Equation models versus agent models: an example about pre-crash bubbles

David S. Brée
ISI, Torino, Italy and
University of Manchester, U.K.

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The work on pre-crash bubbles is joint with
Nathan L. Joseph, Aston University, U.K.

With grateful thanks to:
Dr Anders Johansen for the considerable assistance that he provided.

2 bedroom semi-detached house to let:
Kings Lynn, Norfolk

2 Bedroom home set a short distance from the town centre. This home offers spacious accommodation, gas central heating and a court yard garden area! Call 01553 773077 to view!

Ref: FAP/28117_KLYWINFARTH16

In 2005:

Lynn, a hard working free-lance translator,
inherits a small amount of capital.
So that when she has to stop working
she will have the rent as a pension,
she decides to invest the capital by buying houses-to-let
in the attractive nearby town of Kings Lynn, Norfolk, U.K.

Then and now ...?

Classical scientific method

Observations

|

Laws

|

Theories
and conjectures

|

Models

|

Falsifiable Predictions

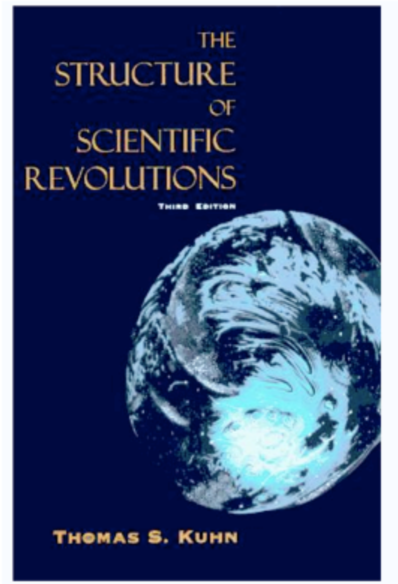
|

Experiments

|

Results

Karl Popper,
The Logic of Scientific Discovery, 1934/1959

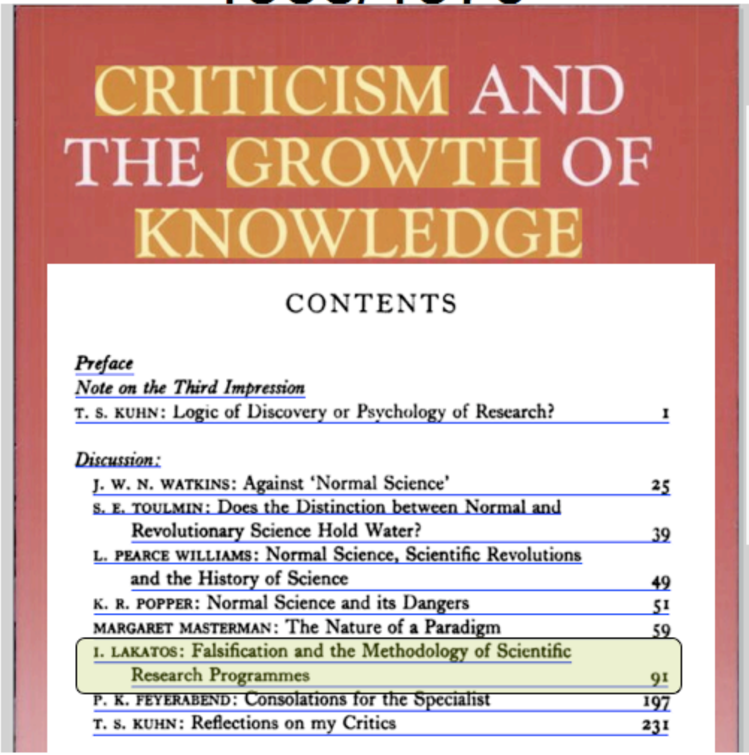


1962

Michael Polanyi,
Professor of Physical
Chemistry (later Social
Sciences),
Manchester University,
around 1937

Thomas
Kuhn

1965/1970



CRITICISM AND THE GROWTH OF KNOWLEDGE

CONTENTS

Imre Lakatos in 1961, dressed for his Cambridge PhD award ceremony

Stylized facts

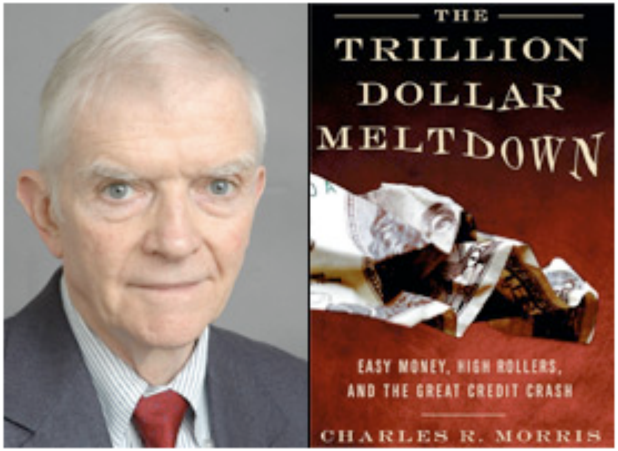
Nicholas Kaldor, 1908-1986.

A Model of Economic Growth, 1957, *Economic J.*

E.G. The shares of national income received by labor and capital are roughly constant over long periods of time

Understanding versus prediction

- Prediction IS understanding: Milton Friedman [video clip]
- Understanding without prediction: the literature on the current financial crisis.
  But see: e.g. Charles Morris,

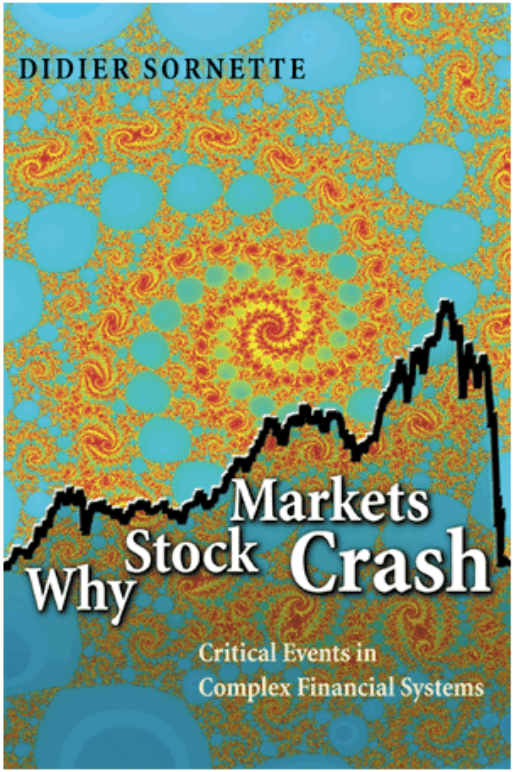


Charles R. Morris (Andrew Popper)

*Charles Morris is one of the sages of Wall Street. A former lawyer and banker, Morris published a book in January that predicted, as the title puts it, "**The Trillion Dollar Meltdown**." We talked to Morris about why he now thinks he underestimated the problem, why the government is "doing the whole thing wrong," and where he's putting his cash now.*

Obviously, looking at the behavior of the stock and credit markets, there's already worry that the bailout is failing and...you're laughing...

- Prediction with understanding:
  e.g. van der Waals gas laws $\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$
  where:
  p is the pressure of the fluid
  V is the total volume of the container containing the fluid
  T is the absolute temperature
  a is a measure of the attraction between the particles $a = N_{\mathrm{A}}^2 a'$
  a' is a measure for the attraction between the particles
  b is the volume excluded by a mole of particles $b = N_{\mathrm{A}} b'$
  b' is the average volume excluded from v by a particle
  n is the number of moles
  R is the gas constant, $R = N_{\mathrm{A}} k$
  N_{A} is Avogadro's constant
  k is Boltzmann's constant


DIDIER SORNETTE
Why Stock Markets Crash
Critical Events in
Complex Financial Systems

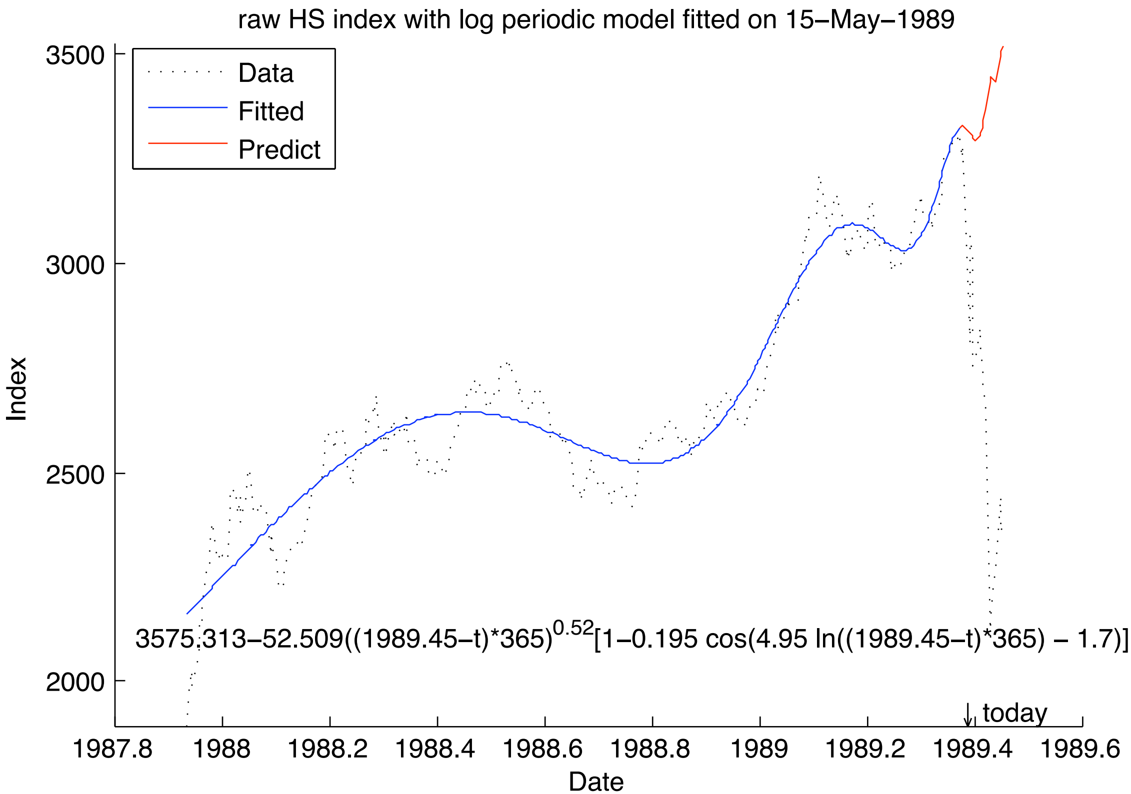


Figure 1: LPPL fit to the bubble preceding the 1989 crash on Hang Seng

1 The Log Periodic Power Law:

$$y_t = A + B(t_c - t)^{\beta}\Big\{1 + C\cos(\omega\log(t_c - t) + \phi)\Big\} \tag{1}$$

where:

$y_t > 0$	is the price (index), or the log of the price;
$A > 0$	the value of y_{t_c} at the critical time;
$B < 0$	the increase in y_t over the time unit before the crash, if C were to be close to zero;
$\lvert C\rvert < 1$	is the proportional magnitude of the fluctuations around the exponential growth;
$t_c > 0$	is the critical time;
$t < t_c$	is any time into the bubble, preceding t_c;
$\beta = 0.33 \pm 0.18$	is the exponent of the power law growth;
$\omega = 6.36 \pm 1.56$	is the frequency of the fluctuations during the bubble;
$0 \leq \phi \leq 2\pi$	is a shift parameter.

Overview:

- What is the underlying mechanism?
- Is the LPPL with fitted parameters a precursor of crashes?
- Are these LPPL parameters sufficient to distinguish between fits that precede a crash from those that do not?
  [For another occasion]

2 The underlying mechanism

The martingale condition put forward by Johansen, Ledoit and Sornette,[1] is that the expected price rise must be just sufficient to compensate for the known risk of a crash:

$$dp = \kappa.p(t).h(t).dt \tag{2}$$

where:	dp	is the change in price over the time interval dt;
	κ	is the proportion by which the price is expected to drop;
	$p(t)$	is the price at time t;
	$h(t)$	is the hazard rate at time t, i.e. the chance that the crash occurs in the next unit of time, given that it has not occurred already.

NB: All the terms on the right hand side of Equation 2 are necessarily positive.

[1] Crashes as critical points. *International Journal of Theoretical and Applied Finance*, **3**, 219–225, 2000.

Reordering Equation (2):

$$dp = \kappa.p(t).h(t).dt$$

gives us:

$$\frac{1}{p(t)}dp = \kappa h(t)dt \tag{3}$$

and integrating:

$$\log p(t) = \kappa \int_{t_0}^{t} h(t')dt' \tag{4}$$

Now the behaviour of the hazard rate, $h(t)$, needs to be specified.

2.1 Traders' behaviour

Johansen et al posit a model in which each trader is in one of two states, either bull (+1) or bear (-1). At the next time step the state, s_i, of trader i is given by:

$$s_i = \mathrm{sign}\Big(K \sum_{j \in N(i)} s_j + \sigma \epsilon_i\Big) \tag{5}$$

where:	K	is an imitation factor;
	$N(i)$	is the set of neighbouring traders who influence trader i;
	σ	is the tendency towards idiosyncratic behaviour amongst all traders;
	ϵ_i	is a random draw from a zero mean unit variance from the normal distribution.

Further they assume that:

- during a bubble the value of K increases, until it reaches a critical value K_c;
- this increase is such that $K_c - K(t)$ depends on $t_c - t$;
- the hazard rate h varies in the same way as K/K_c, the susceptibility of the collection.

With these reasonable, but unfortunately untestable, assumptions, we have the dynamics of the hazard rate:

$$h(t) \approx B'(t_c - t)^{-\alpha}[1 + C' \cos(\omega \log(t_c - t) + \phi')] \tag{6}$$

Some subsititutions and integration gives us:

- Substituting for h in (4) from (6) gives:

$$\log p(t) = \kappa \int_{t_0}^{t} B'(t_c - t')^{-\alpha}\{1 + C' \cos(\omega \log(t_c - t') + \phi')\} dt' \quad (7)$$

- Substituting $\beta = 1 - \alpha$ and $\psi(t) = \omega \log(t_c - t) + \phi'$ in the integral

$$\begin{aligned} \int (t_c - t)^{-\alpha} \cos(\omega \log(t_c - t) + \phi') dt &= \int (t_c - t)^{\beta - 1} \cos \psi(t) dt \\ &= \frac{-(t_c - t)^{\beta}}{\omega^2 + \beta^2}\{\omega \sin \psi(t) + \beta \cos \psi(t)\} \quad (8) \end{aligned}$$

- Integrating (7) using (8) and substituting:
$A = \log p(t_c)$, $B = -\kappa B'/\beta$, and $C = \beta^2 C'/(\omega^2 + \beta^2)$, gives us:

$$\log p(t) \approx A + B(t_c - t)^{\beta}[1 + C \cos(\omega \log(t_c - t) + \phi)] \quad (9)$$

which is the LPPL of equation (1) with $y_t = \log(p_t)$.

2.2 Index: raw versus log

Johansen et al often fit the LPPL to the raw index rather than the log.

- This is justified by replacing the condition (2) by:

$$dp = \kappa(p(t) - p_1)h(t)dt \tag{10}$$

  where p_1 is some 'fundamental' value.

- Integrating (10) from the moment when the bubble starts, t_0, and assuming that $p(t) - p(t_0) \ll p(t_0) - p_1$, gives:

$$\begin{aligned} p(t) &= p(t_0) + \kappa \int_{t_0}^{t} (p(t') - p_1)h(t')dt' \\ &= p(t_0) + \kappa(p(t_0) - p_1) \int_{t_0}^{t} h(t')dt'. \end{aligned} \tag{11}$$

- This assumption is both untestable and questionable.
- There seems to be no underlying justification here for the raw price, rather than its log, to fluctuate as an LPPL.

2.3 Tests of the underlying mechanism

Chang and Feigenbaum[2] tested the mechanism underlying the LPPL

- To do so they first extended the LPPL model by:
  - adding a random term with zero mean and variance estimated from the data.
  - adding a positive upward drift term, again estimated from the data.

- They computed the likelihood, given the extended LPPL model, of the price rises observed for each day.

- And not suprisingly found that:
  the mechanism underlying their adaptation of the LPPL, is not to be preferred above a random walk model.

[2] A Bayesian analysis of log-periodic precursors to financial crashes, *Quantitative Finance*, **6**, 15–36, 2006.

But there is another test.

Recall that, from Equation 2:

$$dp = \kappa.p(t).h(t).dt$$

the **predicted** price must always rise throughout the bubble.
In later work Sornette and Zhou used this as a constraint on the parameters
But not in the early studies.
So was the constraint met in the earlier studies?

Recall that it didn't for the 1989 crash on the Hang Seng:

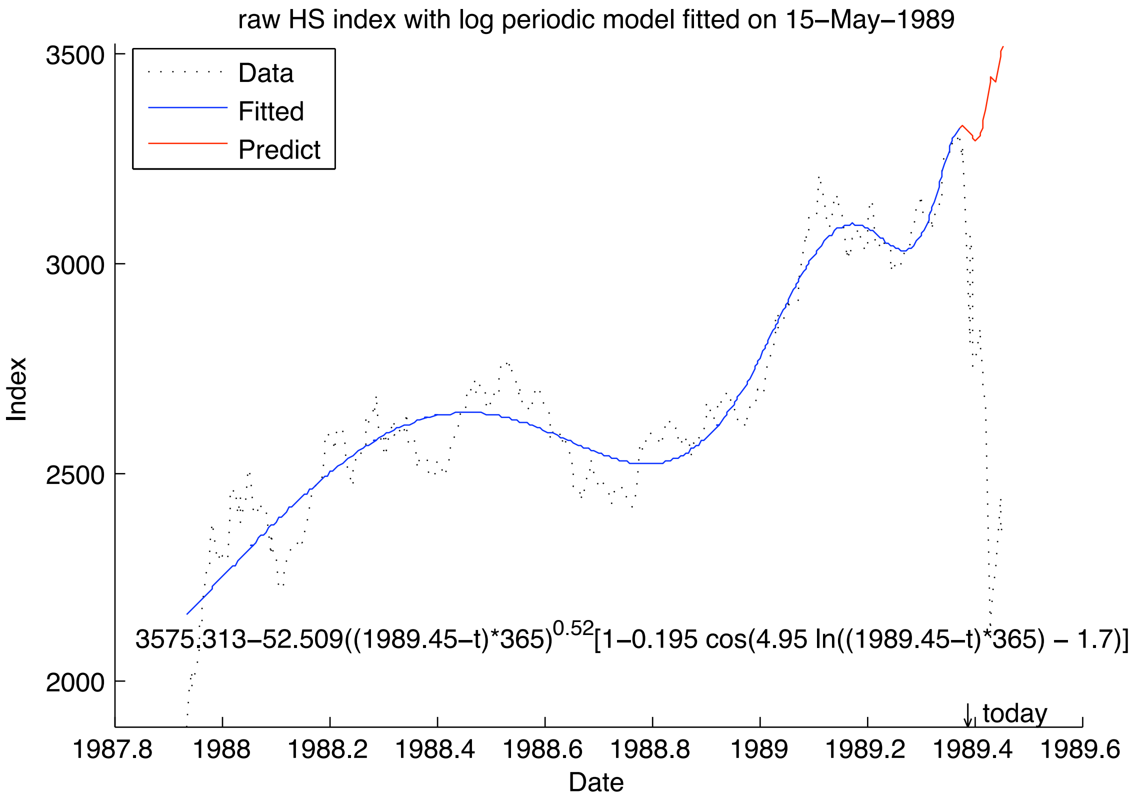


Figure 2: LPPL fit to the bubble preceding the 1989 crash on Hang Seng

Table 1: Slope of the LPPL fit to the bubbles preceding various crashes, by market and year, from Johansen et al

	Bubbles ending in crashes where the LPPL fit:	
Market	has a positive slope throughout	sometimes has a negative slope
Dow Jones		1929, 1962
S&P		1937, 1987
Hang Seng	’71, ’73, ’78, ’80, ’87, ’97	1989, 1994
Korean		1994
Indonesian		1994, 1997
Malaysian		1994
Philippine		1994
Thailand	1994	
Argentinian	1994	1991, 1992, 1997
Brazilian	1997	
Chilean	1991, 1993	
Mexican	1994	1997
Peruvian	1993	1997
Venezuelan	1997	
Total number:	14	16

So half of the LPPLs fitted to 30 crashes by Johansen et al have a negative slope.

This implies that the underlying mechanism is incorrect;
but not that the LPPL, with suitable parameters, is a poor fit to bubbles.

A potential for saving the underlying mechanism is to insist that the slope is never negative, which is guaranteed by:

$$\beta - |C|\sqrt{\beta^2 + \omega^2} \geq 0$$

This feature has been used in later work to decide whether or not a LPPL fit was a crash precursor.[3]

[3] D. Sornette and W.-X. Zhou, Predictability of large future changes in major financial indices. *International Journal of Forecasting*, **22**, 153-168, 2006.

But insisting on this constraint here requires that LPPL fits to half of the bubbles preceding crashes were not crash precursors!

3 Is the LPPL with fitted parameters a precursor of crashes?

- What's a crash?
- Troughs and bubble beginnings.
- Fitting the LPPL parameters. [skip]
- The 'best' fits of the LPPL to the 10 Hang Seng bubbles.

3.1 What's a crash?

Three parameters for defining a crash:

1. The period prior to the peak for which there is no higher value than the peak's;
   one year of weekdays (262).

2. The size of the drop;
   25%, i.e. down to 0.75 of the peak price.

3. The period within which this drop needs to occur;
   60 weekdays.

The eight crashes on the HS identified by Johansen and Sornette[4] are shown in Figure 3, together with three additional crashes: in 1981, 2000 and 2007.

[4] Bubbles and anti-bubbles in Latin-American, Asian and Western stock markets: an empirical study, *International Journal of Theoretical and Applied Finance*, **4**, 853–920, 2001.
Significance of log-periodic precursors to financial crashes, *Quantitative Finance*, **1**, 452–471, 2001.

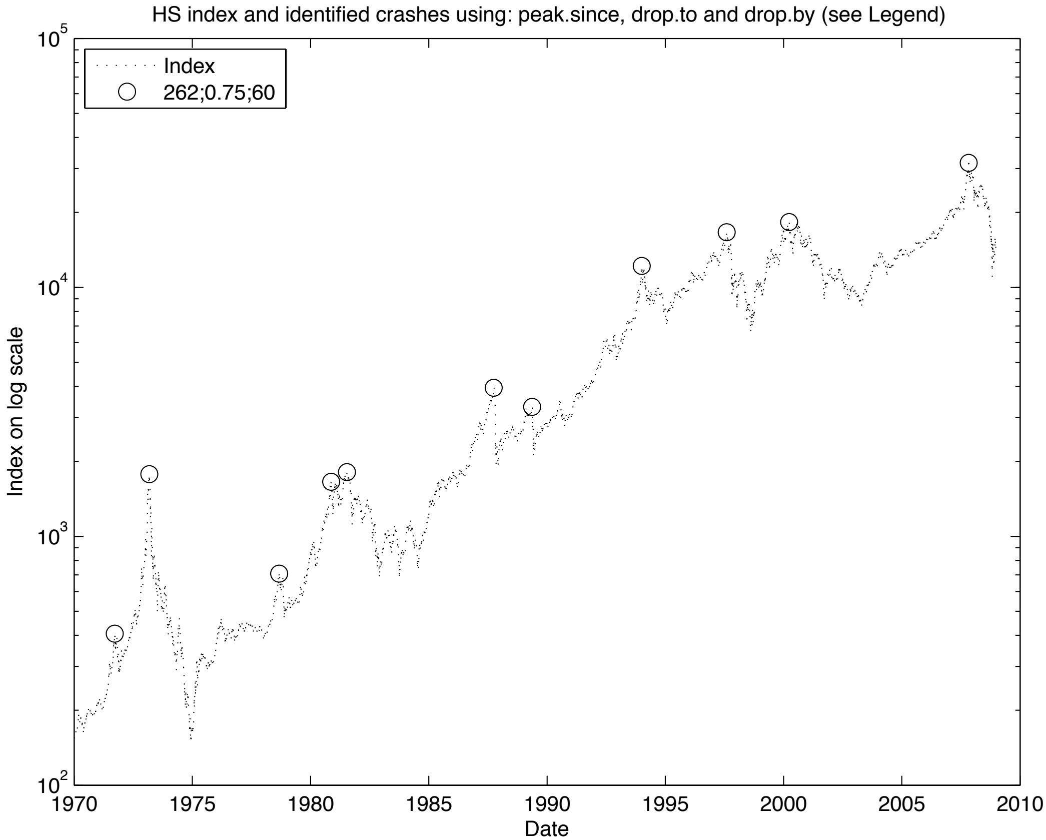


Figure 3: Crashes on Hang Seng index 1970 to 2008

Why was the 1981 crash not included in the original study?
Possible ways of excluding the 1981 peak from being a crash:

- by increasing the drop-to criterion or reduce the drop-by criterion;
  but this excludes other peaks from being crashes: 1978, 1994, 1997.
- as the preceding bubble is too short (7 months);
  however, another crash, that of 1971, uses only 6 months data.

So the peak of 1981 should be included as a crash.

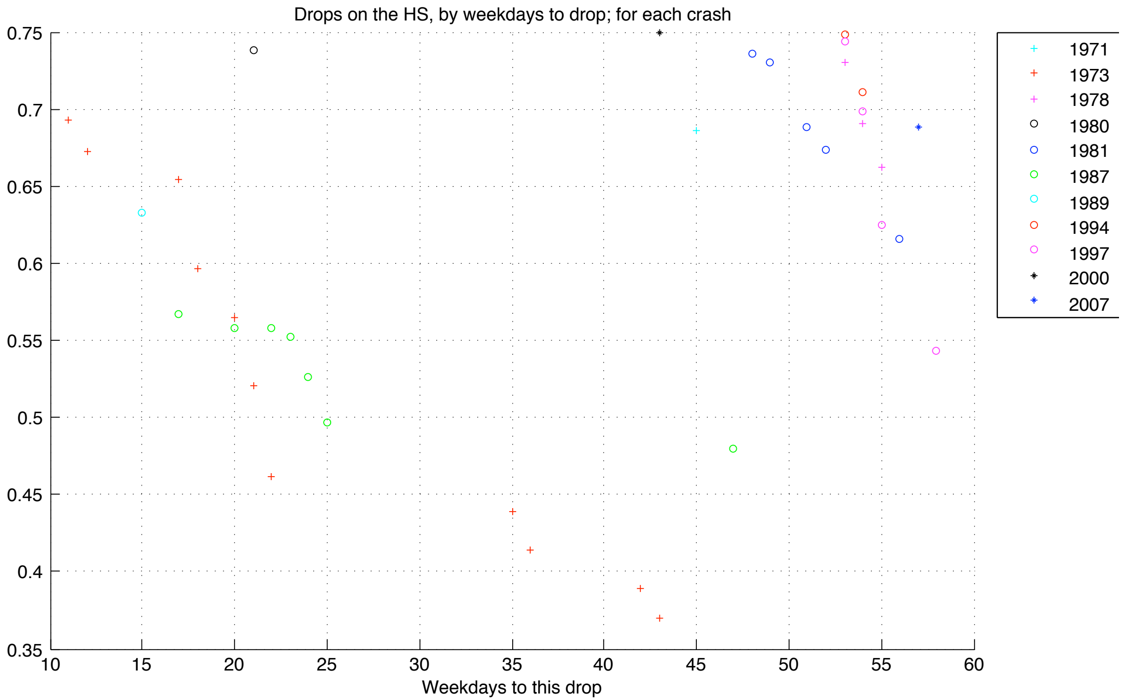


Figure 4: Crashes on Hang Seng index 1970 to 2008

3.2 Troughs and bubble beginnings

Bubbles begin at the lowest point after the previous peak.

BUT Johansen & Sornette moved the beginning of the bubble to a later time for 4 of the 8 HS crashes:[5]

- 1971 crash: forward 2 months;
- 1978 crash: forward 3 years and 1 month -
  a long period of stable prices which is clearly not part of a bubble;
- 1987 crash: forward 1 year and 8 months -
  period characterised by two mini bubbles and two peaks;
- 1994 crash: forward 2 years and 2 months.

They are shown by green squares in Figure 5.
It is not so clear why the other two bubble beginnings (1971 and 1994) were moved forward.

[5] This was done if at the trough the next bubble had not yet begun (personal communication).

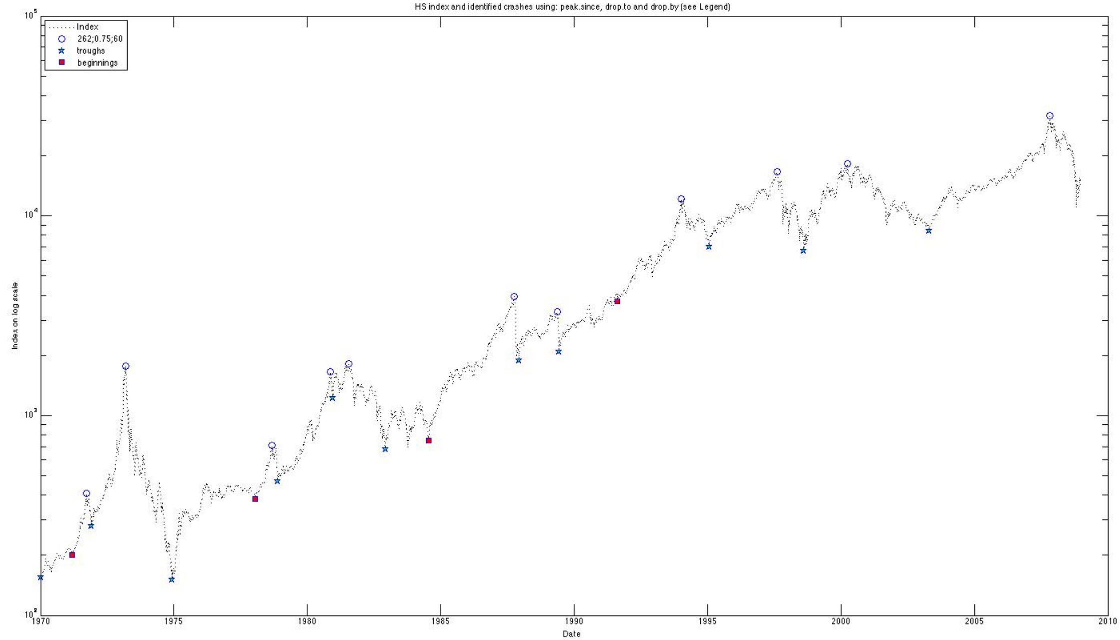


Figure 5: Troughs (red circles) and other beginnings of bubbles (green squares) on Hang Seng 1970 to 2008

3.3 Fitting the LPPL parameters

The squared error between the prediction from the LPPL (1) and the data is:

$$SE = \sum_{t=t_1}^{t_n} (y_t - \hat{y}_t)^2 = \sum_{t=t_1}^{t_n} \left\{ y_t - A - B(t_c - t)^{\beta} \Big(1 + C \cos(\omega \log(t_c - t) + \phi \Big) \right\}^2 \tag{12}$$

where:

y_t	is the data point, either the price (index) or its log;
$\hat{y}_t$	is the data point as predicted by the model;
t_i	is the i^{th} weekday from the beginning of the bubble ;
n	is the number of weekdays in the bubble.

Partially differentiating (12) with respect to A, B and C gives us three linear equations for the A, B and C that minimise the RMSE,
given the other four parameters: β, ω, t_c and ϕ.

The method used by Sornette et al for their searches was:

- First to make a grid of points for the parameters ω and t_c, from each a Taboo search was conducted to find the best value of β and ϕ.
- To select from these points just those for which $0 < \beta < 1$.
- From these points to conduct a Nelder-Mead Simplex search, with all the four search parameters free, but A, B and C chosen to minimise the RMSE.

Our search method:

- We use a preliminary search procedure based on a grid to provide seeds for the Nelder-Mead Simplex method.
- It is based on choosing different values for the two parameters critical to determining whether the fitted LPPL is a crash precursor or not: ω and β.

- The algorithm is shown in Table 3;
  the parameter values used in the algorithm are shown in Table 2.
- Note that instead of the crash date, t_c, we use $t2c$, the number of days between the day on which the estimate is being made, here the end of the bubble, and the predicted critical date.

Table 2: Initial bounds on the four parameters for selecting seeds

	β	ω rads	$t2c$ days	ϕ rads
lower	0	0	1	0
upper	2	20	260	π
minimum width	0.2	2	–	–

Table 3: Our search algorithm

0. For each of the four parameters $\beta, \omega, t2c$ and ϕ, fix the lower L and upper U bounds for the seeds. For a subset $\mathcal{P}$ of selected parameters (β and ω), fix the minimum width W to continue searching.

1. Choose as the current seed $S1 \leftarrow (L+U)/2$, the mid point of the current lower and upper bounds.

2. Run the unbounded Nelder-Mead Simplex search from the current seed $S1$, which will return a solution $S2$.

3. Construct a hypercube in the space of $\mathcal{P}$ using $S1$ and $S2$, with their minimum as the bottom corner: $B \leftarrow \min(S1, S2)$; and their maxima as the top corner: $T \leftarrow \max(S1, S2)$.

4. Set $p = 1$.

5. For $p \leftarrow 1 : \text{size}(\mathcal{P})$, i.e. for each of the selected parameters, do:

   if $B_p L_p < W_p$ i.e. if there is too little space under the hypercube on the p^{th} dimension in $\mathcal{P}$, set $B_p \leftarrow L_p$, i.e. set the bottom of the hypercube on the p^{th} dimension to its lower bound,
   else recursively search from step 1, with $L' \leftarrow L$ and $U' \leftarrow U, U'_p \leftarrow B_p$, i.e. search under the hypercube;

   if $U_p T_p < W_p$, i.e. if there is too little space above the hypercube on the p^{th} parameter, set $T_p \leftarrow U_p$, i.e. set the top of the hypercube on the p^{th} parameter to its upper bound,
   else recursively search from step 1, with $L' \leftarrow L, L'_p \leftarrow T_p$ and $U' \leftarrow U$, i.e. search above the hypercube.

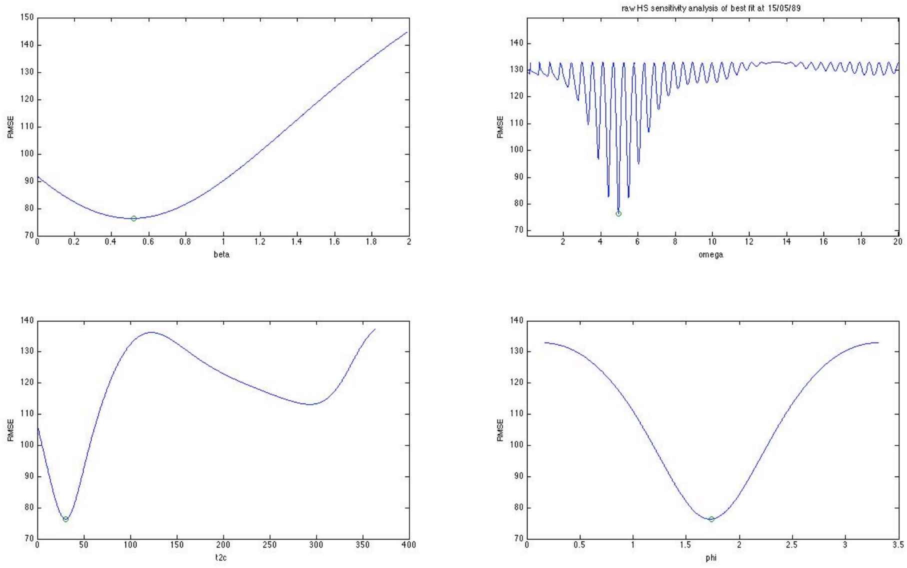


Figure 6: Sensitivity of the RMSE of the parameters of the LPPL fit to the bubble preceding the crash on Hang Seng, 1989; circles indicate the chosen value

Table 4: The bubbles preceding crashes of the Hang Seng index

		A	B	C	β	ω	$t2c$	ϕ	RMSE
		HSI	HSI			rads	days	rads	HSI
Bubble:	Ref	low:			0.15	4.80	1	0	
from/to		high:	0		0.51	7.92	?	π	
*10-Mar-1971	SJ	594	−132	−0.033	0.20	4.30	7	0.50	7.58
20-Sep-**1971**		539	−101	−0.047	0.22	4.30	3	0.25	6.11
22-Nov-1971	SJ	11	−3	0.003	0.11	8.70	2	0.05	0.0722
09-Mar-**1973**	*log*	65	−56	−0.001	**0.01**	**11.1**	20	1.32	0.0538
	log	8	−0	−0.177	0.57	**1.47**	2	3.14	0.0549
	raw	2443	−485	−0.114	0.26	**1.45**	2	3.14	40.91
*13-Jan-1978	SJ	816	−50	−0.053	0.40	5.90	6	0.17	10.09
04-Sep-**1978**		741	−23	0.072	0.51	5.30	1	0.00	10.12
20-Nov-1978	SJ	1998	−231	−0.044	0.29	7.24	3	1.80	46.72
13-Nov-**1980**		41164	−38080	0.001	**0.01**	7.51	52	3.06	35.02
		7929	−5352	0.008	**0.05**	6.79	26	1.55	35.55
		1998	−231	−0.044	0.29	7.24	3	2.63	37.00
12-Dec-1980									
17-Jul-**1981**		1753	−0	−0.890	**2.41**	**3.02**	1	3.14	40.46
		1817	−3	−0.567	**1**	4.75	12	0.35	49.24
		1946	−11	−0.399	**0.76**	5.89	36	0.00	54.95
*23-Jul-1984	JS	5262	−542	−0.007	0.29	5.60	22	1.60	133.86
01-Oct-**1987**		5779	−711	0.048	0.27	5.68	34	2.63	68.47
07-Dec-1987	SJ	3403	−32	−0.023	0.57	4.90	34	0.50	133.21
15-May-**1989**		3575	−53	−0.195	0.52	4.95	31	1.74	76.33
*19-Aug-1991	JS	21421	−7614	0.024	0.12	6.30	4	0.60	322.80
04-Jan-**1994**		212635	−194575	−0.002	0.27	5.95	1	3.13	272.82
		14038	−1717	−0.028	0.26	6.43	4	3.14	281.36
23-Jan-1995	JS	20359	−1149	−0.019	0.34	7.50	51	0.80	531.79
07-Aug-**1997**		20255	−1201	−0.048	0.33	7.47	51	2.29	438.79
13-Aug-1998									
28-Mar-**2000**		21918	−16	0.073	**1.00**	**18.35**	290	0.00	710.99
		24095	−97	−0.057	**0.76**	**17.51**	264	3.14	720.17
		19503	−372	0.111	0.52	5.7	9	2.07	744.15
23-Apr-2003									
30-Oct-**2007**		38876	−6388	0.018	0.2	5.53	1	2.35	697.72

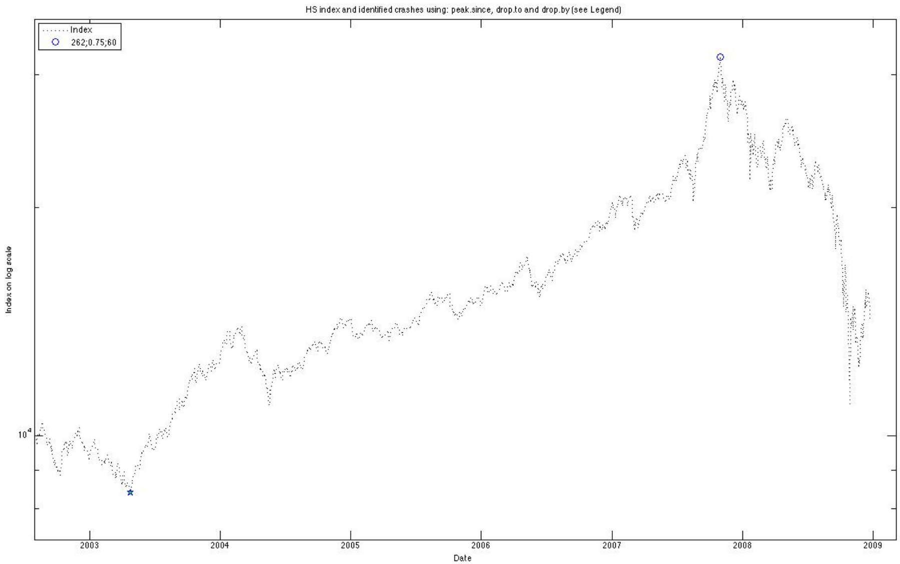


Figure 7: The Hang Seng bubble and crash of 2007-8

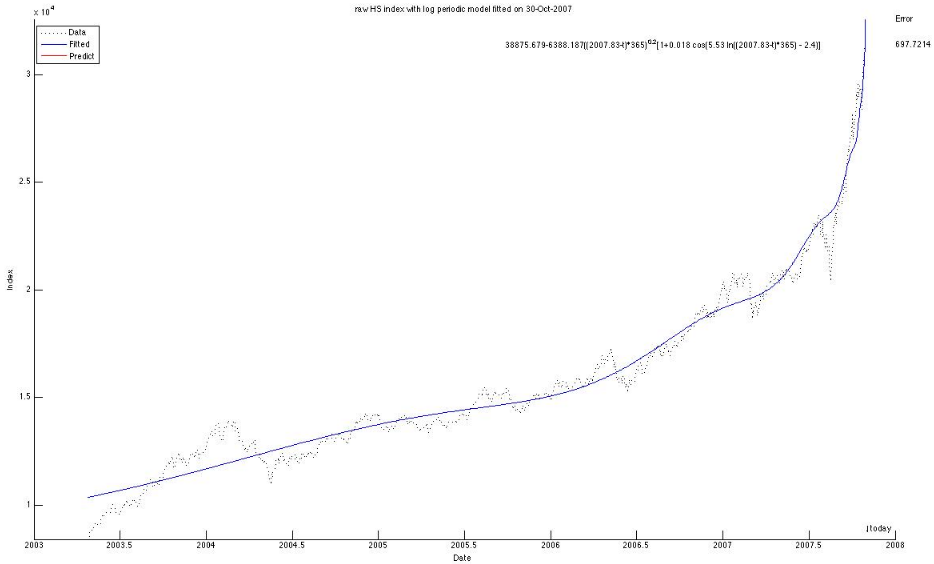


Figure 8: The Hang Seng bubble preceding the 2007-8

Of the eight bubbles which are fitted by JS:2001 and SJ:2001 we find virtually the same parameters for the LPPL on six bubbles: '71, '78, '87, '89, '94, '97. But:

1973: SJ:2001 fit to the log rather than to the raw index. The fit to the log of the index has ω outside the acceptable range.
For the raw index, $\omega = 1.45$ is well below the lower bound.

1980: Our best fit predicted a crash after 52 days, but $\beta = 0.01$.
The fit reported in SJ:2001 was not the best fit.

1981: This crash was not fitted in the original study.
Our best fit has $\beta > 1$ and ω outside their acceptable ranges.
As $\beta > 1$, this would have been rejected in the original study.
The first fit with $\beta <= 1$ has $\omega = 4.75$, just acceptable,
but with a $\beta = 1$, i.e. no power law, so well outside its acceptable range.

Crashes that occurred after the original study took place.

2000: Best fit has both critical parameters well outside their respective acceptable ranges
predicts a crash after 290 days, i.e. one that could be ignored.
There is a fit that does have these parameters within their acceptable ranges, and predicts a crash after only 9 days.
But it is not the best fit.

2007: The LPPL would have predicted this crash. Moreover it had a slow beginning, not dropping seriously until 57 days after the peak, so giving time to react.

4 Summary

- As in half the 30 studies reported the LPPL fitted to the index (or its log) decreases at some point during the bubble **another underlying mechanism (read ABM) is needed.**

- For four HS bubbles an LPPL could be found for two crashes if: ... However, these LPPLs did not have the best fits. For the remaining two crashes (1973 and 2000), there seems to be no saving strategy.
  So we need a non-dertministic predictor.

- For many values of the parameters the fit is about equally good. Sloppiness. **A reduced parameter space is called for.**

- The LPPL fit indicates that a bubble may just be exponential growth coupled with increasing volatility.
  This is a simple and necessary null hypothesis.

5 What to expect to find in an ABM of crashes:

- a visible positive feedback loop to cause the bubble, e.g. availability of cheap (initially) mortgages
  → increasing house prices
  → increasing willingness to lend mortgages
  → increasing pool of house buyers

- a visible build up of a diffusion network, e.g. increase in borrowing requirements
  → increasing demand for investment capital
  → increasing the network of suppliers (internationally)
  → decreasing regulation and oversight
  → increasing risk

- a saturation to stop the bubble, e.g. eligible borrowers, available investors,

- another visible feedback loop to fuel the crash, e.g. the last buyers are the most risky so defaults are on house purchased at the top prices
  $\rightarrow$ forced sale of these houses
  $\rightarrow$ drop in house prices
  $\rightarrow$ more xmortgage defaults, etc.

- using the diffusion network to spread the crash, e.g. default of a mortgage lender
  $\rightarrow$ write off of commercial paper
  $\rightarrow$ financial loss of investors
  $\rightarrow$ unloading of risky paper
  $\rightarrow$ decrease in value of commercial paper
  $\rightarrow$ bankruptcies of investors