Dickey–Fuller Test. Whenever the test statistic is less than the critical value, then the null hypothesis of is rejected and stationary accepted.

In R the ADF test requires the package *tseries* installed and launched and it is performed on time series x by the following code:

```
adf.test(x)
```

C. The software toolbox: Netlogo, R and RServe

Netlogo is a modelling environment to simulate natural and social phenomena agent-based particularly well-suited to study complex systems that develops over time. It was authored by Uri Wilensky in 1999 and has been in continuous development ever since at the Center for Connected Learning and Computer-Based Modeling.

The Agent-Based modelers have the opportunity to instruct hundreds or thousands of single agents to operate independently. This is incredibly powerful since it allows to researchers to investigate both micro-level behaviors of agents and macro aggregated products arising from their interactions.

Moreover it is possible to implement multiple conditions that may or not affect agent's behaviors according to the modeler's will. Thanks to its high versatility it has been successfully applied in different fields such as biology and medicine, physics and chemistry, mathematics and computer science, and economics and social psychology.

Netlogo is the evolution of the series of multi-agent modeling languages including StarLogo and StarLogoT. NetLogo runs on the Java virtual machine

R is a programming language and software environment designed for statistical and graphical analysis. In fact, the R language is widely used among statisticians and data miners for developing statistical software and data analysis. It has its roots on the programming language S developed by John Chambers at the Bell Laboratories. On its modern form R was created by Ross Ihaka and Robert Gentleman at the University of Auckland, New Zealand, and is currently developed by the R Development Core Team.

R is a free and open-source software that is becoming more and more popular among researchers especially thanks to its libraries with a wide variety of statistical and graphical techniques, including linear and nonlinear modeling, classical statistical tests, time-series analysis, classification, clustering, and others. Additionally, R is easily extensible through functions and extensions, and the R community is noted for its active contributions in terms of packages.

Another strength of R is static graphics, which can produce publication-quality graphs, including mathematical symbols. Dynamic and interactive graphics are available through additional packages.

Among the hundreds of library available for R, the following have been used in this work:

- “stats”: that contains functions for statistical calculations and random number generation.
- “tseries”: that provides the functions and tools for Time series analysis and computational finance.
- “scatterplot3d”: that gives the opportunity to plot a three dimensional (3D) point cloud.
- “rgl” : a 3D real-time rendering system for R
- “RColorBrewer”: that creates nice looking color palettes especially for thematic maps.
- “Rserve” a Server providing R functionality to applications via TCP/IP or local unix sockets. It allows efficient communication between R and Netlogo

NetLogo-Rserve-Extension provides primitives to use the statistical software R via the Rserve package (based on TCP/IP connection) within a NetLogo model.

There are primitives to create R-Variables with values from NetLogo variables or agents and others to evaluate commands in R with and without return values. The Rserve server can run on your local machine or on a different machine also connected via TCP/IP. It is very easy to setup but has, compared to the NetLogo-R-Extension less functionalities.

Once successfully installed the Rserve extension has to be added in the Netlogo model simply by pasting *extensions[rServe]* at the top of the Procedures Tab. First of all, it is necessary connect Netlogo to the Rserve server thanks to the *rserve:init* primitive. After these preliminary steps are done it is possible to send Netlogo variables to R using new primitives (Thiele, Kurth and Grimm, 2012). The main primitives used also in our model are:

- *rserve:putdataframe* > Consider a Netlogo model containing two lists *mylist1* and *mylist2*, with the same number of entries, a call of (*rserve:putdataframe "df1" "v1" mylist1 "v2" mylist2*) would create an R data.frame with the name *df1* and two columns *v1* and *v2*.
- *rserve:putagent* > Assuming the NetLogo turtles have two turtle-own variables *v1* and *v2*, one could create a data.frame with the same structure as before by executing (*r:putagentdf "df1" turtles "v1" "v2"*).
- *rserve:eval primitives* > it allows to execute an R function primitive available. For instance to provide a box plot representation of the above dataset in R the Netlogo code has to be *rserve:eval "boxplot(df1)*. It is also possible to recall R statistical functions otherwise non available in Netlogo as for the Spearman's correlation coefficient and the other statistical tools used in our work.
- *rserve:get variables* > it provides values or variables from R into Netlogo. For instance in our code the line *rserve:get "adf.test(PriceData\$VAR)\$statistic* has been executed to receive the statistics associate with the Augmented Dickey-Fuller test on *PriceData\$VAR*.

D. The code

The code presented in the following pages correspond to the one behind model 3.0 and it includes also the procedures used to call R functions, tests and graphs.

The Netlogo code is presented divided in logical subsets according to the *nls* inner divisions of the algorithm itself that has been characterizing the model since its previous version 2.0.

D.1 Setup

```
breed [randomAgents randomAgent]
randomAgents-own [buy sell pass price cash stocks wealth tmp_wealth leader ID_leader
temp_leader Skill Utility p_sell p_buy p_pass h_strategy discriminability attractiveness n_stock
Gain Loss Ug Ul]
globals [logB logS exePrice dailyPrice h_prices UB LB sdMemory p_fallacy sdMemoryLong
BBprice_memory ra x_sell x_buy x_pass n_random n_BB n_imit n_GF n_rnd PminusN
N_leaders Port_I Port_BB Port_GF Port_ZI TCZI TCI TCBB TCGF Var_price ]
extensions [rserve]
```

to **setup**

```
clear-all
set exePrice [100]
set dailyPrice [100]
set logB []
set logS []
set sdMemory []
set sdMemoryLong []
set BBprice_memory[]
reset-ticks
create-randomAgents nRandomAgents
let side sqrt nRandomAgents
let step max-pxcor / side
set p_fallacy [100]
set n_random 0
set n_BB 0
set n_rnd 0
set n_imit 0
set n_GF 0
set PminusN []
set N_leaders []
set TCI []
set TCZI []
set TCBB []
set TCGF []
set Var_price []
set ra random nRandomAgents
```

```

set h_prices      [100]
set Port_I       [1000]
set Port_BB      [1000]
set Port_GF      [1000]
set Port_ZI      [1000]
ask randomAgents
[set shape "person"
  set size        2
  set stocks      0
  set cash        1000
  set leader      false
  set wealth      [1000]
  set tmp_wealth  [1000]
  set gain        0
  set loss        0
  set Skill random-float 1
  set temp_leader  0
  set p_sell      0
  set p_buy       0
  set p_pass      0
  set Utility     1000
  set Ug          0
  set Ul          0
  set h_strategy  []
ifelse prospect_theory [
  set attractiveness 0
  while [ attractiveness < 0.21]
  [ set attractiveness random-float 1.52
    set discriminability ((attractiveness * 0.2965) + (random-normal 0.2079 0.02 ))
  ]
]
[ set attractiveness 1 set discriminability 1 set n_stock 1]
]

let an 0
let x 0 let y 0
while [an < nRandomAgents]
  [if x > (side - 1) * step
  [set y y + step set x 0]
  ask randomAgent an [setxy x y]
  set x x + step set an an + 1]

end

```

```

to go
ask randomAgents
CDA_pricesformation
Bstrategy
herd_behavior
Gambler_fallacy
prospect_thr
CDA_bargain
graphBB
mcps
set dailyPrice lput last exePrice dailyPrice
if ticks = Simulation_length

    [let T (length(h_prices) - 1)
    let n 0
    while [n < T]

        [let a ( ( ln (item (0 + 1 + n) h_prices )) - ( ln (item (0 + n ) h_prices )) )
        set Var_price lput a Var_price
        set n n + 1
        ]
    ]

end

```

```

to graph
set-current-plot "exePrice"
plot last exePrice
end

```

```

to graphBB
if ticks > 1

    [let sd standard-deviation dailyPrice
    set UB (mean dailyprice) + (2 * sd) set LB (mean dailyprice) - (2 * sd)
    set sdMemory lput sd sdMemory
    set sdMemoryLong lput sd sdMemoryLong
    set h_prices lput last exePrice h_prices
    if ticks > 21 [set sdMemory remove-item 0 sdMemory ]
    if ticks > 19 [set dailyPrice remove-item 0 dailyPrice ]
    ]

end

```

```

to mcps
ask randomAgents
[if member? "I" h_strategy      [ set TCI lput cash TCI]
 if member? "BB" h_strategy    [ set TCBB lput cash TCBB]
 if member? "GF" h_strategy    [ set TCGF lput cash TCGF]
 if member? "ZI" h_strategy    [ set TCZI lput cash TCZI]
]
ifelse (count randomagents with [member? "I" h_strategy ] ) > 0
  [ set Port_I lput ( mean (TCI )) Port_
    ask randomAgents
      [ if member? "I" h_strategy [set h_strategy remove "I" h_strategy]
      ]
  ]
  [ let a last Port_I          set Port_I lput a Port_I]
ifelse (count randomagents with [member? "BB" h_strategy ] ) > 0
  [set Port_BB lput ( mean ( TCBB )) Port_BB
    ask randomAgents
      [ if member? "BB" h_strategy[set h_strategy remove "BB" h_strategy]
      ]
  ]
  [let bbb last Port_BB      set Port_BB lput bbb Port_BB]
ifelse (count randomagents with [member? "GF" h_strategy ] ) > 0
  [set Port_GF lput ( mean ( TCGF )) Port_GF
    ask randomAgents
      [if member? "GF" h_strategy [set h_strategy remove "GF" h_strategy]
      ]
  ]
  [ let c last Port_GF      set Port_GF lput c Port_GF]
ifelse (count randomagents with [member? "ZI" h_strategy ] ) > 0
  [ set Port_ZI lput ( mean ( TCZI )) Port_ZI
    ask randomAgents
      [ if member? "ZI" h_strategy [set h_strategy remove "ZI" h_strategy]
      ]
  ]
  [ let d last Port_ZI      set Port_ZI lput d Port_ZI ]
set TCI []   set TCBB []   set TCGF []   set TCZI []
end

```

D.2 CDA Price formation

to CDA_pricesformation

```
ask randomAgents [
  set p_sell      0
  set p_buy      0
  set p_pass     0
  set p_sell random-float 0.10000000000000001
  set p_buy  random-float 0.10000000000000001
  set p_pass random-float 0.10000000000000001
  set n_rnd      n_rnd + 1
  let my-price ((last exePrice) + (random-normal 0 10))
  set price      my-price
  set h_strategy lput "ZI" h_strategy
  if p_sell > p_buy and p_sell > p_pass
    [ set sell true set pass false set buy false]
  if p_buy > p_sell and p_buy > p_pass
    [set buy true set pass false set sell false]
  if p_pass > p_sell and p_pass > p_buy
    [set pass true set sell false set buy false]
  if pass      [set color gray ]
  if buy       [set color red]
  if sell      [set color green]
]
set logB []
set logS []
end
```

D.3 Imitative behavior

```
to herd_behavior
if Imitative_behavior
  [let side sqrt nRandomAgents
  let step max-pxcor / side let n 0
  let leader_price 0
  let olead[]
  let center[]
  let m 0
while [n < nRandomAgents]
  [ ask randomAgent n
    [set center lput cash center
    set center lput who center
    ask randomAgents in-radius (step * 1.5)
      [let lead[]
      set lead lput cash lead
      set lead lput who lead
      set olead lput lead olead
      set olead remove center olead
      set olead sort-by [item 0 ?1 > item 0 ?2] olead
      ]
    if (item 0 center) > item 0 (item 0 olead)
      [set leader true set shape "star"]
    if (item 0 center) < item 0 (item 0 olead)
      [set leader false set shape "person"]
    ]
  set n n + 1
  set olead[]
  set center[]
  ]
while [m < nRandomAgents]
  [ ask randomAgent m
  [if leader = true
    [ set leader_price price
    if pass
      [ ask randomAgents in-radius (step * 1.5)
      [ if Skill < Skill_threshold and leader = false and
      Imit_sensibility >= random 11 and temp_leader = 0
        [ set p_pass p_pass + ( 0.20 + ( Imit_Impact / 1000 ) )
        set n_imit n_imit + 1
        set temp_leader temp_leader + 1
        set h_strategy lput "I" h_strategy
        ]
      ]
    ]
  ]
  ]
```

```

if buy
  [ ask randomAgents in-radius (step * 1.5)
    [ if Skill < Skill_threshold and
      leader = false and Imit_sensibility >= random 11 and temp_leader = 0
        [ set price ( leader_price + ( ((random 21) - 10 )/ 10))
          set p_buy p_buy + ( 0.20 + ( Imit_Impact / 1000 ))
          set n_imit n_imit + 1
          set temp_leader temp_leader + 1
          set h_strategy lput "I" h_strategy
        ]
      ]
    ]
  ]
if sell
  [ ask randomAgents in-radius (step * 1.5)
    [ if Skill < Skill_threshold and leader = false and
      Imit_sensibility >= random 11 and temp_leader = 0
        [ set price ( leader_price + ( ((random 21) - 10 )/ 10))
          set p_sell p_sell + ( 0.20 + ( Imit_Impact / 1000 ))
          set n_imit n_imit + 1
          set temp_leader temp_leader + 1
          set h_strategy lput "I" h_strategy
        ]
      ]
    ]
  ]
]
]
set m m + 1 ]
ask randomAgents [ set temp_leader 0 ]
let nl ((count randomAgents with [leader] ) / (NrandomAgents))
set N_leaders lput nl N_leaders ]
end

```

D.4 Gambler's Fallacy

to **Gambler_fallacy**

if Gambler's_Fallacy

 [**set** p_fallacy **lput last** exePrice p_fallacy

if ticks > 4

 [

let At item 5 p_fallacy

let At-1 item 4 p_fallacy

let At-2 item 3 p_fallacy

let At-3 item 2 p_fallacy

let At-4 item 1 p_fallacy

set p_fallacy **but-first** p_fallacy

let n 0

while [n < nRandomAgents]

 [**ask** randomAgent n

 [**if** Skill < (1 - Skill_threshold) **and** GF_sensibility >= random 11

 [**if** At < At-1 **and** At-1 < At-2 **and** At-2 < At-3 **and** At-3 < At-4

 [**let** A 0.20 + (GF_Impact / 1000)

set p_buy p_buy + A

set n_GF n_GF + 1

set h_strategy **lput** "GF" h_strategy

]

if At > At-1 **and** At-1 > At-2 **and** At-2 > At-3 **and** At-3 > At-4

 [**let** B 0.20 + (GF_Impact / 1000)

set p_sell p_sell + B

set n_GF n_GF + 1

set h_strategy **lput** "GF" h_strategy

]

]

]

set n n + 1]

]

]

end

D.5 Technical Analysis

to **BStrategy**

```
set BBprice_memory lput last exePrice BBprice_memory
if ticks > 19 [set BBprice_memory but-first BBprice_memory]
if BBStrategy and ticks > 19[
  ask randomAgents[
    if last dailyPrice >= (UB * ( 1 + ((random 11)) / 1000) ) [
      if Skill > Skill_threshold[
        let A 0.20 + ( BB_Impact / 1000)
        set p_sell p_sell + A
        set n_BB n_BB + 1
        set h_strategy lput "BB" h_strategy
      ]
    ]
  ]
ask randomAgents[
  if last dailyPrice <= (LB * ( 1 - ((random 11)) / 1000) ) [
    if Skill > Skill_threshold[
      let B 0.20 + ( BB_Impact / 1000)
      set p_buy p_buy + B
      set n_BB n_BB + 1
      set h_strategy lput "BB" h_strategy
    ]
  ]
]
ask randomAgents[
  let a random 6
  if last dailyPrice < ( mean BBprice_memory * ( 1 + (a / 1000) ) )
  and last dailyPrice > ( mean BBprice_memory * ( 1 - (a / 1000) ) ) [
    if Skill > Skill_threshold
      [ let C 0.20 + ( BB_Impact / 1000)
        set p_sell p_sell + C
        set n_BB n_BB + 1
        set h_strategy lput "BB" h_strategy
      ]
  ]
]
]
end
```

D.6 Prospect Theory

to **prospect_thr**

```
ifelse prospect_theory [  
  ask randomAgent RA [  
    set x_sell p_sell  
    set x_buy p_buy  
    set x_pass p_pass  
  ]  
  ask randomAgents [  
    if p_sell > 0 and p_sell < 1 [  
      set p_sell (( attractiveness * ( p_sell ^ discriminability ) ) / ( ( attractiveness * ( p_sell ^ discriminability ) ) + ((1 - p_sell) ^ discriminability) ) )  
    ]  
  
    if p_buy > 0 and p_buy < 1 [  
      set p_buy (( attractiveness * ( p_buy ^ discriminability ) ) / ( ( attractiveness * ( p_buy ^ discriminability ) ) + ((1 - p_buy) ^ discriminability) ) )  
    ]  
  
    if p_pass > 0 and p_pass < 1 [  
      set p_pass (( attractiveness * ( p_pass ^ discriminability ) ) / ( ( attractiveness * ( p_pass ^ discriminability ) ) + ((1 - p_pass) ^ discriminability) ) )  
    ]  
  ]  
  ask randomagent RA [  
    if p_sell > 0 [  
      let prob p_sell  
      set-current-plot "Weighting function of a random agent"  
      set-plot-x-range 0 1  
      set-plot-y-range 0 1  
      set-plot-pen-mode 2  
      set-plot-pen-color 55  
      plotxy x_sell prob  
    ]  
    if p_buy > 0 [  
      let prob p_buy  
      set-current-plot "Weighting function of a random agent"  
      set-plot-x-range 0 1  
      set-plot-y-range 0 1  
      set-plot-pen-mode 2  
      set-plot-pen-color 15  
      plotxy x_buy prob  
    ]  
  ]  
]
```

```

if p_pass > 0[
  let prob p_pass
  set-current-plot "Weighting function of a random agent"
  set-plot-pen-color 5
  plotxy x_pass prob
]
]
]
[ask randomagent RA[
  if p_sell > 0[
    let prob p_sell
    set-current-plot "Weighting function of a random agent"
    set-plot-x-range 0 1
    set-plot-y-range 0 1
    set-plot-pen-mode 2
    set-plot-pen-color 55
    plotxy p_sell prob
  ]
  if p_buy > 0[
    let prob p_buy
    set-current-plot "Weighting function of a random agent"
    set-plot-x-range 0 1
    set-plot-y-range 0 1
    set-plot-pen-mode 2
    set-plot-pen-color 15
    plotxy p_buy prob
  ]
  if p_pass > 0[
    let prob p_pass
    set-current-plot "Weighting function of a random agent"
    set-plot-pen-color 5
    plotxy p_pass prob ]
  ]
]
]
end

```

D.7 CDA Bargain process

to CDA_bargain

ifelse prospect_theory

```
[
  ask randomAgents [
    ifelse not Var_stock
      [set n_stock 1]
      [
        if attractiveness >= 0.21 and attractiveness < 0.472
          [ set n_stock 1 ]
        if attractiveness >= 0.472 and attractiveness < 0.743
          [set n_stock 1 + random 2 ]
        if attractiveness >= 0.743 and attractiveness < 0.996
          [set n_stock 2 + random 2 ]
        if attractiveness >= 0.996 and attractiveness < 1.258
          [ set n_stock 3 + random 2 ]
        if attractiveness >= 1.258 and attractiveness < 1.52
          [ set n_stock 4 + random 2 ]
      ]
  ]
]

[
  ask randomAgents [
    ifelse not Var_stock
      [set n_stock 1]
      [
        let a random 5
        if a = 0      [ set n_stock 1 ]
        if a = 1     [ set n_stock 1 + random 2 ]
        if a = 2     [ set n_stock 2 + random 2 ]
        if a = 3     [ set n_stock 3 + random 2 ]
        if a = 4     [ set n_stock 4 + random 2 ]
      ]
  ]
]

ask randomAgents [
  if last exePrice < 30 + ( random 6 ) and p_pass > p_sell and p_pass > p_buy [
    set p_buy p_buy + random-float 0.15000000000000001
    set p_pass p_pass - random-float 0.15000000000000001
  ]
]

ask randomAgents[
  if p_sell > p_buy and p_sell > p_pass      [ set sell true set pass false set buy false]
  if p_buy > p_sell and p_buy > p_pass      [ set buy true set pass false set sell false]
  if p_pass > p_sell and p_pass > p_buy     [ set pass true set sell false set buy false]
  if pass = true                          [set color gray]
  if buy = true                            [set color red]
  if sell = true                           [set color green]
]
```

```

tick
ask randomAgents [
  if pass = false
  [
    let tmp[]
    set tmp lput price tmp
    set tmp lput who tmp
  if buy
  [
    let alfa n_stock
    while [alfa > 0][
      set logB lput tmp logB
      set alfa (alfa - 1)
    ]
  ]
set logB reverse sort-by [item 0 ?1 < item 0 ?2] logB
let alfa n_stock
while [alfa > 0]
  [
    if (not empty? logB and not empty? logS) and
    item 0 (item 0 logB) >= item 0 (item 0 logS)
    [
      set exePrice lput (item 0 (item 0 logS)) exePrice
      let agB item 1 (item 0 logB)
      let agS item 1 (item 0 logS)
      ask randomAgent agB
      [
        set stocks stocks + 1
        set cash cash - last exePrice
      ]

      ask randomAgent agS
      [
        set stocks stocks - 1
        set cash cash + last exePrice
      ]

      set logB but-first logB
      set logS but-first logS
    ]
    set alfa (alfa - 1)
  ]
if sell
  [
    let beta n_stock
    while [beta > 0][
      set logS lput tmp logS
      set beta (beta - 1)
    ]
  ]
]

```

```

    set alfa n_stock
    while [alfa > 0][
        set logS sort-by [item 0 ?1 < item 0 ?2] logS
        if (not empty? logB and not empty? logS) and
        item 0 (item 0 logB) >= item 0 (item 0 logS)[
            set exePrice lput (item 0 (item 0 logB)) exePrice
            let agB item 1 (item 0 logB)
            let agS item 1 (item 0 logS)
            ask randomAgent agB
                [set stocks stocks + 1
                 set cash cash - last exePrice]
            ask randomAgent agS
                [set stocks stocks - 1
                 set cash cash + last exePrice]
            set logB but-first logB
            set logS but-first logS
        ]
        set alfa (alfa - 1)]
    graph
]
ask randomAgents [
    set wealth lput cash wealth
    set tmp_wealth lput cash tmp_wealth
    if length tmp_wealth > 2
        [set tmp_wealth remove-item 0 tmp_wealth ]
    if item 1 tmp_wealth > item 0 tmp_wealth
        [
            set gain ((item 1 tmp_wealth) - (item 0 tmp_wealth))
            set loss 0
        ]
    if item 1 tmp_wealth = item 0 tmp_wealth
        [
            set gain 0
            set loss 0
        ]
    if item 1 tmp_wealth < item 0 tmp_wealth
        [
            set gain 0
            set loss ((item 0 tmp_wealth) - (item 1 tmp_wealth))
        ]
]

```

```

if gain > 0
[
set Ug ((log gain 100) * 10)
set Ul 0
set Utility Utility + Ug
]
if loss > 0
[
set Ul ((log loss 10) * (-10))
set Ug 0
set Utility Utility + Ul
]
if gain = 0 and loss = 0 [
set Ul 0
set Ug 0
]
]
ask randomagent RA
[
set-current-plot "Value function"
set-plot-pen-mode 2
plotxy gain Ug
let yl ( (-1) * loss)
plotxy yl Ul
]
let Pos count randomAgents with [cash > 0 ]
let P% ( Pos / ( NrandomAgents))
set PminusN lput P% PminusN
end

```

D.8 Rserve Connection

to `go_with_R`

```
if ticks < Simulation_length [go]
if ticks = Simulation_length [if rserve:isConnected[
  (rserve:putagent "agentlist" turtles "who" "cash" "skill" "leader" "discriminability"
"attractiveness" "n_stock" "utility" )
  ; rserve:eval "View(agentlist)"
  rserve:eval "library(scatterplot3d)"
  rserve:eval "library(rgl)"
  rserve:eval "library(RColorBrewer)"
  rserve:eval "library(tseries)"
  rserve:eval "library(stats)"
  rserve:eval "  setwd('C:/Users/Davide/Desktop/rServe plots') "
  rserve:eval "MA <- function (x,n){ filter (x, rep (1/n, n), sides =1) }"
  rserve:eval "dat <- data.frame(ID = agentlist$who, Cash = agentlist$cash, Skill =
agentlist$skill, leader=agentlist$leader, discriminability=agentlist$discriminability,
attractiveness=agentlist$attractiveness, n_stock = agentlist$n_stock, Utility =
agentlist$utility)"
  rserve:eval "skilldata <- data.frame ( Skill = agentlist$skill, Cash = agentlist$cash )"
  rserve:eval "skilldata <- skilldata [ skilldata$Skill > 0.6,]"
  rserve:eval "dev.new()"
  rserve:eval "par(mfrow=c(2,2))"
  rserve:eval "plot(agentlist$skill,agentlist$cash, xlim=c(0,1), ylab = 'Final cash
amount', xlab = ' Skill', las=1, main='Cash and Skill on population')"
  rserve:eval "boxplot(agentlist$cash[agentlist$skill <= 0.25],
agentlist$cash[agentlist$skill > 0.25 & agentlist$skill<= 0.5 ], agentlist$cash[agentlist$skill >
0.5 & agentlist$skill<= 0.75 ], agentlist$cash[agentlist$skill > 0.75 & agentlist$skill<= 1 ],
las=1, ylab='Final cash amount', xlab='Skill quartiles', main=' Cash and Skill quartiles: Boxplot
', names= c('Q1', 'Q2', 'Q3', 'Q4'))"
  rserve:eval "boxplot(agentlist$cash[agentlist$skill <= 0.10],
agentlist$cash[agentlist$skill > 0.10 & agentlist$skill<= 0.20], agentlist$cash[agentlist$skill >
0.20 & agentlist$skill<= 0.30], agentlist$cash[agentlist$skill > 0.30 & agentlist$skill<= 0.40 ],
agentlist$cash[agentlist$skill > 0.40 & agentlist$skill<= 0.50 ], agentlist$cash[agentlist$skill >
0.50 & agentlist$skill<= 0.60 ], agentlist$cash[agentlist$skill > 0.60 & agentlist$skill<= 0.70 ],
agentlist$cash[agentlist$skill > 0.70 & agentlist$skill<= 0.80 ], agentlist$cash[agentlist$skill >
0.80 & agentlist$skill<= 0.9 ], agentlist$cash[agentlist$skill > 0.90 & agentlist$skill<= 1 ],las=1,
ylab='Final cash amount', xlab='Skill deciles', main='Cash and Skill deciles: Boxplot', names=
c(0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0))"
  rserve:eval "dev.copy(jpeg,filename='Cash&Skill.jpg')"
  rserve:eval "dev.off ()"
  rserve:eval "dev.new()"
  rserve:eval "par(mfrow=c(2,2))"
  rserve:eval "plot(agentlist$attractiveness, agentlist$n_stock, las=1, main='N Stocks
and Risk Attractiveness', xlab= 'Risk-Attractiveness', ylab='N of stocks' )"
  rserve:eval "plot(agentlist$attractiveness, agentlist$cash, las=1, main='Cash and Risk-
Attractiveness', xlab= 'Risk-Attractiveness', ylab = 'Final cash amount' )"
  rserve:eval "plot(agentlist$n_stock, agentlist$cash, las=1, main='Cash and N stock',
xlab= 'N of stock', ylab = 'Final cash amount' )"
```

```

rserve:eval "dev.copy(jpeg,filename='Risk&NStocks.jpg')"
rserve:eval "dev.off ()"
rserve:eval "plot3d( agentlist$cash, agentlist$attractiveness, agentlist$skill, xlab='Net
profit/loss', zlab='Skill', ylab='Risk-attractiveness', col =
brewer.pal(5,'Dark2')[unclass(agentlist$n_stock)], las=1, size=6)"
rserve:eval "dev.copy(jpeg,filename='Plot3d.jpg')"
rserve:eval "dev.off ()"
( rserve:putdataframe "PriceData" "DailyPrice" h_prices "Volatility" sdMemoryLong
"TickPrice" exePrice "Portfolios" PminusN "Leaders" N_leaders "VAR" Var_price )
rserve:eval "dev.new()"
rserve:eval "par(mfrow=c(2,1))"
rserve:eval "plot(PriceData$DailyPrice, xlab='Days', ylab='Price', type= 'l',
main='Daily Stock Price')"
rserve:eval " MAP <- MA( PriceData$DailyPrice , 20 )"
rserve:eval " lines(MAP, col='2' )"
rserve:eval " BB <- MA( PriceData$Volatility , 20 )"
rserve:eval " lines( MAP + 2 * BB , col='3' )"
rserve:eval " lines( MAP - 2 * BB , col='3' )"
rserve:eval "plot(PriceData$VAR, xlab='time', ylab='Volatility (ln) ' , type= 'l',
main='Price Variation ') "
rserve:eval "dev.copy(jpeg,filename='Price&Volatility.jpg')"
rserve:eval "dev.off ()"
rserve:eval "dev.new()"
rserve:eval "par(mfrow=c(2,1))"
rserve:eval "plot(PriceData$Volatility, xlab='time', ylab='Price Standard deviation' ,
type= 'l', main='Daily prices standard deviations over t') "
rserve:eval "plot (PriceData$Portfolios, main=' Positive Wealth ratio in pop', type = 'l'
, xlab = 'time', ylab='Ratio', las = 1 )"
rserve:eval "dev.copy(jpeg,filename='PosPort&StandDev.jpg')"
rserve:eval "dev.off ()"
rserve:eval "dev.new()"
rserve:eval "par(mfrow=c(2,2))"
rserve:eval "qqnorm(PriceData$VAR, main='Normal Q-Q Plot of Price Variations' )"
rserve:eval " qqline(PriceData$VAR )"
rserve:eval " hist(PriceData$VAR, main = 'PV Histogramm', xlab='Price Variations',
prob=T, ylim = c (0, 20) )"
rserve:eval "lines(density(PriceData$VAR), lwd=2) "

rserve:eval " acf(PriceData$VAR, main= 'Price variations ACF') "
rserve:eval "dev.copy(jpeg,filename='Normalità&ACF.jpg')"
rserve:eval "dev.off ()"
rserve:eval "dev.new()"
rserve:eval "par(mfrow=c(2,2))"
rserve:eval "plot(agentlist$skill, agentlist$utility, xlab = 'Skill', ylab='Utility', las = 1,
main='Utility and Skill' )"
rserve:eval "plot(agentlist$cash, agentlist$utility, xlab = 'Final amount of cash',
ylab='Utility', las = 1, main='Utility and cash' )"
rserve:eval "boxplot(agentlist$utility[agentlist$skill <= 0.25],
agentlist$utility[agentlist$skill > 0.25 & agentlist$skill<= 0.5 ], agentlist$utility[agentlist$skill
> 0.5 & agentlist$skill<= 0.75 ], agentlist$utility[agentlist$skill > 0.75 & agentlist$skill<= 1 ],

```

```

las=1, ylab='Utility', xlab='Skill quartiles', main='Utility and skill quartiles: boxplots', names=
c('Q1', 'Q2', 'Q3', 'Q4'))"
  rserve:eval "dev.copy(jpeg,filename='Utility.jpg')"
  rserve:eval "dev.off ()"
  ( rserve:putdataframe "OtherData" "Imit_porfolio" Port_I "Port_Bollinger" Port_BB
"Port_Fallacy" Port_GF "Port_ZI" Port_ZI )
  rserve:eval "dev.new()"
  rserve:eval "par(mfrow=c(3,1))"
  rserve:eval "plot( OtherData$Imit_porfolio, type = 'l' , xlab='time', main = 'Mean
wealth for Imitators', ylab = 'Value',las=1 )"
  rserve:eval "plot( OtherData$Port_Bollinger, type = 'l' , xlab='time', main = 'Mean
wealth for Techn Analysts with Bollinger Bands', ylab = 'Value',las=1 )"
  rserve:eval "plot( OtherData$Port_Fallacy, type = 'l' , xlab='time', main = 'Mean wealth
for Gambler fallacy', ylab = 'Value',las=1)"
  rserve:eval "dev.copy(jpeg,filename='PortStrategy1.jpg')"
  rserve:eval "dev.off ()"
if Imitative_behavior [
  rserve:eval "dev.new()"
  rserve:eval "par(mfrow=c(3,2))"
  rserve:eval "plot (PriceData$Leaders, main=' Leaders Ratio on pop ', type = 'l' ,
xlab = 'time', ylab='Abs Values', las = 1 )"
  rserve:eval "boxplot(agentlist$cash[agentlist$leader == TRUE],
agentlist$cash[agentlist$leader == FALSE], names = c('Leaders', 'Followers'), main='Cash :
Leaders VS Followers' )"
  rserve:eval "plot(agentlist$attractiveness[agentlist$leader == TRUE],
agentlist$cash[agentlist$leader == TRUE], main = 'Cash & risk among leaders', xlab = 'risk',
ylab='Net Profits' )"
  rserve:eval "abline(lm(agentlist$attractiveness[agentlist$leader == TRUE] ~
agentlist$cash[agentlist$leader == TRUE]))"
  rserve:eval "plot(agentlist$attractiveness[agentlist$leader == FALSE],
agentlist$cash[agentlist$leader == FALSE], main = 'Cash & risk among Followers', xlab =
'risk', ylab='Net Profits' )"
  rserve:eval "boxplot(agentlist$cash[agentlist$skill <= 0.25 & agentlist$leader
== TRUE], agentlist$cash[agentlist$skill > 0.25 & agentlist$skill<= 0.5 & agentlist$leader ==
TRUE ], agentlist$cash[agentlist$skill > 0.5 & agentlist$skill<= 0.75 & agentlist$leader ==
TRUE ], agentlist$cash[agentlist$skill > 0.75 & agentlist$skill<= 1 & agentlist$leader ==
TRUE], las=1, ylab='Final cash amount', xlab='Skill quartiles', main='Profits and skill
distribution Leaders ', names= c('Q1', 'Q2', 'Q3', 'Q4'))"
  rserve:eval "boxplot(agentlist$cash[agentlist$skill <= 0.25 & agentlist$leader
== FALSE], agentlist$cash[agentlist$skill > 0.25 & agentlist$skill<= 0.5 & agentlist$leader ==
FALSE ], agentlist$cash[agentlist$skill > 0.5 & agentlist$skill<= 0.75 & agentlist$leader ==
FALSE ], agentlist$cash[agentlist$skill > 0.75 & agentlist$skill<= 1 & agentlist$leader ==
FALSE], las=1, ylab='Final cash amount', xlab='Skill quartiles', main='Profits and skill
distribution Followers ', names= c('Q1', 'Q2', 'Q3', 'Q4'))"
  rserve:eval "dev.copy(jpeg,filename='Imitation.jpg')"
  rserve:eval "dev.off ()"
]

print "Shapiro-Wilk Test on price variations"
print rserve:get "shapiro.test ( PriceData$VAR )"

```

```

print "Value of the Shapiro-Wilk test statistic"
print rserve:get "shapiro.test ( PriceData$VAR )$statistic"
print "Approximate p-value for the test"
print rserve:get " shapiro.test ( PriceData$VAR )$p.value"
print ""
print "Jarque-Bera Test on price variations"
;print rserve:get "jarque.bera.test( PriceData$VAR )"
print "Value of the Jarque-Bera test statistic"
print "Approximate p-value for the test"
print rserve:get " jarque.bera.test(PriceData$VAR)$p.value"
print ""
print "Augmented Dickey-Fuller Test"
print "data: Price Variations"
print "Dickey-Fuller ="
print rserve:get "adf.test( PriceData$VAR )$statistic"
print "Lag order ="
print rserve:get "adf.test( PriceData$VAR )$parameter"
print "P-Value for the test "
print rserve:get "adf.test( PriceData$VAR )$p.value"
Print "Alternative HP"
print rserve:get "adf.test( PriceData$VAR )$alternative"
print ""
print ""
print "Augmented Dickey-Fuller Test"
print "data: Price "
print "Dickey-Fuller ="
print rserve:get "adf.test( PriceData$DailyPrice )$statistic"
print "Lag order ="
print rserve:get "adf.test( PriceData$DailyPrice )$parameter"
print "P-Value for the test "
print rserve:get "adf.test( PriceData$DailyPrice )$p.value"
Print "Alternative HP"
print rserve:get "adf.test( PriceData$DailyPrice )$alternative"
rserve:eval " datatestsc <-data.frame ( Skill = agentlist$skill, Cash = agentlist$cash )"
rserve:eval "set1 <- subset(datatestsc, agentlist$skill <= quantile(agentlist$skill,
0.25))"
rserve:eval "set2 <- subset(datatestsc, agentlist$skill >= quantile(agentlist$skill,
0.75))"
rserve:eval "set3 <- subset(datatestsc, agentlist$skill <= quantile(agentlist$skill,
0.10))"
rserve:eval "set4 <- subset(datatestsc, agentlist$skill >= quantile(agentlist$skill,
0.90))"
rserve:eval " datatestsc2 <-data.frame ( Skill = agentlist$skill, utility =
agentlist$utility)"
rserve:eval "set5 <- subset(datatestsc2, agentlist$skill <= quantile(agentlist$skill,
0.25) )"
rserve:eval "set6 <- subset(datatestsc2, agentlist$skill >= quantile(agentlist$skill,
0.25) )"
print "Welch Two Sample t-test"
print "data: Cash Agent skill fourth quartile Vs skill first quartile "

```

```

rserve:eval "t.test(set2$Cash,set1$Cash)"
print " T test statistic value"
print rserve:get "t.test(set2$Cash,set1$Cash )$statistic "
print "Degree of freedom"
print rserve:get "t.test(set2$Cash, set1$Cash)$parameter"
print "P-Value"
print rserve:get "t.test(set2$Cash, set1$Cash)$p.value"
print "Mean estimates for set2 and set1 "
print rserve:get "t.test(set2$Cash, set1$Cash)$estimate"
print "Alternative HP"
print rserve:get "t.test(set2$Cash, set1$Cash)$alternative"
print ""
print "Welch Two Sample t-test"
print " data: Utility Agent utility skill quartile Vs skill first quartile "
rserve:eval "t.test(set6$utility,set5$utility)"
print " T test statistic value"
print rserve:get "t.test(set6$utility,set5$utility )$statistic "
print "Degree of freedom"
print rserve:get "t.test(set6$utility,set5$utility)$parameter"
print "P-Value"
print rserve:get "t.test(set6$utility,set5$utility)$p.value"
print "Mean estimates for set6 and set5 "
print rserve:get "t.test(set6$utility,set5$utility)$estimate"
print "Alternative HP"
print rserve:get "t.test(set6$utility,set5$utility)$alternative"
print ""
print "Welch Two Sample t-test"
print "data: : Cash Agent skill fourth quartile Vs skill first quartile "
rserve:eval "t.test(set2$Cash,set1$Cash, alt ='g' )"
print " T test statistic value"
print rserve:get "t.test(set2$Cash,set1$Cash, alt ='g' )$statistic "
print "Degree of freedom"
print rserve:get "t.test(set2$Cash, set1$Cash, alt ='g')$parameter"
print "P-Value"
print rserve:get "t.test(set2$Cash, set1$Cash, alt ='g')$p.value"
print "Mean estimates for set2 and set1 "
print rserve:get "t.test(set2$Cash, set1$Cash, alt ='g')$estimate"
print "Alternative HP"
print rserve:get "t.test(set2$Cash, set1$Cash, alt ='g')$alternative"
print ""
print "Welch Two Sample t-test"
print ": Cash Agent skill fourth quartile Vs skill first quartile "
rserve:eval "t.test(set3$Cash,set4$Cash, alt ='g' )"
print " T test statistic value"
print rserve:get "t.test(set3$Cash,set4$Cash, alt ='g' )$statistic "
print "Degree of freedom"
print rserve:get "t.test(set3$Cash,set4$Cash, alt ='g')$parameter"
print "P-Value"
print rserve:get "t.test(set3$Cash,set4$Cash, alt ='g')$p.value"
print "Mean estimates for set3 and set4 "

```

```

print rserve:get "t.test(set3$Cash,set4$Cash, alt ='g')$estimate"
print "Alternative HP"
print rserve:get "t.test(set3$Cash,set4$Cash, alt ='g')$alternative"
print ""
print "Cash-Skill correlation in the population"
print rserve:get "cor(agentlist$cash, agentlist$skill, method='spearman')"
print ""
print "Cash-Skill correlation for 25% highest skilled agents"
print rserve:get "cor(agentlist$cash[agentlist$skill >= quantile(agentlist$skill, 0.75)],
agentlist$skill[agentlist$skill >= quantile(agentlist$skill, 0.75)], method='spearman')"
print ""
print "Cash-Skill correlation for 10% highest skilled agents"
print rserve:get "cor(agentlist$cash[agentlist$skill >= quantile(agentlist$skill, 0.90)],
agentlist$skill[agentlist$skill >= quantile(agentlist$skill, 0.90)], method='spearman')"
print ""
; print "Mean for Q10"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.90)])"
; print "Mean for Q09"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.80) & agentlist$skill < quantile(agentlist$skill, 0.90) ])"
; print "Mean for Q08"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.70) & agentlist$skill < quantile(agentlist$skill, 0.80) ])"
; print "Mean for Q07"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.60) & agentlist$skill < quantile(agentlist$skill, 0.70) ])"
; print "Mean for Q06"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.50) & agentlist$skill < quantile(agentlist$skill, 0.60) ])"
; print "Mean for Q05"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.40) & agentlist$skill < quantile(agentlist$skill, 0.50) ])"
; print "Mean for Q04"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.30) & agentlist$skill < quantile(agentlist$skill, 0.40) ])"
; print "Mean for Q03"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.20) & agentlist$skill < quantile(agentlist$skill, 0.30) ])"
; print "Mean for Q02"
; print rserve:get " mean (agentlist$cash[agentlist$skill >=
quantile(agentlist$skill, 0.10) & agentlist$skill < quantile(agentlist$skill, 0.20) ])"
; print "Mean for Q01"
; print rserve:get " mean (agentlist$cash[agentlist$skill < quantile(agentlist$skill,
0.10) ])"
; print ""
]
stop ]
end

```

```
to r-idle
rserve:eval "Sys.sleep(0.01)"
end
```

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