# Financially Constrained Fluctuations in an Evolving Network Economy

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# Introduction

- Motivation:
  - We study the properties of a **credit-network economy** characterized by credit relationships connecting downstream and upstream firm (**trade credit**) and firms and banks (**bank credit**).
  - It is straightforward to think of *agents* as **nodes** and of *debt contracts* as **links**
  - The **network topology** changes over time due to an endogenous process of **partner selection** in an **imperfect information** decisional context.
  - The bankruptcy of one agent can bring about the bankruptcy of one or more other agents possibly leading to **avalanches of bankruptcies.**
  - We investigate the interplay between **network evolution** and **business fluctuations** (bankruptcy propagation)
  - “The **high rate of bankruptcy** is a cause of the **high interest rate** as much as a consequence of it” (Stiglitz and Greenwald, 2003: 145)
    - Agents' defaults → bad loans → deterioration of lenders' financial conditions → credit restriction (increase of the interest rate)
    - credit restriction (increase of the interest rate) → deterioration of borrowers' financial conditions → agents' defaults...

# Introduction

- Related literature:
  - Financial contagion in the **interbank market**: Allen and Gale (2000), Freixas et al. (2000), Furfine (2003), Boss et al. (2004), Iori et al., (2006), Nier et al. (2007) → interbank lending, liquidity management, network structure and financial crises.
  - **Credit interlinkages:** Stiglitz and Greenwald (2003, Ch. 7) → a "circle" of connected firms (trade credit) linked to a bank (bank credit).
  - Delli Gatti, Gallegati, Greenwald, Russo, Stiglitz (2006): business fluctuations (and bankruptcy propagation) in a three-sector economy (downstream firms, upstream firms and banks); **static network**
  - The specific contribution of the present work is the introduction of a mechanism for the **endogenous evolution of the network**

# The environment

- **Multi-sector** network economy:
  - Downstream sector ( $i = 1,2,\ldots,I$ firms )
  - Upstream sector ( $j = 1,2,\ldots,J$ firms )
  - Banking sector ( $z = 1,2,\ldots,Z$ banks )
- **Discrete time** steps ( $t = 1,2,\ldots,T$ )
- **Two goods**: consumption and intermediate goods
- **Two inputs**: labour and intermediate goods

- **Downstream (D) firms** produce a perishable consumption good using labour and intermediate goods
- **Upstream (U) firms** produce intermediate goods "on demand" using only labour as input

# The environment

- We rule out (by construction) the possibility of ***avalanches of output*** due to the mismatch of demand and supply of intermediate goods along the supply chain (Bak, Chen, Scheinkman and Woodford, 1993)

- The financial side of the economy is characterized by ***two lending relationships***:
  - D and U firms obtain credit from banks
  - D firms buy intermediate goods from U firms by means of a commercial credit contract

- *Endogenous network formation* according to the ***preferred-partner choice***:
  - In every period each D firm looks for the U firm with the lowest price of intermediate goods; at the same time each firm searches for the bank with the lowest interest rate
  - The number of potential partners an agent can check in each period is limited (*imperfect information*)

# Firms

- The **core assumption** of the model is that the scale of activity of the *i-th* D firms at time $t$ is an increasing concave function of its financial robustness, proxied by net worth ($A_{it}$):

$$Y_{it} = \phi A_{it}^{\beta}$$

  where $\phi > 1$ and 0 < _ < 1 are parameters, uniform across D firms.

- Two rationales for the *financially constrained output function*:
  - A **simple rule of thumb** in a world in which
    - ***Bounded rationality*** prevents the elaboration of optimizing decision-making processes and
    - ***Asymmetric information*** between lenders and borrowers yields a financing hierarchy in which net worth ranks first.
  - Alternatively one can think of this equation as the solution of a firm's **optimization problem** (Greenwald and Stiglitz, 1993):
    - Max expected profits minus bankruptcy costs: increase of ***financial fragility*** (reduction of netw worth) --> increase of ***bankruptcy probability***

# Firms

- Labour and intermediate goods **requirement functions** for D firms:
  - $N_{it} = \delta_d Y_{it}$ (demand for labour)
  - $Q_{it} = \gamma Y_{it}$ (demand for intermediate goods)

  where $\delta_d > 0$ and $\gamma > 0$.

- Final goods are sold at a **stochastic price** $u_{it}$, a random variable distributed in the interval (0,2).

- In each period a U firm receives **orders** from a set of D firms ($\_{j}$)
  - $\_{j}$ depends on the price $p_{jt} = 1 + r_{jt}$, where $r_{jt}$ is the interest rate on trade credit
  - The lower the price the higher the number of D firms placing order to *j-th* U firm
  - We assume that the interest rate depends on the firm's financial conditions:

$$r_{jt} = \alpha A_{jt}^{-\alpha}$$

where _ >0.

# Firms

- The **scale of production** of U firms is "demand constrained": $Q_{jt} = \gamma \sum_{i \in \Phi_j} Y_{it}$
- Labour **requirement function** for U firms: $N_{jt} = \delta_u Q_{jt}$, where $\delta_u > 0$.
- **Financing hierarchy**: the financing gap (the difference between the firm's expenditures and internal finance) is filled by means of credit
  - U and D firms: wage bill – net worth
  - D firms: intermediate goods → trade credit
- **Demand for credit**: $B_{xt} = W_{xt} - A_{xt}$

  where $W_{xt} = wN_{xt}$ is the wage bill ($x$=$i$ for D firms, $j$ for U firms)_
- **Self-financed firms** (firms with a sufficient level of net worth to finance the wage bill) do not demand credit
- The **real wage** $w$ is constant and uniform across firms

# Banks

- In each period of time a set of (D and U) firms, denoted by $\_z$ , demands **credit** to the *z-th* bank (the lower the interest rate the larger the number of customers)_

- The **interest rate** on the loan to the *x-th* borrower is: $r^x_{zt} = \sigma A_{zt}^{-\beta} + \theta \left( l_{xt} \right)^\theta$

  where $A_{zt}$ is the *net worth* of the bank and $l_{xt}=B_{xt}/A_{xt}$ is the *leverage ratio* of the *x-th* firm, _ and _ are positive parameters.

  - According to this rule:
    - Financially sound banks can extend credit at better conditions (they reduce the interest rate and attract more firms)
    - Banks penalizes financially fragile firms (the interest rate charged by the lender incorporates an *external finance premium,* increasing with leverage and therefore inversely related to the borrower's net worth)

# Partner choice

- Each D firm has a (productive and credit) relationship with a U firm.
- At the beginning, links are established at random.

- In subsequent periods the network changes endogenously according to a **preferred-partner choice rule** (with noise):
  - with a (small) probability _, the D firm chooses a partner at random;
  - with probability (**1** – _), it looks at the prices of a randomly selected number (M) of U firms
    - if the minimum observed price is lower than the price of the previous partner, it will switch to the new U firm
    - otherwise, it will stick to the previous partner

- The preferred-partner choice also applies to the relationships between firms (both D and U) and banks

# Profits, net worth and bad debt

- The **profit** of *i-th* **D firm** is: $\pi_{it} = u_{it}Y_{it} - (1 + r^{i}_{zt})B_{it} - (1 + r_{jt})Q_{it}$
- The **profit** of the *j-th* **U firm** is: $\pi_{jt} = (1 + r_{jt})Q_{jt} - (1 + r^{j}_{zt})B_{jt}$
- The **profit** of the *z-th* **banks** is: $\pi_{zt} = \sum_{i \in Iz}(1 + r^{i}_{zt})B_{it} + \sum_{j \in Jz}(1 + r^{j}_{zt})B_{jt}$

- At the end of the period, the **net worth** of the *x-th* agent (x=i for D firms, j for U firms and z for banks) is: $A_{xt+1} = A_{xt} + \pi_{xt} - BD_{xt}$

  where BD is **bad debt** (non-performing loans).

  - In the case of **U firms**: $BD_{jt} = (1 + r_{jt})\gamma \sum_{i \in \Phi_j^B} Y_{it}$
  - In the case of **banks**: $BD_z = \sum_{i \in \Phi_z^B} (1+r^{i}_{zt})B_{it} + \sum_{j \in \Phi_z^B} (1+r^{j}_{zt})B_{jt}$

- The agent goes **bankrupt** if $A_{xt+1} < 0$.

# Simulations

- **Agents**: $I$ = 500 (D firms); $J$ = 250 (U firms), and $Z$ = 100 (banks).
- **Time span**: $T$ = 1000.
- **Parameter setting**:
  - Financially constrained output of D firms: $\phi = 1.5$; _ = 0.8;
  - Labour requirement of D and U firms: $\delta_d = 0.5$; $\delta_u = 1$;
  - Intermediate goods requirement of D firms: $\gamma = 0.5$;
  - Interest rate on trade credit: _ = 0.1;
  - Interest rate on bank credit: _ = 0.1; _ = 0.05;
  - Real wage: $w = 1$;
  - Number of potential partners: $M = 5$; $N = 5$;
  - Probability of preferred-partner choice: $1 - \varepsilon = 0.99$.
- **Initial conditions**: new worth is set to 1 for all agents
- **Entry-exit process**:
  - One-to-one replacement: net worth of new entrants is drawn from a uniform distribution with support (0,2)

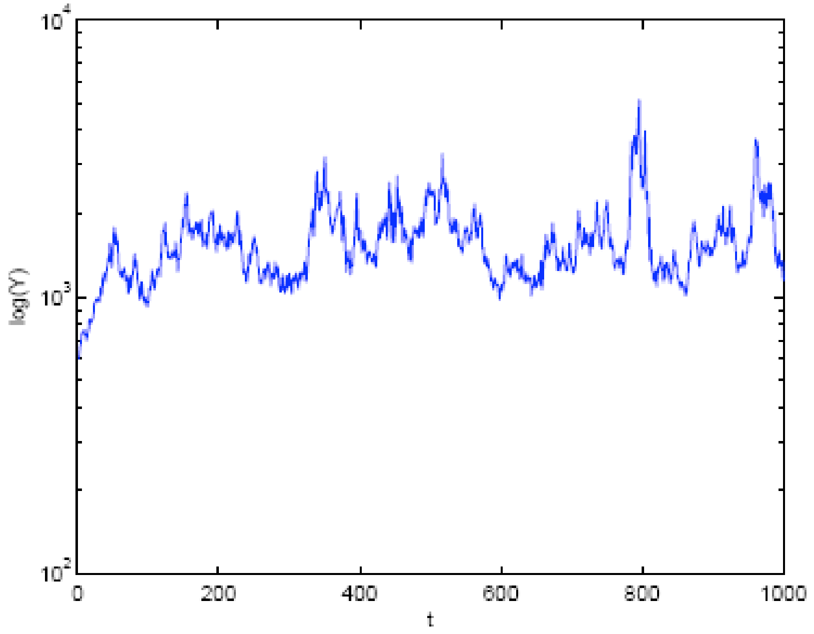



Figure 1: Aggregate Production

- **Aggregate production of D firms**: As expected in complex adaptive systems, **fluctuations are irregular** (amplitude and periodicity vary from period to period)
- **Aggregate production of U firms** follows the same dynamic pattern since U suppliers produce intermediate goods for D firms "on demand".

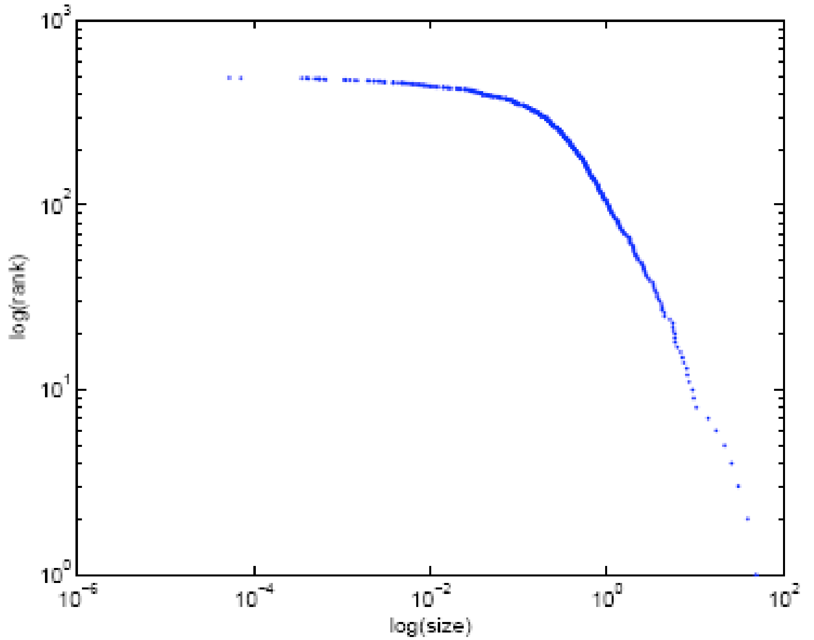
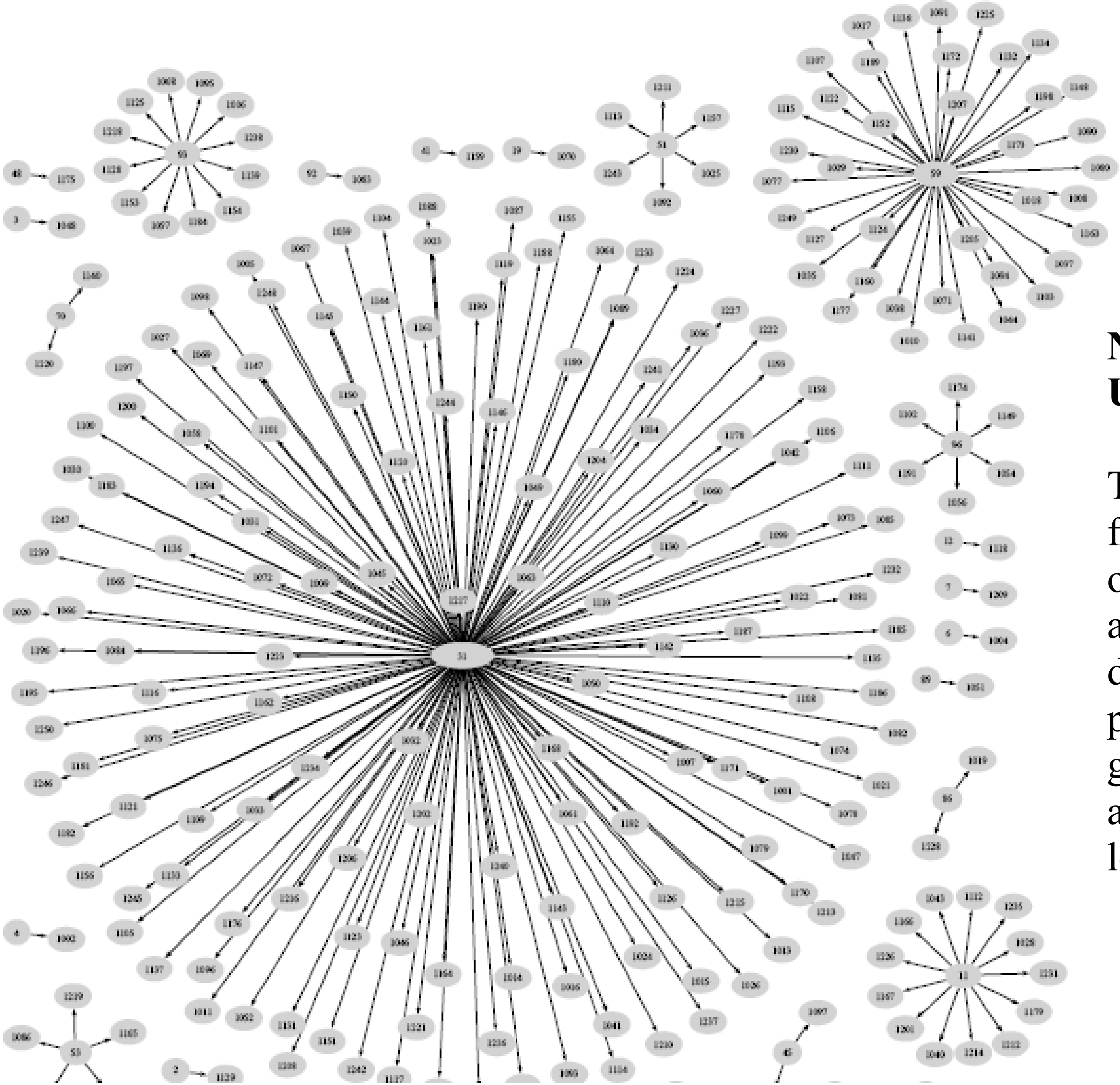



Figure 2: Firm size distribution (DS firms' net worth)

- Starting from **identical initial conditions** agents become rapidly heterogeneous
- **Firm size distribution** tends to a power law



**Network structure:**
**U firms vs. banks**

The number of links for each lender (U firm or bank) becomes asymmetric over time due to the preferred-partner choice governing interaction among borrowers and lenders

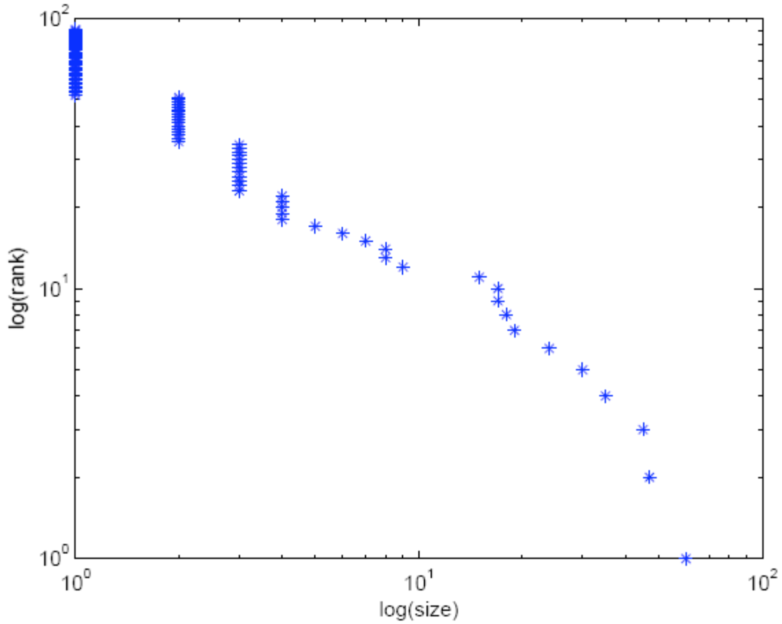



Figure 3: Degree distribution of the network: DS firms vs. US firms

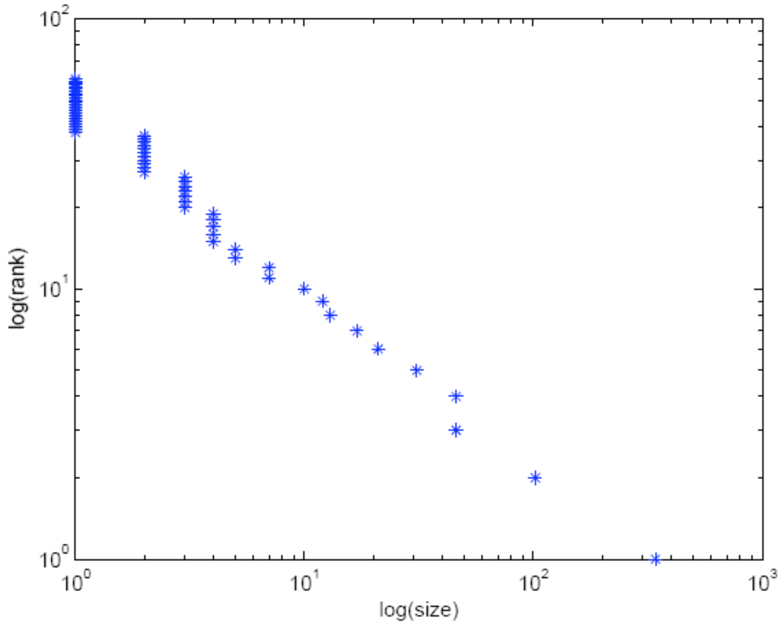



Figure 4: Degree distribution of the network: DS and US firms vs. banks

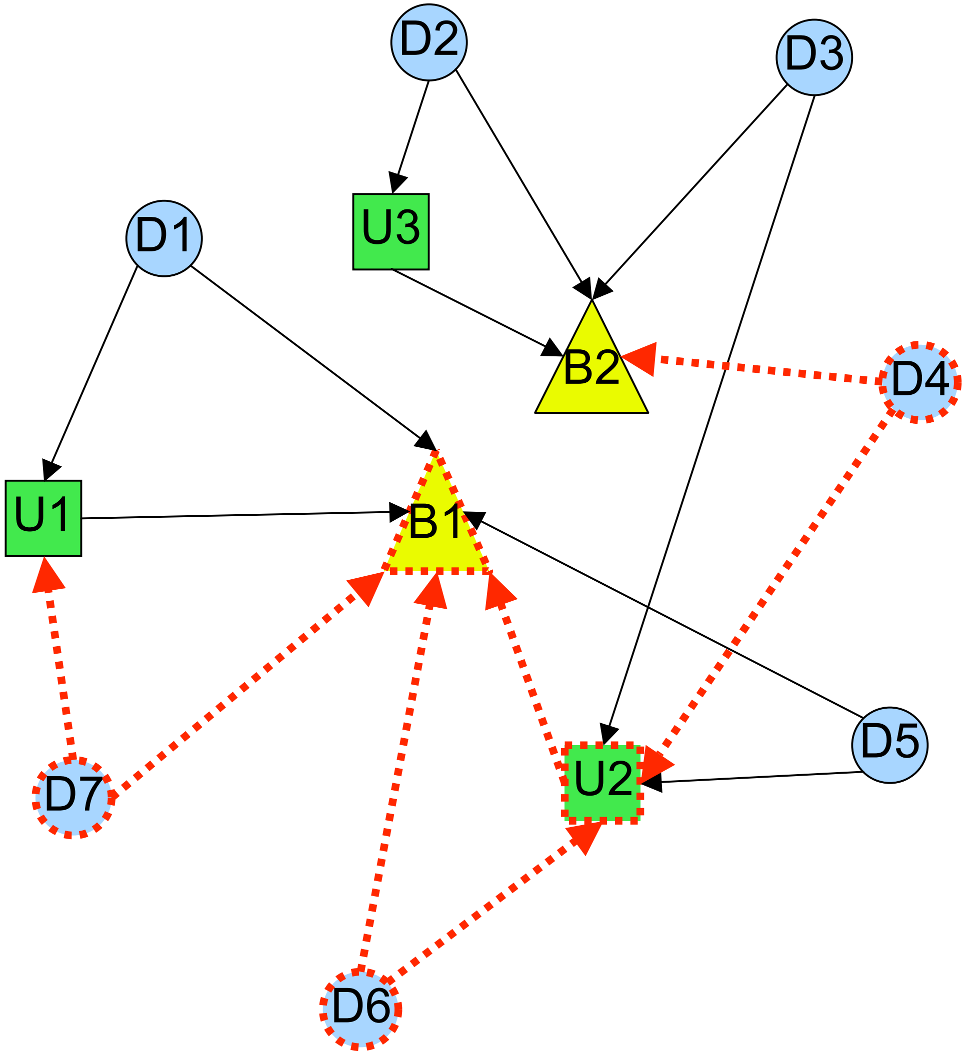
- The **degree distribution** of the interaction network tends to a power law

- The preferred-partner choice rule makes ***preferential attachment*** endogenous through a mechanism similar to the ***fitness model***

- Economic behaviour, financial conditions and network evolution: **financially robust lenders** can supply credit at better conditions and therefore increase their market share. The opposite holds true for financially fragile agents. As a consequence, the corporate and the banking sectors become **polarized** and the degree distribution becomes **asymmetric**.

- **Robustness**: the network is robust to random shock.

- **Vulnerability**: the network is vulnerable to targeted shocks, because the default of a highly connected agent (rare event) may produce other defaults...




**A typical story**:

- D4, D6 and D7 go bankrupt due to idiosyncratic shocks
- They do not fulfill debt commitments
- The financial conditions of lenders deteriorate due to bad loans...
- In this case, U2 and B1 go bankrupt, while U1 and B2 survive to the failure of their partners
- Channel of bankruptcy propagation:
  - The failure of D4 and D6 provokes the default of U2
  - The failure of D6, D7 and, in particular, of U2 provokes the default of B1
- The deterioration of the financial conditions of U1 and B2 may produce an increase of the interest rate...
- The high rate of bankruptcy is a cause of the high interest rate as much as a consequence of it!

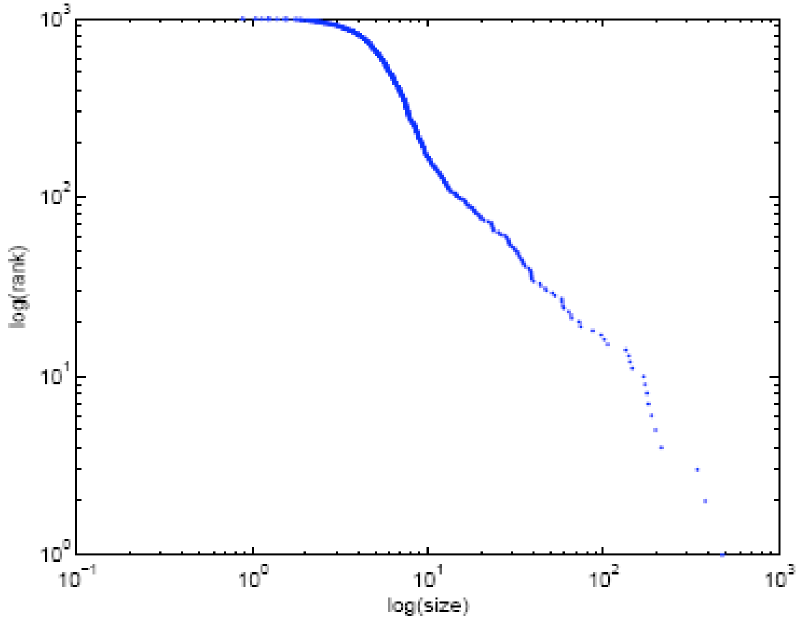



Figure 5: Bad debt distribution

- The extent of bankruptcy depends on the amount of **bad debt**
- The deterioration of lenders' financial condition due to borrowers' bankruptcies may be absorbed if the size of the non-performing loans is “small enough” or the lenders' net worth is “high enough”

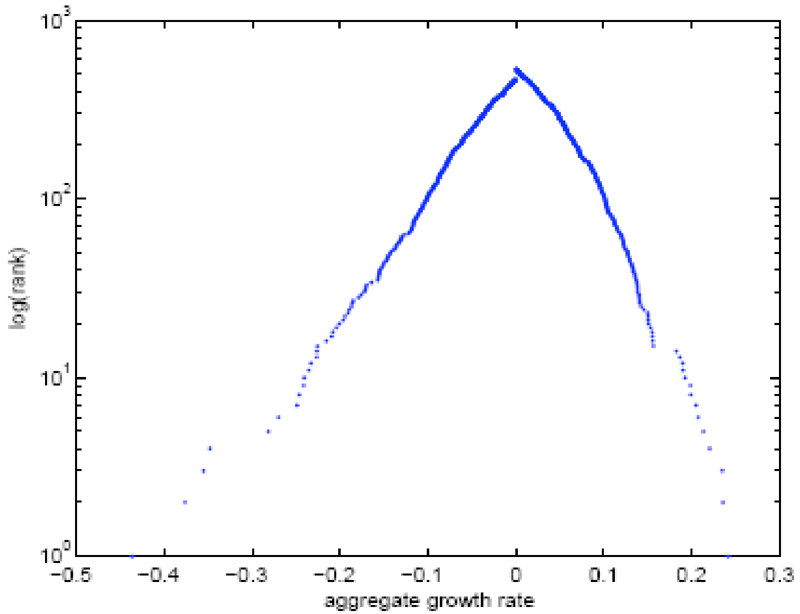


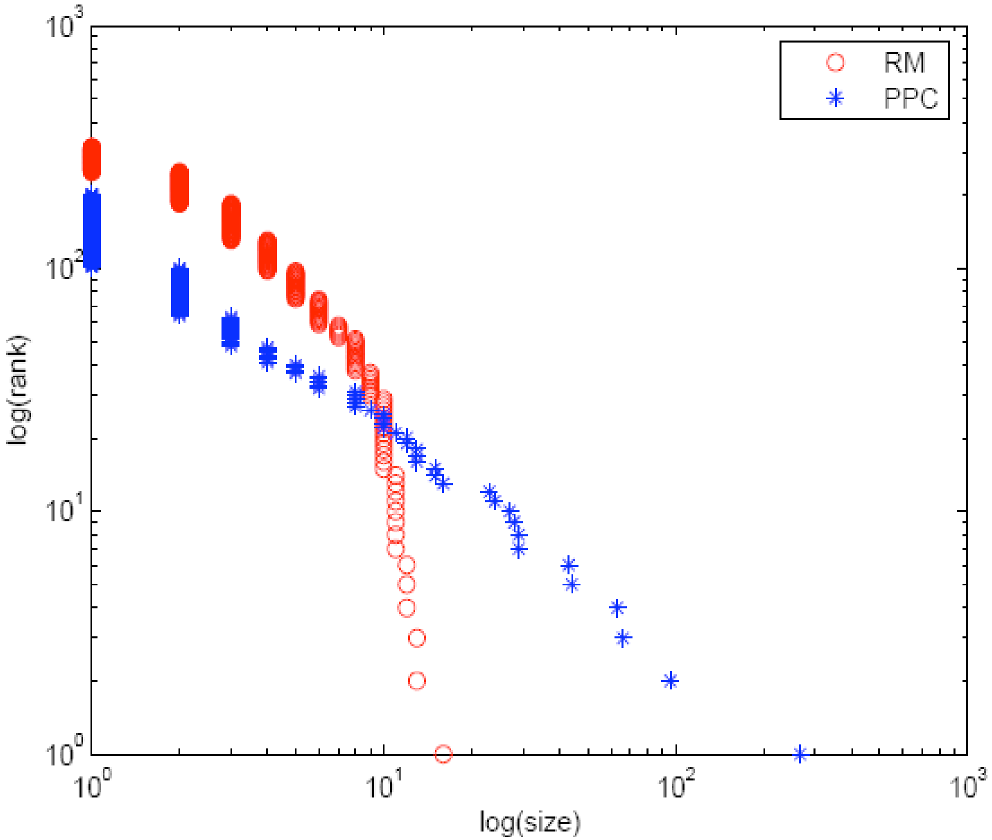

Figure 6: The distribution of aggregate growth rates

- The distribution of aggregate **growth rates** is far from being Gaussian (tent-shaped or double exponential)
- Asymmetry for **negative events**

# PPC vs. RM

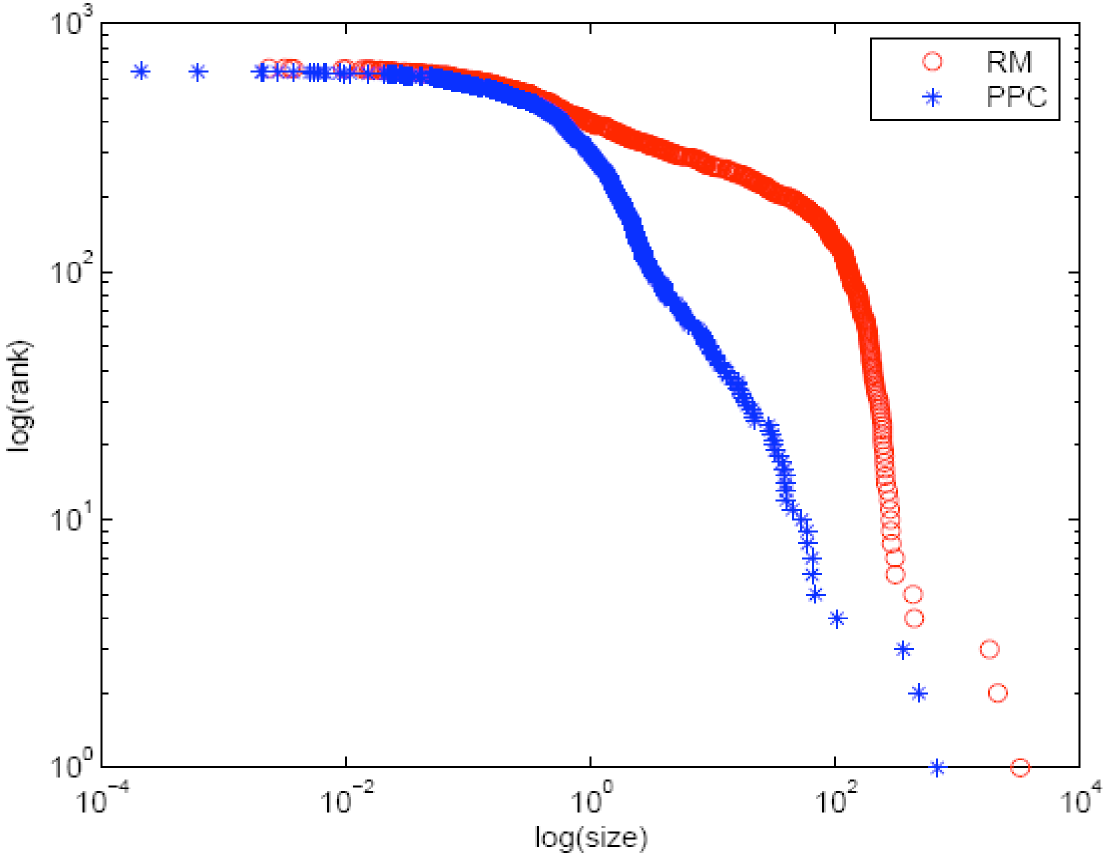
- Random network:
  - in every period each agent chooses a partner at random (model simulation with unchanged parameters but for **$\varepsilon$ = 1**)
- Major findings:
  - Degree distribution of the network (credit interlinkages)
  - Firm size distribution (agents' heterogeneity)
  - Bankruptcy propagation (correlation structure across sectors)
  - Bankruptcy probability (systemic risk)

# PPC vs RM




- Degree distribution of the network
  - with PPC, the distribution of links tends to a power law

# PPC vs RM




- Firm size distribution
  - with PPC, FSD tends to a power law shape

*Table 3. Monte Carlo simulations: RM vs. PPC.*

| | *RM* | *PPC* |
|---|---|---|
| mean(GR) | 0.0013 (0.0000) | 0.0013 (0.0000) |
| standard deviation(GR) | 0.0393 (0.0018) | 0.0401 (0.0025) |
| skewness(GR) | −0.1238 (0.0838) | −0.1393 (0.0696) |
| kurtosis(GR) | 2.9386 (0.1510) | 2.9132 (0.1657) |
| median(GR) | 0.0022 (0.0010) | 0.0025 (0.0010) |
| median absolute deviation(GR) | 0.0266 (0.0014) | 0.0273 (0.0020) |
| robust skewness(GR) | −0.0279 (0.0315) | −0.0356 (0.0301) |
| robust kurtosis(GR) | 0.0031 (0.0521) | −0.0018 (0.0567) |
| median(BD) | 37.2940 (1.5381) | 36.8541 (3.8340) |
| median absolute deviation(BD) | 12.0540 (1.2254) | 16.49 (2.7380) |
| robust skewness(BD) | 0.6612 (0.0681) | 0.6518 (0.0880) |
| robust kurtosis(BD) | 0.4279 (0.1523) | 0.4031 (0.1918) |
| bankruptcy rate: corr(D,U) | 0.2683 (0.0327) | 0.2895 (0.0261) |
| bankruptcy rate: corr(D,B) | 0.2044 (0.0324) | 0.2196 (0.0295) |
| bankruptcy rate: corr(U,B) | 0.7970 (0.0240) | 0.7472 (0.0194) |
| bankruptcy rate: corr(D+U,B) | 0.4320 (0.0327) | 0.4200 (0.0284) |
| bankruptcy probability | 0.1009 (0.0000) | 0.1093 (0.0011) |
| bankruptcy probability: D | 0.1656 (0.0000) | 0.1670 (0.0000) |
| bankruptcy probability: U | 0.0104 (0.0000) | 0.0253 (0.0014) |
| bankruptcy probability: B | 0.0035 (0.0000) | 0.0310 (0.0022) |
| corr(Debt-to-Equity,Y) | 0.6024 (0.1540) | −0.0453 (0.2187) |

- **Bankruptcy rate**: Correlations of bankruptcy rates are similar in RM vs. PPC
- **Bankruptcy probability**: The bankruptcy probability of U firms and banks is higher in PPC than in RM
- **Systemic risk**: even though the correlation among bankruptcies is similar in the two scenarios, the greater incidence of defaults in the U and banking sectors means that the endogenous network increases the likelihood of bankruptcy propagation, starting from idiosyncratic shocks regarding D firms

# Concluding remarks

- Modelling of productive and credit interlinkages: Endogenous network formation through preferred-partner choice

- Credit relationships (network structure), bankruptcy propagation, business fluctuations: bankruptcy rate ↔ interest rate

- Skew distributions: Firm size distribution, degree distribution of networks, bad debt, negative asymmetry for growth rates, etc.

- Endogenous network (PPC) vs. random matching (RM): systemic risk

- Empirical analysis: validating simulation results

- Towards a “complete” credit-network economy
  - Remove the hypothesis of (exogenous) stochastic price
  - Remove the hypothesis of (exogenous) constant number of agents
  - Remove the hypothesis of “on demand” production of U firms
  - Introduce the interbank market to investigate monetary policy issues