Financially Constrained Fluctuations in an Evolving Network Economy

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Introduction

• Motivation:
  - We study the properties of a credit-network economy characterized by credit relationships connecting downstream and upstream firm (trade credit) and firms and banks (bank credit).
  - It is straightforward to think of agents as nodes and of debt contracts as links.
  - The network topology changes over time due to an endogenous process of partner selection in an imperfect information decisional context.
  - The bankruptcy of one agent can bring about the bankruptcy of one or more other agents possibly leading to avalanches of bankruptcies.
  - We investigate the interplay between network evolution and business fluctuations (bankruptcy propagation).
  - “The high rate of bankruptcy is a cause of the high interest rate as much as a consequence of it” (Stiglitz and Greenwald, 2003: 145)
    - Agents' defaults → bad loans → deterioration of lenders' financial conditions → credit restriction (increase of the interest rate)
    - credit restriction (increase of the interest rate) → deterioration of borrowers' financial conditions → agents' defaults...
Introduction

- Related literature:
  - **Credit interlinkages**: Stiglitz and Greenwald (2003, Ch. 7) → a “circle” of connected firms (trade credit) linked to a bank (bank credit).
  - Delli Gatti, Gallegati, Greenwald, Russo, Stiglitz (2006): business fluctuations (and bankruptcy propagation) in a three-sector economy (downstream firms, upstream firms and banks); **static network**
  - The specific contribution of the present work is the introduction of a mechanism for the **endogenous evolution of the network**
The environment

- **Multi-sector** network economy:
  - Downstream sector \((i = 1,2,...,I \text{ firms})\)
  - Upstream sector \((j = 1,2,...,J \text{ firms})\)
  - Banking sector \((z = 1,2,...,Z \text{ banks})\)

- **Discrete time** steps \((t = 1,2,...,T)\)

- **Two goods**: consumption and intermediate goods

- **Two inputs**: labour and intermediate goods

- **Downstream (D) firms** produce a perishable consumption good using labour and intermediate goods

- **Upstream (U) firms** produce intermediate goods “on demand” using only labour as input
The environment

- We rule out (by construction) the possibility of *avalanches of output* due to the mismatch of demand and supply of intermediate goods along the supply chain (Bak, Chen, Scheinkman and Woodford, 1993)

- The financial side of the economy is characterized by *two lending relationships*:
  - D and U firms obtain credit from banks
  - D firms buy intermediate goods from U firms by means of a commercial credit contract

- *Endogenous network formation* according to the *preferred-partner choice*:
  - In every period each D firm looks for the U firm with the lowest price of intermediate goods; at the same time each firm searches for the bank with the lowest interest rate
  - The number of potential partners an agent can check in each period is limited (*imperfect information*)
Firms

- The core assumption of the model is that the scale of activity of the i-th D firms at time $t$ is an increasing concave function of its financial robustness, proxied by net worth ($A_{it}$):

$$Y_{it} = \phi A_{it}^\beta$$

where $\phi > 1$ and $0 < _- < 1$ are parameters, uniform across D firms.

- Two rationales for the financially constrained output function:
  - A simple rule of thumb in a world in which
    - Bounded rationality prevents the elaboration of optimizing decision-making processes and
    - Asymmetric information between lenders and borrowers yields a financing hierarchy in which net worth ranks first.
  - Alternatively one can think of this equation as the solution of a firm's optimization problem (Greenwald and Stiglitz, 1993):
    - Max expected profits minus bankruptcy costs: increase of financial fragility (reduction of net worth) --> increase of bankruptcy probability
Firms

- Labour and intermediate goods requirement functions for D firms:
  - \( N_{it} = \delta_d Y_{it} \) (demand for labour)
  - \( Q_{it} = \gamma Y_{it} \) (demand for intermediate goods)
  where \( \delta_d > 0 \) and \( \gamma > 0 \).

- Final goods are sold at a stochastic price \( u_{it} \), a random variable distributed in the interval (0,2).

- In each period a U firm receives orders from a set of D firms (\( _j \))
  - \( _j \) depends on the price \( p_{jt} = 1 + r_{jt} \), where \( r_{jt} \) is the interest rate on trade credit
  - The lower the price the higher the number of D firms placing order to \( j-th \) U firm
  - We assume that the interest rate depends on the firm's financial conditions:

\[
    r_{jt} = \alpha A_{jt}^{-\alpha}
\]

where \( \alpha > 0 \).
Firms

- The **scale of production** of U firms is “demand constrained”: 
  \[ Q_{jt} = \gamma \sum_{i \in \Phi_j} Y_{it} \]

- Labour **requirement function** for U firms: 
  \[ N_{jt} = \delta_u Q_{jt} \] , where \( \delta_u > 0 \).

- **Financing hierarchy**: the financing gap (the difference between the firm's expenditures and internal finance) is filled by means of credit
  - U and D firms: wage bill – net worth
  - D firms: intermediate goods \( \rightarrow \) trade credit

- **Demand for credit**: 
  \[ B_{xt} = W_{xt} - A_{xt} \]
  
  where \( W_{xt} = wN_{xt} \) is the wage bill (\( x=i \) for D firms, \( j \) for U firms)

- **Self-financed firms** (firms with a sufficient level of net worth to finance the wage bill) do not demand credit

- The **real wage** \( w \) is constant and uniform across firms
Banks

- In each period of time a set of (D and U) firms, denoted by \( z \), demands credit to the \( z \)-th bank (the lower the interest rate the larger the number of customers).

- The **interest rate** on the loan to the \( x \)-th borrower is:

\[
 r_{x}^{z} = \sigma A_{zt}^{-\beta} + \theta (l_{xt})^{\theta}
\]

where \( A_{zt} \) is the net worth of the bank and \( l_{xt} = B_{xt}/A_{xt} \) is the **leverage ratio** of the \( x \)-th firm, \( \sigma \) and \( \theta \) are positive parameters.

- According to this rule:
  - Financially sound banks can extend credit at better conditions (they reduce the interest rate and attract more firms)
  - Banks penalizes financially fragile firms (the interest rate charged by the lender incorporates an *external finance premium*, increasing with leverage and therefore inversely related to the borrower's net worth)
Partner choice

- Each D firm has a (productive and credit) relationship with a U firm.
- At the beginning, links are established at random.

- In subsequent periods the network changes endogenously according to a preferred-partner choice rule (with noise):
  - with a (small) probability $\_\_$, the D firm chooses a partner at random;
  - with probability $(1 - \_\_)$, it looks at the prices of a randomly selected number (M) of U firms
    - if the minimum observed price is lower than the price of the previous partner, it will switch to the new U firm
    - otherwise, it will stick to the previous partner

- The preferred-partner choice also applies to the relationships between firms (both D and U) and banks
Profits, net worth and bad debt

- The profit of the \( i \)-th D firm is: 
  \[
  \pi_{it} = u_{it}y_{it} - (1 + r_{zt})B_{it} - (1 + r_{jt})Q_{it}
  \]

- The profit of the \( j \)-th U firm is: 
  \[
  \pi_{jt} = (1 + r_{jt})Q_{jt} - (1 + r_{zt})B_{jt}
  \]

- The profit of the \( z \)-th banks is: 
  \[
  \pi_{zt} = \sum_{i \in I_z} (1 + r^{i}_{zt})B_{it} + \sum_{j \in J_z} (1 + r^{j}_{zt})B_{jt}
  \]

- At the end of the period, the net worth of the \( x \)-th agent (\( x = i \) for D firms, \( j \) for U firms and \( z \) for banks) is: 
  \[
  A_{xt+1} = A_{xt} + \pi_{xt} - BD_{xt}
  \]
  where \( BD \) is bad debt (non-performing loans).

- In the case of U firms: 
  \[
  BD_{jt} = (1 + r_{jt})\gamma \sum_{i \in \Phi^B_j} y_{it}
  \]

- In the case of banks: 
  \[
  BD_z = \sum_{i \in \Phi^B_z} (1 + r^{i}_{zt})B_{it} + \sum_{j \in \Phi^B_z} (1 + r^{j}_{zt})B_{jt}
  \]

- The agent goes bankrupt if \( A_{xt+1} < 0 \).
Simulations

- **Agents:** $I = 500$ (D firms); $J = 250$ (U firms), and $Z = 100$ (banks).
- **Time span:** $T = 1000$.
- **Parameter setting:**
  - Financially constrained output of D firms: $\phi = 1.5; _\varphi = 0.8$;
  - Labour requirement of D and U firms: $\delta_d = 0.5; \delta_u = 1$;
  - Intermediate goods requirement of D firms: $\gamma = 0.5$;
  - Interest rate on trade credit: $_\varphi = 0.1$;
  - Interest rate on bank credit: $_\varphi = 0.1; _\varphi = 0.05$;
  - Real wage: $w = 1$;
  - Number of potential partners: $M = 5; N = 5$;
  - Probability of preferred-partner choice: $1 - \varepsilon = 0.99$.
- **Initial conditions:** new worth is set to 1 for all agents
- **Entry-exit process:**
  - One-to-one replacement: net worth of new entrants is drawn from a uniform distribution with support $(0, 2)$
- **Aggregate production of D firms**: As expected in complex adaptive systems, **fluctuations are irregular** (amplitude and periodicity vary from period to period)

- **Aggregate production of U firms**
  follows the same dynamic pattern since U suppliers produce intermediate goods for D firms “on demand”.

- Starting from **identical initial conditions** agents become rapidly heterogeneous

- **Firm size distribution** tends to a power law
Network structure: U firms vs. banks

The number of links for each lender (U firm or bank) becomes asymmetric over time due to the preferred-partner choice governing interaction among borrowers and lenders.
- The **degree distribution** of the interaction network tends to a power law.

- The preferred-partner choice rule makes *preferential attachment* endogenous through a mechanism similar to the *fitness model*.

- Economic behaviour, financial conditions and network evolution: *financially robust lenders* can supply credit at better conditions and therefore increase their market share. The opposite holds true for financially fragile agents. As a consequence, the corporate and the banking sectors become *polarized* and the degree distribution becomes *asymmetric*.

- **Robustness**: the network is robust to random shock.

- **Vulnerability**: the network is vulnerable to targeted shocks, because the default of a highly connected agent (rare event) may produce other defaults...

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**Figure 3:** Degree distribution of the network: DS firms vs. US firms

**Figure 4:** Degree distribution of the network: DS and US firms vs. banks
A typical story:

- D4, D6 and D7 go bankrupt due to idiosyncratic shocks
- They do not fulfill debt commitments
- The financial conditions of lenders deteriorate due to bad loans...
- In this case, U2 and B1 go bankrupt, while U1 and B2 survive to the failure of their partners
- Channel of bankruptcy propagation:
  - The failure of D4 and D6 provokes the default of U2
  - The failure of D6, D7 and, in particular, of U2 provokes the default of B1
- The deterioration of the financial conditions of U1 and B2 may produce an increase of the interest rate...
- The high rate of bankruptcy is a cause of the high interest rate as much as a consequence of it!
- The extent of bankruptcy depends on the amount of **bad debt**

- The deterioration of lenders' financial condition due to borrowers' bankruptcies may be absorbed if the size of the non-performing loans is "small enough" or the lenders' net worth is "high enough"

- The distribution of aggregate **growth rates** is far from being Gaussian (tent-shaped or double exponential)

- Asymmetry for **negative events**
PPC vs. RM

- Random network:
  - in every period each agent chooses a partner at random (model simulation with unchanged parameters but for $\varepsilon = 1$)

- Major findings:
  - Degree distribution of the network (credit interlinkages)
  - Firm size distribution (agents' heterogeneity)
  - Bankruptcy propagation (correlation structure across sectors)
  - Bankruptcy probability (systemic risk)
Degree distribution of the network

- with PPC, the distribution of links tends to a power law
- Firm size distribution
  - with PPC, FSD tends to a power law shape
### Table 3. Monte Carlo simulations: RM vs. PPC.

<table>
<thead>
<tr>
<th></th>
<th>RM</th>
<th>PPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean(GR)</td>
<td>0.0013 (0.0000)</td>
<td>0.0013 (0.0000)</td>
</tr>
<tr>
<td>standard deviation(GR)</td>
<td>0.0393 (0.0018)</td>
<td>0.0401 (0.0025)</td>
</tr>
<tr>
<td>skewness(GR)</td>
<td>−0.1238 (0.0838)</td>
<td>−0.1393 (0.0696)</td>
</tr>
<tr>
<td>kurtosis(GR)</td>
<td>2.9386 (0.1510)</td>
<td>2.9132 (0.1657)</td>
</tr>
<tr>
<td>median(GR)</td>
<td>0.0022 (0.0010)</td>
<td>0.0025 (0.0010)</td>
</tr>
<tr>
<td>median absolute deviation(GR)</td>
<td>0.0266 (0.0014)</td>
<td>0.0273 (0.0020)</td>
</tr>
<tr>
<td>robust skewness(GR)</td>
<td>−0.0279 (0.0315)</td>
<td>−0.0356 (0.0301)</td>
</tr>
<tr>
<td>robust kurtosis(GR)</td>
<td>0.0031 (0.0521)</td>
<td>−0.0018 (0.0567)</td>
</tr>
<tr>
<td>median(BD)</td>
<td>37.2940 (1.5381)</td>
<td>36.8541 (3.8340)</td>
</tr>
<tr>
<td>median absolute deviation(BD)</td>
<td>12.0540 (1.2254)</td>
<td>16.49 (2.7380)</td>
</tr>
<tr>
<td>robust skewness(BD)</td>
<td>0.6612 (0.0681)</td>
<td>0.6518 (0.0880)</td>
</tr>
<tr>
<td>robust kurtosis(BD)</td>
<td>0.4279 (0.1523)</td>
<td>0.4031 (0.1918)</td>
</tr>
</tbody>
</table>

- **Bankruptcy rate:** Correlations of bankruptcy rates are similar in RM vs. PPC.

- **Bankruptcy probability:** The bankruptcy probability of U firms and banks is higher in PPC than in RM.

- **Systemic risk:** even though the correlation among bankruptcies is similar in the two scenarios, the greater incidence of defaults in the U and banking sectors means that the endogenous network increases the likelihood of bankruptcy propagation, starting from idiosyncratic shocks regarding D firms.
Concluding remarks

- Modelling of productive and credit interlinkages: Endogenous network formation through preferred-partner choice

- Credit relationships (network structure), bankruptcy propagation, business fluctuations: bankruptcy rate ↔ interest rate

- Skew distributions: Firm size distribution, degree distribution of networks, bad debt, negative asymmetry for growth rates, etc.

- Endogenous network (PPC) vs. random matching (RM): systemic risk

- Empirical analysis: validating simulation results

- Towards a “complete” credit-network economy
  - Remove the hypothesis of (exogenous) stochastic price
  - Remove the hypothesis of (exogenous) constant number of agents
  - Remove the hypothesis of “on demand” production of U firms
  - Introduce the interbank market to investigate monetary policy issues