Mark-to-Market and Leveraged Trading in a Speculative Market: A Simulation with Zero Intelligent Agents

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In liquid markets the frequency or timing of settlement (marking-to-market) may not be expected to impact prices. However during illiquid periods with leveraged trading, settlement can have a significant impact on forced liquidations and price volatility. This paper presents an agent-based model of real time settlement in a leveraged market with zero-intelligent traders in an open-outcry double auction, stylized after a futures market. Initial simulations produce the emergent market phenomenon of volatility clustering in returns and serial correlation in prices, which is related to the degree of leverage among traders. These results suggest that mark-to-market real time settlement promotes overshooting in prices, as price changes lead to a cascade of deleveraging among market participants in an illiquid market.

INTRODUCTION

The current financial crisis has once again highlighted the fragility of markets, where interacting market participants create multiplicative processes that are not necessarily stable or optimizing. This is counter to the efficient markets hypothesis (EMH) which argues suggests that out of equilibrium trades are random and exogenous shocks lead to unlimited arbitrage and price convergence to fundamentals. This 'near religious devotion' (Lo 1999) or belief is free markets as the most efficient manner in which to allocate resources takes market prices as the best measure of a security's intrinsic or fair value. Indeed under the guidance of perfect competition, prices are not only informationally efficient but also Pareto efficient, where supply equals demand. This theory infers that markets, with many independent agents, has a kind of inbuilt self-regulating mechanism of convergence, stability, and efficiency.

In contrast to the this information consolidation view of market prices, agent-based modelers have argued that prices should be recognized as an emergent outcome of interacting agents and institutions rather than a thermometer reflecting all available information. In such a setting one can drop the assumptions that individual investors form expectations rationally; markets aggregate information efficiently; that action is independent of others; that prices incorporate all available information; and that prices are at equilibrium.

In studying market results as emergence researchers typically model behavioral inconsistencies or animal spirits (e.g. trend followers) as opposed to rationality, considering how such behavior impacts prices. An alternative is to model random trading in order to isolate the *institutional* determinism of prices. Gode and Sunder (1993) highlighted the impact that the double auction (DA) has on random or zero-intelligent (ZI) noise traders. If one was to define the equilibrium price as the equation of supply and demand, this ZI research program has shown that such prices can be attained just be institutional rules. An efficient market outcome can be a purely institutional derivation rather than one of rational choice and individual optimization. In turn, institutions and rules can be the conduit for instability.

During the ‘great moderation’ there was substantial growth in the shadow banking sector: financial institutions that acted like banks - borrowing short and lending long - but lacked the regulation compared to banks and without a bank bail-out guarantee to save them from bankruptcy.[1] Following the 2007-2008 financial crisis these highly leveraged institutions became an area of focus institution for systemic risk. Such a scenario is typically modeled within a social coordination problem, such as the Diamond and Dybvig (1983) ‘assurance game’ model where there are two Nash equilibria: one that is payoff dominant and the other is risk dominant. In the face of uncertainty such a setting leads to heightened risk aversion and flight to quality - leading to destabilizing dynamics and a recession. The solution offered by certain ethics and accounting boards (e.g. the CFA Institute) was to advocate mark-to-market valuations or apply *fair value accounting* methods to make valuations transparent and to increase market confidence. They argued that the only way to create confidence was to write down assets fully, such that a floor is placed on the value of these assets, at which point, a fair assessment of the situation can be used to asses the appropriate risk aversion and avoid social coordination problems. This view emphasizes the lack of trust between financial market participants and the endogeneity of expectations as the cause for bank runs or security liquidations. It is a micro solution targeted in applying a conservative *value at risk* and transparency.

An alternative to the coordination approach is to consider exogenous rules or institutional architecture as a conduit for price instability. The *Institute of International Finance* put out an interim report for market best practices in April 2008 in light of the 2007-08 global credit crisis. One part of this report was on the practice of fair-value or market to market accounting. They write

> “Over the past decade, fair-value/mark-to-market accounting has generally proven highly valuable in promoting transparency and market discipline, and continues to be an effective and reliable accounting method for securities in liquid markets. When there is

[1] It was recently thought that banks only needed a lender of last resort to save them from bankruptcy, but the mistrust and gridlock in interbank markets have shown that only a guaranteed capital injection can keep a bank solvent. The cause of this has its seeds in the mark-to-market valuations of bank assets and bank capital. Previously, the refinancing of long term assets with short term debt was facilitated by borrowings from the central bank and interbank markets, maintaining bank cash flow. However, now that bank capital is being revalued under new fair value accounting, access to funds is no longer sufficient to maintain bank solvency.

> no or severely limited liquidity in secondary markets, however, it has the potential to create serious and self-reinforcing challenges that both make valuation more difficult and increase the uncertainties around those valuations. … There are questions about whether fair-value accounting approaches have increased the severity of the market stress … [exacerbating] the overall degree of risk aversion in the marketplace … [i]n circumstances where doubts about products and underlying credit quality undermine valuations inducing extensive margin calls, there is the danger of a precipitous and destructive downward spiral, which reinforces the procyclical impact. … While there is no desire to move away from the fundamentals of fair-value accounting, the Committee feels that it is nonetheless essential to consider promptly whether there are viable sound proposals that could limit the destabilizing downward spiral of forced liquidations, writedowns and higher risk and liquidity premia." (2008, pp.15-17)

In this account it is the mechanism rather than human strategic behavior that is being blamed for price instability, price overshooting, and contagion. Indeed some in the press have even called the mark-to-market accounting methods as "dooms day devices" (John Gizard March 2008, *Financial Times*).

In simulating a virtual financial market, a model could contain both behavioral strategy and the existence of institutional rules that guide or override strategic behavior. This paper specifically tries to disentangle these two approaches. Bowles (2004, pp.58-66) divides the design of an evolutionary model into two forms. Either a *biological system* depicting zero-intelligent evolutionary dynamics under the combined influences of chance, inheritance, and natural selection. Or an *adaptive agents* model where forward looking payoff-based calculations exist, but such optimization has 'bounded rationality' and is often dominated by group dynamics and herd behavior. Adaptive behavior that is based on tradition, imitation, experience and even cooperation or morality. Between these two agent-based systems, there is also the evolution of institutions which constrain individual choices or actions and set the landscape upon which agents interact.

While an understanding of behavioral responses is crucial to any policy solutions, this paper takes another aspect of the problem and analyses the impact of institutional and environmental constraints, confining the trader population to random speculation (as opposed to strategic or adaptive behavior that might respond to information). We find evidence that trading rules over leveraged and illiquid markets can have destabilizing effects on the price discovery process.

ZERO-INTELLIGENT AGENTS

Smith (1982) defines three categories that determine the performance of a micro system: the *institutional structure* defines the market protocol and the rules that govern trading; the *environment* (number of markets, agents, and initial conditions for agents' tastes, risk profile and endowments - information and resources); and *agent behavior* in terms of learning or adapting ones trading strategy over time. This last criteria is referred to by Gode and Sunder (1993 & 1997) as "intelligence" and in economics this is equivalent to a rational or a utility optimizing agent.

It is in this regard that we use zero intelligent (ZI) agents, which despite their lack of rationality, are very good at producing stable and efficient (Pareto optimal) stock market outcomes when there is no borrowing (Gode and Sunder 1993, hence forth known as GS; and Gode, Spear and Sunder 2004, hence forth known as GSS). By adapting this model to use risk neutral agents who trade on margin, we can easily reproduce a scenario of leveraged trading and binding capital constraints. While participant expectations are typically sourced as the cause of financial instability, the model presented here isolates the contribution that comes from balance sheet accounting rules.

THE MODEL

A zero-intelligent (ZI) model of *n* risk neutral speculators, who have the same initial endowment of money, m_0^i, and futures contracts, $x_0^i = 0$, for all *i* traders. All cash is kept in a *margin account* which cannot not be added to exogenously. Each speculator or trader will buy low and sell high to make a profit in a futures contract *x*, on a non-storable underlying commodity, liquidated prior to the spot date.[2] Each speculator has their own subjective valuation about the future spot price of the commodity, $p^{i,\theta}$, at some date in the future beyond the period of study. This valuation is drawn from a symmetric Beta[2,2] distribution with finite boundaries. The valuation does not change, characteristic of our ZI agents who have no learning or evolving

[2] A non-storable commodity has a futures valuation equivalent to the expected final spot price of that commodity Commodities that are storable would be complicated by *cash and carry* pricing formula from arbitrageurs. The maturity date of the futures contract is considered to be in the future beyond our analysis and does not figure into any calculations. A model with interest rates and spot markets was considered in previous work (Ussher 2005a).

animal spirits. Simulated time *t* represents the sequence of discrete bilateral transactions in futures contracts at price p_t . At transaction time *t*, the sale or purchase of futures contracts Δx_t^i by speculator *i*, at either the best ask p_t^a, or best bid p_t^b, according to the rules of an open-outcry *double auction* (DA) mechanism which makes the highest bid and the lowest ask the standing prices at which a *crossing limit order* trades. There is no restriction on short selling: trader *i*'s contract position at any one time, x_t^i, can be positive (long contracts) or negative (short contracts).

In this model the futures contract size and price are real values and perfectly divisible, that is, there is no pre-specified tick, and unlike most models the order size is allowed to be different from 1. Since there is no limit order book, the size of each trade is the minimum of the desired amount between the two matching traders, and there is no direct price impact as in markets with limit order books, where large orders 'walk up the book.' But large orders can deplete the desire to trade by the *market makers*, those traders with the best current bid or ask, who drop out. This will be explained in the next section and allows for large price shocks.

The number of futures contracts is endogenous to the trading process, but as in all such markets futures contracts always sum to zero $X_t = \sum_i x_t^i = 0$ for all t. The *futures exchange* charges a percentage transaction cost, $\omega p_t \left| x_t - x_{t-1} \right|$, incurred on each one-way trades (long or short). This creates a threshold around $p^{i,\theta}$ for which demands for changing one's contract position is zero. In other words, speculator *i* wants to hold their current position for all $\left|(p^{i,\theta} - p_t)\right| \leq \omega p_t$. In contrast, speculator *i* will want to take on a longer position when prices are expected to rise by more than the tax: $p^{i,\theta} > (1+\omega)p_t$; and a shorter position when prices are expected to fall by more than the tax: $p^{i,\theta} < (1-\omega)p_t$. Note that a speculator is not forward thinking enough to consider the tax incurred for the future liquidation of the contract position.

Order size, or demand, is dependent on the speculator's collateral or wealth position and the leverage limit placed on traders, by the Exchange. A risk neutral ZI speculator, *i*, uses all his wealth at time *t* to take on a futures position. We can plot a demand function from basic principles of linear optimization.

Since futures are promises in a derivatives market, rather than a realized exchange in goods, there is no trade-off between holding both futures and cash, there is only a margin requirement that is held as cash in a margin account. For simplicity there is no interest paid on cash deposits. In a model with no taxes, the budget set is a rectangle in a 2 goods space, m_t^i and x_t^i, with a positive or negative net amount of futures x_t^i. Figure 1 shows such a budget set, where cash is on the vertical axis and contracts on the horizontal axis. Instead of the horizontal line at m_t we have two sloping lines to accommodate the shrinkage in capital due to transaction taxes on changing ones position from x_{t-1} to x_t. In this figure the speculator is carrying over a long futures position x_{t-1} from the previous price at *t-1*.

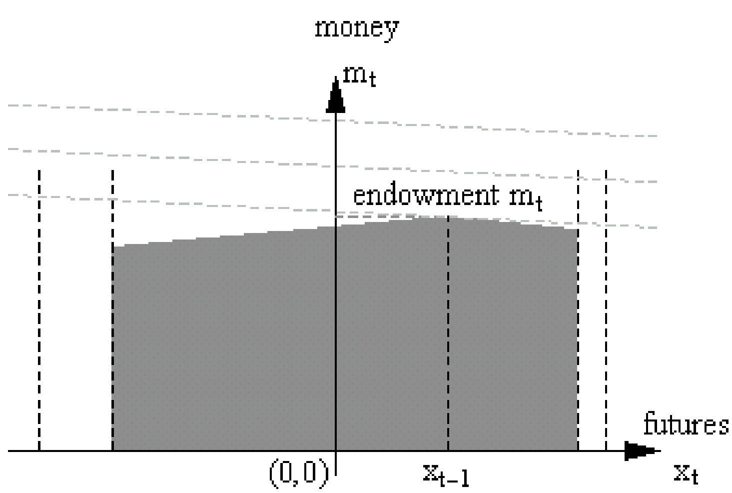


Figure 1: Bullish Speculator's Budget Set and Expected Wealth Objective.
$\left(p^\theta - p_t\right) > 0; x_{t-1} > 0; 0 < \omega < 1; \kappa = 1$

Important in this model is the real time settlement of margin accounts or wealth positions, at every *t* by the Exchange. All traders must meet a *variation margin requirement* on all positions with real-time-settlement (RTS). [3] In practice, the margin requirement allows the exchange to cover all counterparty risk, that is removing the risk of default. The inverse of the margin

[3] *Maintenance margin* is the minimum margin requirement and typically some fraction of the *initial margin* which the trader must maintain to avoid a *margin call*. If the maintenance margin is equivalent to the initial margin then the *margin call* is the *variation margin*.

requirement is also known as the leverage ratio. If the Exchange reduces the margin requirement $m_t^i / p_t x_t^i$, from 50 to 25 percent, then they also double the leverage ratio, $p_t x_t^i / m_t^i$, from 2 to 4. If prices rise and a trader has a long position then the budget set will expand through mark-to-market and margin payments. If prices fall and a trader has a negative position, then the budget set will also increase. As in all futures markets, for every price change there is a winner and a loser between traders with contract positions. This symmetry, along with the symmetry in expectations, and the zero default risk, allows for a neutral testing ground for the impact of market-to-market or RTS on prices, void of other business cycle impacts such as expectations and wealth loss.

Unlike GSS where agents have constant absolute risk aversion (CARA), here agents are risk neutral as in Chan *et al* (1998). Each speculator has a monotonic (linear) objective function for money, shown in Figure 1 as pale grey negatively sloped dashed lines. By speculating in a future market they can generate more (expected) money. This objective function is represented by equation (1), and its slope is determined by whether the speculator expects prices to go up or down. Given the chance to trade, a speculator will either do nothing, plunge long, or plunge short: investing as much as possible.

The amount that a speculator will place as a limit order for a given price can be derived as the speculator's demand curve at time *t*, which is not only affected by transaction costs and the margin requirement, but also by the settlement frequency since the current mid-price is used to mark-to-market wealth and this also determines the amount a trader can afford. There are also margin calls transferring money from a losers account to a winning account. Adjusting the capital base *m* every *t* for RTS valued at p_t^m.

We use all these factors to solve for each ZP speculator's demand function for futures x_t^i by speculator *i* from the following objective and constraints:

Maximize:

$$\pi_{t+1}^e = \left(p^\theta - p_t\right)x_t + m_t \tag{1}$$

Subject to:

$$p_t x_t \geq -\kappa\left(\left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} - \omega\ p_t\left(x_t - x_{t-1}\right)\right) \quad (2)$$

$$p_t x_t \geq -\kappa\left(\left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} + \omega\ p_t\left(x_t - x_{t-1}\right)\right) \quad (3)$$

$$p_t x_t \leq \kappa\left(\left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} - \omega\ p_t\left(x_t - x_{t-1}\right)\right) \quad (4)$$

$$p_t x_t \leq \kappa\left(\left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} + \omega\ p_t\left(x_t - x_{t-1}\right)\right) \quad (5)$$

$$m_t \leq \left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} - \omega\ p_t\left(x_t - x_{t-1}\right) \quad (6)$$

$$m_t \leq \left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} + \omega p_t\left(x_t - x_{t-1}\right) \quad (7)$$

$$0 \leq \left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} - \omega\ p_t\left(x_t - x_{t-1}\right) \quad (8)$$

$$0 \leq \left(p_t^m - p_{t-1}^m\right)x_{t-1} + m_{t-1} + \omega\ p_t\left(x_t - x_{t-1}\right) \quad (9)$$

$$m_t \geq 0 \quad (10)$$

This produces a demand function:

$$x_t^i\left(p_t; p^{i,\theta}, \varepsilon, x_{t-1}^i, m_{t-1}^i, p_t^m, p_{t-1}^m, \kappa, \omega\right) \quad (11)$$

where:

p_{t}	=	Price at tick t, which must be at either a bid p_{t}^b, or an ask p_{t}^a
$p^{i,\theta}$	=	Price valuation of the next futures price p_{t+1} (and long run spot price)
$x_{\mathrm{t-1}}^{\mathrm{i}}$	=	Previous contract position,
$m_{\mathrm{t-1}}^{\mathrm{i}}$	=	Previous cash position in margin account following last transaction,
p_{t}^m	=	Current mid-price, used by exchange to mark-to-market position $x_{\mathrm{t}}^{\mathrm{i}}$ under RTS
$p_{\mathrm{t-1}}^m$	=	Previous mid-price, used by exchange to calculate position change $\Delta p_{\mathrm{t}}^m x_t^i$ under RTS
$1/\kappa$	=	Percentage margin requirement (where κ >1) of futures position $x_t^i p_{\mathrm{t}}^m$ valued at p_{t}^m
ω	=	Percentage transaction tax on a one-way trade (i.e. paid each way).
ε	=	Interpolation parameter to solve for indeterminacy

In summary, the risk-neutral speculator maximizes next periods expected wealth (1). The first four boundary constraints represents the limit on a speculator's investment by the margin requirement when one is short in futures, (2) and (3), versus the extent to which futures can be long, (4) and (5). These are the black dashed vertical lines in Figure 1. We have two each of these restrictions to take into account the one-way tax on both buying and selling

$\omega p_t\left|(x_t - x_{t-1})\right|$. If the transaction tax is positive then one of these boundary constraints will be slack for both short or long positions. The tax imposition also changes the marginal change in ones position $(x_t - x_{t-1})$ creating a maximum point for cash at the hold position, (6) and (7). The bankruptcy conditions, (8) through (10), stop money wealth from voluntarily going below zero.

There is an indeterminant solution when the objective or iso-wealth function is parallel to the budget set, $p_t = \dfrac{p^{i,\theta}}{(1 \pm \omega)}$. To solve this we use an interpolation method that multiplies the slope of the iso-wealth function by a very small amount $(1 \pm \varepsilon)$ to interpolate (x^*, m^*) by drawing a line from the origin, at x_{t-1} and cash position minus maximum potential taxes, to the intersection of these two modified iso-wealth functions. This solution is shown diagrammatically in Figure 2. In our simulations below we use $\varepsilon = 0.001$ to produce an interpolation gap that is very small, a range for p_t of less than 0.0003 on either side of expectations. This retains agents to be effectively risk neutral but removes the discontinuity.

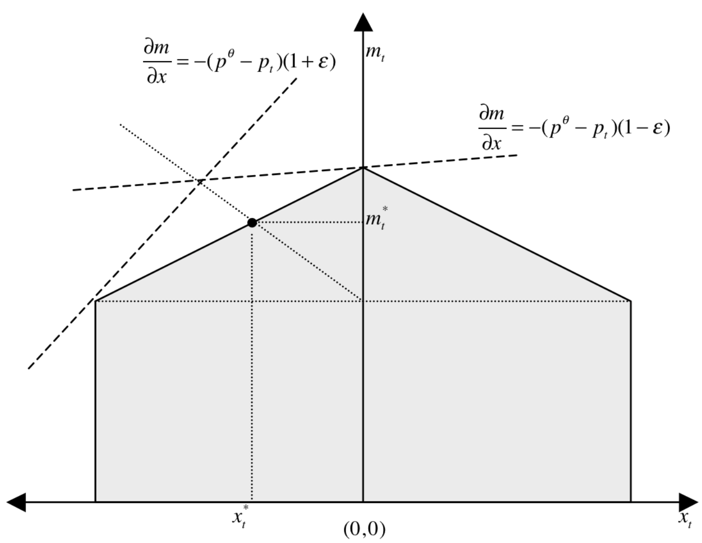


Figure 2: Bearish Speculator using the ε interpolation method to solve for x_t
$x_{t-1} = 0; \kappa = 1$.

The speculative RTS demand schedule can is drawn in Figure 3. Usually a futures contract demand curve represented as a smooth downward sloping line from the top of quadrant II to the bottom of quadrant I in the two dimensional R^2 space. Here, the speculator's demand curve is piecewise due to risk neutrality and the transaction tax ω, and dominated by corner solutions:

either buy, sell or hold. It asymptotically approaches the price and quantity axis due to the income effect on fixed cash holdings m_t^i.

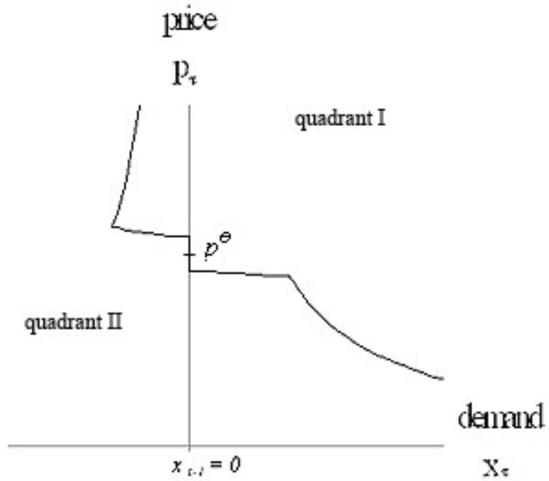


Figure 3: Demand for futures x_t as a function of p_t
$(x_{t-1} = 0)$

The last 6 terms of the demand function in equation (11) are out of the ZI speculator's control. Input into his decision is only based on the current price and his valuation to make his decision to dive into a position or do nothing. The last 6 terms are in effect outside of his behavior – environmental and institutional factors. All money is kept as collateral in the margin account by each speculator.[4] Each speculator can leverage their absolute futures to cash position to a maximum ratio of $\kappa \geq \left| p_t^m x_t^i \right| / m_t^i$, where $\kappa \geq 1$ and set by the exchange. The margin requirement is assessed in real time at the midpoint price. If collateral is less than the real time margin requirement, that is $m_t^i < \left| p_t^m x_t^i \right| / \kappa$, then speculator *i* will be forced to liquidate their position with an offset, purchase or sale, of the required amount to avoid a *margin call*, or injection of new capital.[5]

The competitive bid-ask spread is determined by the difference in expectations between the best bidder and the best offerer, plus the transaction tax approximating $2(1+\omega)p_t^m$ if valuations near

[4] In real markets this is typically held as Treasury bonds earning the risk free rate, hence it is often said that there is no opportunity cost in holding futures.
[5] Deposits of new cash to satisfy margin calls are ruled out as they require additional cash placed into the account from outside the model.

the best bid and ask are far apart. It is possible for one speculator to offer both sides of the market with a spread $2\omega p_t^m$ but even this can be narrowed with two traders if valuations are closer than this distance. An increase of the transaction tax would possibly widen the bid ask spread as shown by a single demand function in Figure 4. Increasing the allowed leverage ratio, κ, will stretch the middle of the single trader's demand curve out, also shown in Figure 4.

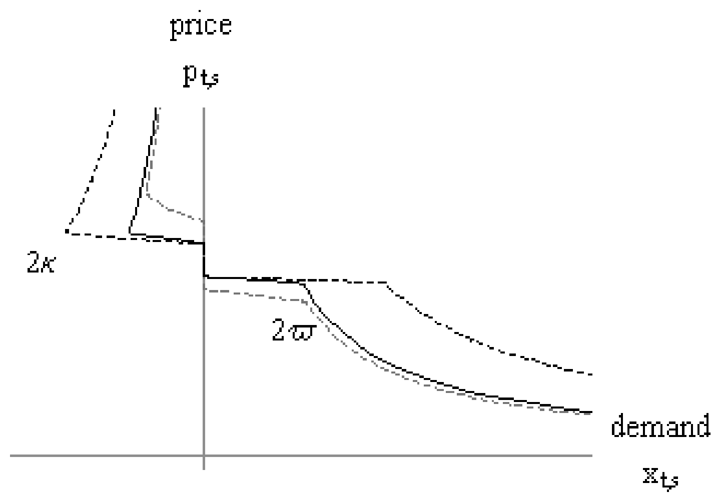


Figure 4: Speculator *i*'s demand curve with the dashed line doubling ω or κ.
($\varepsilon = 0.2$ and $x_{t-1}^i = 0$)

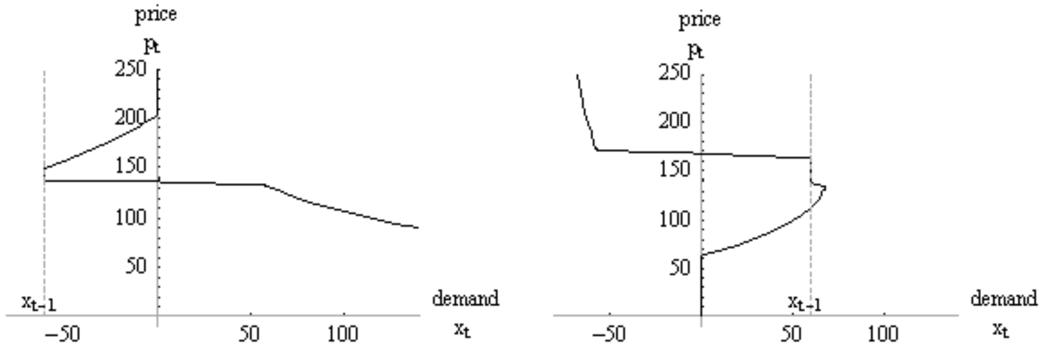
Marking-to-Market in Real Time

Marking to market positions in real time is the duty of the exchange. At each tick time *t* the exchange will simultaneously redistribute profits and losses across all traders valued at the mid-price, which is the average of the best bid and ask: $p_t^m = (p_t^b + p_t^a)/2$.

The profit or loss is calculated with price changes of the mid-price from one tick to the next, and it shows up in our budget conditions (1) through (9): $\left(p_t^m - p_{t-1}^m\right)x_{t-1}^i$. If a speculator currently has a prior futures position, $x_{t-1}^i \neq 0$, then RTS may lead to forced liquidation on losing position to lower the leverage to the maximum, if prices move against expectations. This is the possibility of a backward bending demand function, as in Figure 5. This is typical of markets where collateral that underlies demand for *x*, is priced in the same market.[6] While the margin requirement determines how big a position a speculator can take to reinforce its expectations, it

[6] These demand functions are a slightly simplified version from Ussher (2004) as the interest on money holdings here is zero.

can also force a speculator to trade in order to liquidate a position, in opposition to a traders expectations.



Speculator is short futures $x_{t-1} = -60$ Speculator is long futures $x_{t-1} = +60$

Figure 5: Backward Bending demand curves for short or long starting positions ($m_{t-1} = 5000$, $p^{i,\theta} = 150$, and $\kappa = 2$)

To ease exposition of RTS all traders have their positions valued at the midpoint at each time t. When an exchange occurs between two traders at the bid or ask, this change in their position is immediately revalued to the midpoint. This simplifies the updating of positions in real time for traders who are not trading, from one p_{t-1}^{m}, to the next p_t^m. It also explains the $\left(p_t^m - p_t\right)\left(x_t^i - x_{t-1}^i\right)$ element of equation (9) for the trading agent.

If one were to aggregate the individual demand curves together, along Walrasian lines where all traders could trade simultaneously, then it is easy to see how we can end up with multiple equilibria, see Figure 6. This result is very similar to the analytical model presented by Chowdhry and Nanda (1998) which can have multiple prices at which the market can clear due to margin requirements.

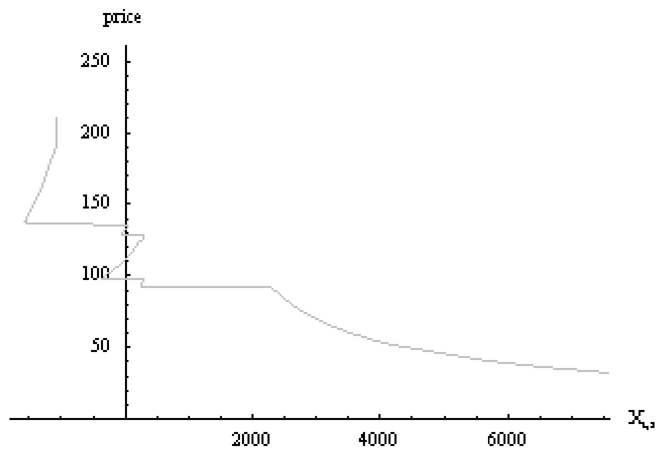


Figure 6: Speculator *i*'s demand curve with the dashed line doubling ω or κ .
($\varepsilon = 0.2$ and $x_{t-1}^{i} = 0$)

BIDDING BY ZERO INTELLIGENT SPECULATORS

The above derivation of demand only explains the quantity exchanged at a transaction price if there is no limit to order size imposed on by the conterparty. What limit order price a trader submits is based on their non-optimizing, satisficing, static behavior rules. Which price is ultimately used for a bilateral trade is determined by the rules of the DA mechanism. The auction consists of a sequence of bidding between each transaction. Speculators are selected randomly with non-replacement in each round to place their limit order. Hence each trader has an equal chance of trading every round. Speculators trade if their bid crosses or is crossed by another limit order according to the DA rules. Quantities traded and their transaction prices are registered at each time *t*.

The DA replicates *open-outcry* on the floor of a futures exchange where "a quote is good only as long as the breath is warm" (Erenburg *et al* 2003, p.7). Only the best bid and best ask prevail at any one time.[7] This is formally known as a limit order-book of size one, as there is one quote on

[7] In a continuous DA open-outcry futures market, as described by Silber (1984), all *bids* and *offers* must be announced publicly to the pit through the outcry of buy or sell orders. Strict priority is kept, where the highest bid price and the lowest offer take precedence, known as the *inside spread*. Lower bidders must keep silent when a higher bid is called out, and higher offers are silenced when a lower offer is announced. All bids and offers, even non-competitive ones, are called *limit orders*. For every limit order there is a quantity attached. The current market bid and ask remain until the quantity they want is filled.

either side. However, since there is no accumulation of verbal limit orders between transactions there is no need for a tally book.[8]

This model casts ZI agents as impatient, or having zero-patience (ZP). Since no order book means that non-competitive limit orders expire instantly, traders instead are characterized as demanding *immediacy*, the desire to trade immediately by posting values close to their reservation prices. LiCalzi and Pellizzari (2007, p. 3573) call these traders "almost truth telling" agents. The random trading element among these zero patient (ZP) traders is the sequence of the order flow and the valuation distribution of the trading population, rather than the limit order as in GS and GSS.[9]

The bidding algorithm for ZP speculators is drawn from Chan *et al* (1998). They use a parameter *s* equal to the half spread to determine the limit order when the ZP valuation $p^{i,\theta}$ is between the best bid and ask. This is explained briefly in the example below (for more detail on this DA mechanism see Ussher (2005).

SIMULATED ZI AND ZP TRADING

Each speculator's reservation valuation is drawn from a symmetric Beta(2,2) distribution bounded between 100 and 130. This distribution is very similar to a Normal distribution: N(115,7.5), but with finite boundaries and thinner tails. The realizations for price valuations $p^{i,\theta}$ for the 200 traders is shown in Figure 7.

[8] In the United States the New York Mercantile Exchange, the Chicago Mercantile Exchange, the Chicago Board of Trade, the Chicago Board Options Exchange, the Minneapolis Grain Exchange (MGEX), and in the United Kingdom the London Metal Exchange still makes use of open outcry.

[9] The ZI algorithm outlined by GS and GSS had random limit orders or *random mark-ups* on a buyer's or seller's reservation price. This random limit price is bound by each trader's fixed reservation price and a floor for buyers, or ceiling for sellers. Limit orders accumulate in the order-book in between transactions. When a transaction occurs the book is emptied.

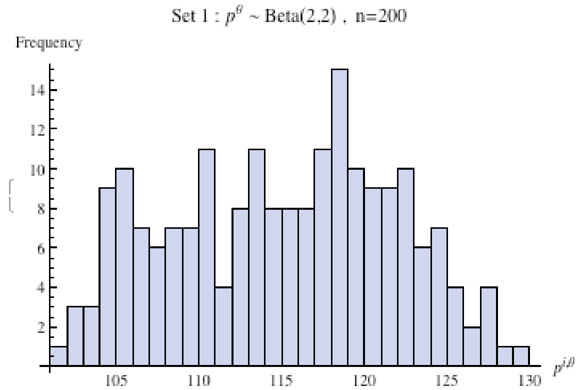


Figure 7: ZI price expectations

Global parameters:

n	=	200	κ	=	8
t	=	2000	ω	=	0.001
m_0^{i}	=	\$10,000 for all i	ε	=	0.001
$p^{i,\theta}$	~	100+30*Beta(2,2)	$p_0^{7,b} : p_0^{5,a}$		(120:130)

This double auction run is initialized with traders $i = \{7,5\}$ with an exogenous bid (120) and ask (130) quote at their desired order size for this price (which could be zero). Shown below is one run that exemplifies the type of results attained from this model for 200 traders for a leverage ratio $\kappa = 8$, otherwise known as a 12.5 per cent margin requirement. The transaction tax is kept small to simplify the results. In a model with retrading, and without wealth constraints prices converge quickly to the equilibrium and remain there (Gode and Sunder 1993). In this model wealth is constrained and marked-to-market in real time, margin trading and short selling is allowed, a small tax is imposed on all one-way trades, and the order size of each trade is real and unrestricted.

In the top left graph of Table 1 we show in black the simulation for a series of 2000 bilateral transactions for a price and quantity at time t. It has the characteristic bid ask bounce of tick data. We have also plotted the midpoint price in yellow. After the simulation run, we solve *ex post* the

Walrasian equilibrium solution p_t^* for the desired demand for all traders at each *t*, using *Mathematica*'s fixedpoint algorithm, which uses *Newton's Method*. At this point t, plotted in red, a Pareto Optimum prevails since gains from trade across all agents are exhausted. In this particular case we find that the equilibrium is quite stable and unique throughout.[10]

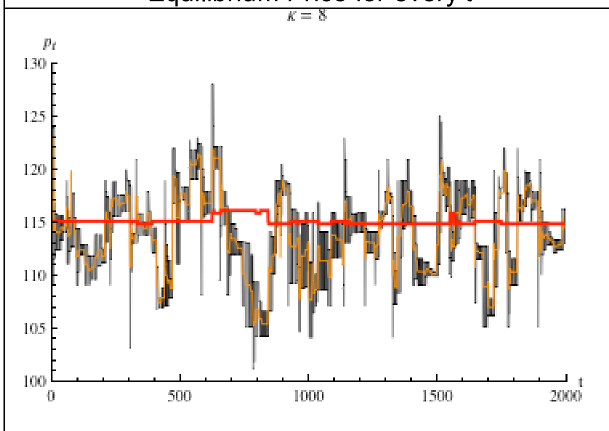
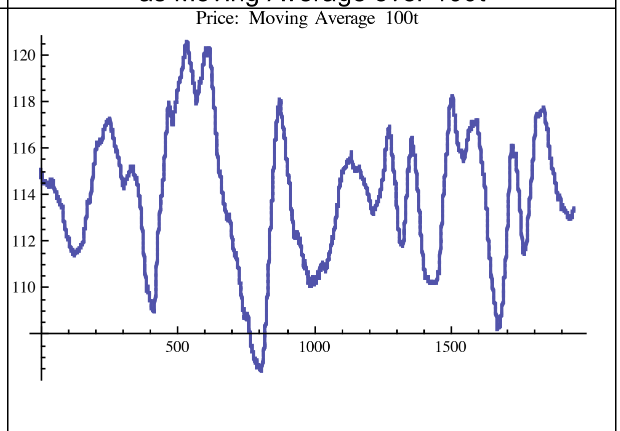
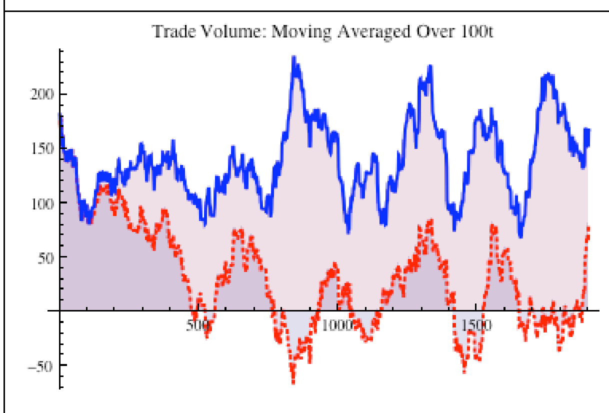
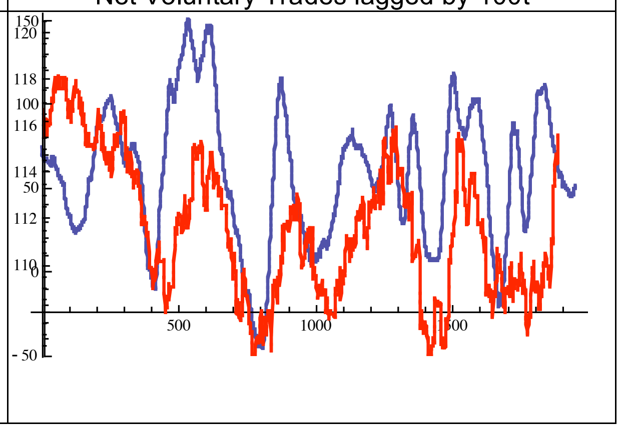
Transaction Price, MidPoint Price, and Walrasian Equilibrium Price for every *t*	Transaction Price as Moving Average over 100t
	
Average: Volume and Net Voluntary Trades	MA Transaction Price and Net Voluntary Trades lagged by 100t
	

Table 1: Simulation with leverage 8 times

Prices are mean reverting around the Walrasian equilibrium, but it takes some time for prices to revert, and they appear to typically overshoot and be positively serially correlated. The next 3 graphs removes some of the noise from this time series and takes a moving average of 100 *t* our

[10] This is not so when there are gaps between trader expectations and the number of traders is smaller, or when the margin requirement is lower (5%) and large wealth distributions occur which can create multiple equilibria.

the time series. The top right hand is the transaction price, and it shows that prices range within one standard deviation from the equilibrium price, within the extreme expectations [100, 130].

The concern with mark-to-market valuation, is thought to be resolved when there are traders who are willing to come in and buy securities putting a floor underneath further price falls. This is what did occur in some cases with the sovereign wealth funds being ready to buy bank shares during the recent subprime crisis. But whether those that step in win from such trades depends on how much capital buffer they have:

"..Geared investors are forced to sell to lower their borrowings, pushing down the market and forcing other leveraged investors to sell – with the potential nightmare being a wave of forced deleveraging…those that have stepped in to catch the falling knife [buying the cheaper assets] are finding themselves seriously injured" (*Financial Times*, 5 March 2008).

If one takes the example of the price from 600*t* to 800*t*. While prices are falling, those traders who are long, will be forced to make margin calls, and liquidate their positions to maintain the 12.5 per cent margin requirement. On the other hand, those traders who are short, will be reaping these payouts, and will be able to go short even more, or start buying, depending on where the price is relative to their static expectations. In the closed model here, there is a symmetrical number of bulls and bears when the price is at its equilibrium value. This includes the changing wealth distribution which gradually gets more skewed as time goes on.

Despite the balance of expectations and supply and demand, there is still quite a good deal of positive serial correlation in prices. If we look at the bottom left hand graph, the blue line is the total volume (either buys or sells), averaged over moving 100*t*. The red line is the net volume of trades that are voluntary as opposed to forced. A voluntary trade is when a bull buys and a bear sells. Involuntary trades are when margin calls force liquidation of positions and bears buy and bulls sell. The red line adds together all the single market orders (crossing limit orders) that are voluntary with a positive sign, and all the forced trades with a negative sign. If voluntary versus forced market orders are, on average, half and half over 100*t* then the red line will be at the zero mark on the y-axis. This graph (which plots the moving average over time) shows that voluntary trading is declining relative to forced trading over time – the market is becoming less liquid. We can see that in the first 200 transactions basically all trades were voluntary as the net amount above forced trades equals total volume.

If we consider a liquidity crisis to be one where fire sales are made and forced deleveraging occurs, then we see that this generally happens directly after large price swings. In the bottom right graph, the price series is plotted again, along with the red line of net voluntary trading lagged by 100*t*. While further investigation is required, a cursory analysis seems to match up the lagged trading with price movements. Further investigation into liquidity crises in a market where expectations do not change is interesting since it emphasizes the manner in which trading rules can produce overshooting in prices when marking-to-market is implemented, although it may not occur simultaneously, as the deleveraging takes time to emerge into a cascade effect.

CONCLUSION

A simple ZI model is presented as a starting point in which to consider the procyclicality of fixed margin requirements, leverage, and RTS. In this preliminary study, it appears that RTS causes strong serial correlation in prices. Future work can extend this analysis to other book keeping and fair value methods. A Monte Carlo approach that can consider different leverage ratios, and different settlement frequency periods. ZI modeling aims to disentangle the institutional rules and trading architecture from the endogenous behavior of agents and their emergent market outcomes. This research hopes to contribute to a more rigorous analysis of procyclical market microstructure.

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