I. Collection, Processing and Transmission of the Information

B. V. Gnedenko

Some Problems of Cybernetics and Statistics

Introductory Comments

In the present day the mathematical method has penetrated deeply into almost all fields of knowledge. Physics and engineering, biology and medicine, astronomy and chemistry, economics and agriculture, linguistics and demography -- all these scientific disciplines and areas of activity to one degree or another are using and basing their conclusions on mathematics and are simultaneously pushing it forward to the development of new means of investigation, the creation of new branches of it. The broad concepts associated with the need for clarification of those regularities to which the processes of efficient control of complex systems are to be subordinated, have led, as is well known, to the idea of distinguishing a particular scientific trend, which has been given the name "cybernetics". Naturally, where we are dealing with the selection of the very best, most expedient line of development, we cannot get by with purely qualitative considerations and general rationalizations. It is necessary, on the basis of precise formulation of problems and strict calculations, to find this best line of development. Otherwise, it will never be possible to eliminate subjective estimation and along with it aimless and useless disputes. Therefore, mathematical methods are the natural and essential constituent of cybernetics.

The experience of the history of mathematics teaches us that each new field of its application represents a source of formulation of new mathematical problems and, by the same token, constitutes an impetus to the creation of new trends in mathematical study. This situation occurred at the end of the 17th-beginning of the 18th centuries, when the development of industrial production confronted science with the problem of creating methods of studying processes of change, processes of movement. Mathematics responded to this by the creation of mathematical analysis. In exactly the same way persistent need for the study of the rules and regulations of diffusion of heat, brought about partly by the vigorous development of application of steam engines to industry, led to the occurrence of a new branch of mathematics, mathematical physics. In precisely the same way, now we are witnesses of the birth of many mathematical disciplines. It is sufficient to bring to mind such new names as information theory, linear and dynamic programming, the theory of automata , the theory of mass service, the theory of reliability, programming for electronic computers, in order to convince the reader of what has been said. Simultaneously, a powerful impetus for further development has been given to many mathematical sciences which existed previously, chiefly those such as mathematical logic, mathematical statistics, theory of probability, theory of differential equations. While some 20 years ago it was believed that mathematical logic is far from actual practical application, now it is deservedly considered that it has literally burst into the front lines and is being utilized extensively for the solution of many problems of current importance.

We should like to note that cybernetics, as follows from its very definition, which was given in the Introduction to the present collection, cannot be considered one of the mathematical disciplines. It only utilizes mathematics extensively as a method of investigation and, particularly, makes systematic use of the method of mathematical modelling for the study of those processes for which it has not yet been possible to construct a satisfactory quantitative theory.

In the present article we are limiting ourselves to a presentation of the way in which during the formulation of certain cybernetic problems inevitably the need arises for mathematical formulation of them. Thereby, we shall not even begin to attempt to describe the content of all the chapters of mathematics important for cybernetics or evaluate their roles.

It is well known that the appearance of electronic computers has not only made possible such calculations which recently were considered impossible because of the tremendous volume of the necessary operations; of even greater importance for the future is the fact that it is possible to hand over the accomplishment of a number of logical operations to electronic computers, including operations for the control of industrial processes. Naturally, before handing over to the electronic machine the control of one process or another it is essential to study the characteristics of this process, to clarify which operations and in which order the machine could accomplish them under different conditions as well as which information should be put into it so that the machine be in a position to follow the course of the process at every moment and to work out sound solutions. At this stage a special role is played by logical analysis and, by the same token, by the method of mathematical logic. In order to learn the form and the information necessary from the controlled system for the purpose of substantiated action on it by the controlling organ it is essential inevitably to bring in mathematical statistics and the information theory.

Undoubtedly, the mathematical apparatus being used will be used only after a detailed study of the contents of the matter, stemming from the characteristics of the process being controlled and the control system. This is why, not uncommonly, it would appear different mathematical methods have to be applied and different methods of solution used for similar phenomena: new situations may require new mathematical systems.

In this article the author is not touching on many problems and is dealing chiefly with those which he himself has studied. This fact naturally makes an indelible impression upon its contents.

Problems of Medical Diagnosis

The problem of rapid and accurate determination of the disease apparently is one of the oldest. Nevertheless, until recently the solution of it was furthest from mathematics. Medical intuition and experience accumulated by medicine for thousands of years were the only things, apparently, which make it possible for a treating physician to draw conclusions about the patient. The general impression which the patient makes on the physician, acquaintance with his life, nature of work, case history, and, certainly, personal examination, undoubtedly put exceptionally valuable material into the hands of the physician. It is well known that this information affords one physician exhaustive basis for making substantiated conclusions; for another it remains useless weight, to be utilized only for filling out the case history. Moreover, the same information dealing with the same patient not uncommonly leads to essentially different conclusions even by experienced physicians.

When in 1956 I and my colleagues at the Institute of Mathematics of the Academy of Sciences UkrSSR became interested in the problems of medical diagnosis and clarified the possibilities of utilization of modern high-speed electronic computers for this purpose we had a multitude of meetings and talks with physicians. We were interested in the following question: how does the physician act to make diagnostic judgments? Frequently, the answers received were of approximately the same nature: "We do not go into the diagnosis of the disease by any definite route, because the variety of diseases and their manifestations are exceptionally great, and even in the presence of apparently the same symptoms not uncommonly we have to draw different conclusions. We make extensive use of our knowledge and medical intuition, which every good physician should possess". Other answers were of a more definite character: "Every physician begins with physical examination and questioning the patient. Not uncommonly, even from the undetectable signs of the manner in which the patient enters the physician's office it is possible to learn the nature of his disease. If these initial observations are inadequate, the case history, listening to the patient's complaints, information about diseases suffered in the past, laboratory analyses and clinical observation give him the right to reject many possibilities and concentrate his attention on a comparatively small group of diseases. In any case, we calculate nothing but know from experience that in the presence of a given sign there may be different diseases, but in the great majority of cases a certain disease is observed. It goes without saying that in simple cases every physician correctly diagnoses the disease. However, in complicated cases additional studies are needed and the greater the quantity of data we obtain about the patient the more precisely we can make the diagnosis. As a rule, medical errors are associated with too hasty conclusions, without complete examination of the patient".

The second answer certainly gives a more thorough picture of the way in which the physician reasons in making the diagnosis. First of all, he bases himself on his own experience and on the experience of all medicine. We must look upon the experience of medicine chiefly as a qualitative (perhaps, semiquantitative) processing of tremendous statistical material. Observations of a large number of patients permit us to derive the relative frequencies of various diagnostic symptoms in the presence of one disease or another as well as the frequency of occurrence of one disease or another in the presence of various groups of symptoms. However, in medicine these observations do not reach the point of being numerical, and customarily therein the verbal evaluation of the frequency is used: "as a rule", "in the great majority of cases", "very rarely", and others.

At the present time, verbal evaluations of this kind in physics, chemistry, and engineering are permissible only on initial acquaintance with the phenomenon. Undoubtedly, for precise conclusions in medicine more complete quantitative data are needed of the following type: in the presence of a given symptom complex the probability of disease B_1=0.98; of disease B_2, 0.37, whereas B_3 has a probability of only 0.02. This is why we have begun to look for estimations of this kind in the literature. Unfortunately, we fail to find them. Either the quantitative data were derived from too little material or treatment of them were completely unsatisfactory, and here clinical material, collected with tremendous difficulty, almost completely lost its value. The difficulties which stand in the way of obtaining well grounded quantitative data of this kind are understandable. However, we were convinced that further progress in medicine of necessity leads to the need not only of collecting statistical data but also of correct statistical processing of them, obtaining different kinds of quantitative and qualitative conclusions from them. Specifically along this line it will be possible to obtain estimates of probabilities of different kinds of events essential for diagnosis, sanitary hygiene, labor prophylaxis, etc. By this method specifically it will be possible to obtain objective estimates of the significance of different symptom complexes, objective comparison of the quality of different diagnostic methods and therapeutic procedures. All this should inevitably attract qualified specialists in the field of the theory of probability and mathematical statistics to work in the field of medicine.

Mathematical statistics and the probability-theory style of thinking in making a diagnosis should win for itself the right to a position in medicine with full rights, both in medical theory and in medical practice. This should be done not only for the sake of external appearance but also for the sake of actual knowledge of truth. Undoubtedly, essential aid therein can also be given by mathematical logic. Its utilization at least will serve for greater clarity of judgments as well as for the elimination of the subjectivism which now reigns in making diagnostic conclusions.

The problem just outlined is one of the many which occurs in medicine and insistently requires solution. If we take into consideration the fact that among the objective characteristics which physicians use for diagnosing diseases there are electrocardiograms, phonocardiograms, ballistocardiograms and others, that is, tracings of the realization of stochastic processes, we must come to the conclusion that it is necessary not only to calculate the means and the dispersions but also to use incomparably more complicated techniques for the statistical treatment of medical data. Specifically, since we can look upon the blood flow as a stationary process, the characteristics just mentioned (at least to a good enough approximation) represent realizations of stationary stochastic processes. Therefore, now we cannot consider that medicine must be limited to some meager store of statistical facilities. Conversely, dealing with the actual problems of medical practice shows us convincingly that modern medicine needs rational utilization of the entire arsenal of measures developed by mathematical statistics.

The viewpoints just presented have been made the basis for an approach by our small group to the utilization of mathematical methods in medical diagnosis. We, engineers and medical personnel, went together along lines which mutually supplemented one another. On the other hand, we attempted to reveal those logical premises and conclusions which guide the physician in his recognition of disease. On the other hand, we attempted to evaluate the statistical significance of the diagnostic characteristics as well as symptom complexes. Simultaneously, different detectors and diagnostic apparatuses were worked out. Programs were made up for formulating the problem of diagnosis on an electronic computer, and in accordance with these programs actual patients were diagnosed. Finally, on this basis the first variant of a diagnostic machine was created. The observed signs of heart disease were put into the machine, and the machine either gave out a ready-made diagnosis or indicated the need for additional study. In a number of cases the machine gave several possible diagnoses for the same patient rather than a single one.

Checking of the operation of the machine on real patients showed us that the original diagnostic schema which we used was imperfect not only because it took into consideration too small a number of characteristics but also because it provided for the indication of only the observed characteristics to the machine. Actually, no less essential are those characteristics which were not observed.

Our first experience, despite all its imperfection, showed that the utilization of mathematics, and chiefly the mathematical method of thinking, can be useful even at such a primitive stage at which we began our work. The diagnostic machine interested the medical personnel and we received many suggestions for constructing similar machines for diagnosing and studying other diseases. Simultaneously, it happened that a number of physicians whom we considered confirmed antimathematicians showed exceptional interest in our activities and offered their collaboration in future work. It was made clear that the diagnostic machine even in its original form can be useful for teaching medical students. If suitable, this machine should be a standard below which the medical student cannot go. It is interesting to note that after putting almost two-score case histories into the memory unit the machine in one case indicated the possibility of two diseases in the same patient. On one diagnosis the attending physicians working in different clinics of Kiev agreed unanimously, whereas the other diagnosis was just as unanimously denied. However, during the course of the discussion which developed it was made clear that this possibility indicated by the machine was still conceivable and had simply escaped the attention of those present.

Our plans for the future lie in the fact that the diagnostic machine will receive the initial data directly from the patient through a system of detectors and diagnostic instruments, and only part of the data associated with evaluation of the patient's external appearance is to be taken from the case history.

Problems of utilization of mathematical methods and electronic computers for diagnostic purposes are at present interesting many research groups in the Soviet Union and outside it. In a number of journals articles on these topics have appeared which merit attention. I am pleased to note that in two articles by American authors, R. S. Ledly and L. B. Lasteda (Leningrad, 1, 2) the same guiding ideas are presented which motivated our small group.

Perhaps it is not superfluous to note that now, in the era of space conquest, the ideas mentioned here acquire particular poignancy and interest. Here, more than anywhere else, it is particularly important to obtain exhaustive information from without and decode it automatically. Thereby, we are limited in the volume of information which can be transmitted.

Problems of Organization of Production and Problems of Mass Service Theory

One of the characteristic features of modern industrial production is its mass nature. On the one hand, many articles are produced and, on the other, production of them is associated with a large number of operations performed in a certain sequence. Every operation requires, on the average, its own definite period of time. In connection with this, the need arises for calculating the number of mechanisms for every stage of processing the article. A rough estimate is that the average productivity of a group of mechanisms accomplishing one operation is equal to the average productivity of the mechanisms accomplishing another operation. However, such a primitive calculation is rarely satisfactory and frequently leads to serious miscalculation.

For example, in world practice cases are known where for the purpose of supplying shaft-type mechanisms special electric power stations were built, and their power was calculated from the total power of the mechanisms set up. Practice showed the defect of such calculations: the power of the electric stations could not be utilized completely, because in normal operation of the equipment it was found that it requires less power than was expected from such a primitive calculation. The cause of this phenomenon is almost obvious: every mechanism has a certain work cycle during which the power required changes. The duration of the cycle does not remain constant either, but rather undergoes quite considerable random variations. This is caused by many factors: the inhomogeneity of the material being processed, change in the voltage, temperature variations, changes in the condition of the instrument, etc. The power required, incidentally, changes from cycle to cycle. As a result, at every moment of time t it represents a random function P (t). The total power, used by all mechanisms, represents the sum of these functions. In summation, the maximum loads of some mechanisms almost inevitably will be superimposed on the small and average loads of others. As the result the actual requirement of power will be much less than the sum of the prescribed powers of all the connected mechanisms.

The problem of efficient calculation of electrical loads in industrial enterprises is of tremendous interest for the national economy. In the press it has been mentioned repeatedly (see, for example, 3) that substantiated calculation methods on a nation-wide scale will lead to a tremendous saving in nonferrous metal, which unjustifiably is now being put into excessively strong cables and transformers. We see, therefore, that optimized control of resources in the area of energetics inevitably leads to the need for utilization of mathematical methods. In the example which we have presented we have come across the utilization of methods of mathematical statistics and the theory of stochastic processes (Leningrad, 4). Simultaneously, here we have an opportunity to analyze another problem: how it frequently happens that variously large numbers of mechanisms simultaneously begin to require maximum power and, by the same token, cause overvoltage in the leading-in cables.

At the present time, such problems arise in many fields of activity, and systematic solution of them has led to the creation of a new scientific discipline known by the name of "the theory of mass service", or, as it is called outside of the Soviet Union, the theory of priority.

In the 1930's, in connection with automation in industry, a transition was planned to operation of several machines by a single worker. At random moments of time the machines, for various reasons, lose their normal rhythm of operation and require care by the repairman servicing them. The duration of the operation needed for starting the machine, generally speaking, is not constant and represents a random magnitude. The question is asked how frequently in a given routine of operation will service be required to a certain number of machines of the total number under the care of the repairman? Further natural questions important for practice are the following: What is the average time that the machines have to be idle when different total numbers are under the care of a repairman? How many machines is it economical and advisable to entrust to a single repairman under a given work organization? How is the work time of the machines and of the repairman utilized, given one service organization or another? How can the service be organized more efficiently: by giving n machines to a single repairman or ns machines to a brigade of s repairmen?

In the organization of constant-flow production and automatic production lines a multitude of problems arises similar to those just presented. We shall dwell on one of them. For the purpose of assuring continuous operation of all successive machines, in the event one of the groups goes out of commission and stops supplying the line with its production, bins are set up in front of the machine in which a reserve supply of articles is accumulated which have passed through all the previous stages and which are awaiting processing at the given machine. How great should the reserve supply of articles be in the bins in order to provide for continuous operation of the line?

The problems associated with calculating the capacity of the bins are of tremendously broader significance than simply providing for the continuous operation of an automatic line. The same situation arises in calculating the volume of storehouse premises as well as in the determination of efficient reserves of parts and units of machines which are needed for the normal operation of a group of machines. A problem which is important and arises continuously in practice is the following: at an automobile factory (this is said only to be definite; we could with equal success speak about an airport, mine or others) there is a store of supply material. How many and which parts need to be brought to the storehouse at the beginning of the year in order to maintain the existing mechanisms in working condition with sufficiently great dependability for all 12 months?

Calculation of the automatic line is at the present time the work of a large number of investigators. Solution of this problem is confronted with a number of analytic difficulties which frequently do not permit obtaining calculated formulas. In these cases work is done along the line of modelling the process of work of the automatic line and, by the same token, the necessary information is obtained. We are presenting the actual idea of such modelling in an exclusively schematic form and we will not elucidate numerous essential details. We should like to note only that in such modelling the utilization of mathematical logic and the theory of mass service go side by side continuously.

The entire mass of parts going into the first operation is divisible into two parts after processing: one part consists of those suitable for further processing; the second part, of those which are discarded. This second part is eliminated from further processing. In order that the group of machines operate continuously and in order that there be no unproductive periods of idleness it is essential to find out the expedient number of machines in it as well as the volume of bins in which the reserve of parts is being created.

In connection with the fact that the modelling method is utilized extensively in cybernetics we shall dwell on the idea of modelling the operation of an automatic line in greater detail. Let us suppose that the raw material for transformation into the final product has to go through the following stages of processing: A_1, A_2, ... A_n. The processing at stage A_s takes up the time γ_s, which depends on many chance circumstances. Let us designate the distribution function γ_s by $F_s(t)$. After each operation a discarded article is possible which subsequently is no longer processed; at stage A_s the output of discarded material occurs with a probability p_s. What should the work output be on such an automatic line? How many machines should be set up for each operation? What volume of the bin should be established in front of each machine? How should the production be distributed among the subsequent machines after each operation? The situation is complexified even further if we take into consideration the possibility of machines going out of commission at each of the operations, the change in the operation of the instrument, overfilling of bins (it is essential to solve the problem of where to send the incoming products in this case) and others. In the case of automated production all this information must come to the controlling system, as must also information about the quality of the raw material, quality of processing at each stage and others. How will various automatic lines, at present only in the planning stage, operate? This question constantly motivates the creators of new technical equipment. At the present time, for the purpose of answering it there is no need to prepare an experimental specimen or model in metal. It is possible to construct a mathematical model of the work process of an automatic line and test its action on an electronic computer. For this purpose the program is made up, in which each of the operations to which the articles to be processed are subjected is taken into consideration the randon time which is necessary for accomplishment of them is worked out on randon number data units. Usually, a uniform distribution is worked out and then it is converted into the necessary stochastic figure with a given probability distribution. Provision is made for a given probability of rejection of poor quality material after each operation. The possibility of the machine's going out of commission and of restoration of it in a random time with a distribution $H(t)$ is also introduced. At the output all the necessary work characteristics of the line are obtained. It goes without saying that each work cycle will have in it randon components, and by a single cycle it is impossible to judge the quality of work of the line and its individual units. This program has to be tried out several times, and thereby the work of the line can be judged for a long period of time. High-speed computers make it possible to play back every cycle for a short time and, by the same token, carry out the entire work cycle of the line in several minutes. It is possible to become acquainted with programs of this kind from the periodical literature as well as from a book (Leningrad, 5).

Systems of mathematical modelling, which are used for the construction of models for automatic lines, can be transferred almost unchanged to many other problems which apparently are very distant in their actual nature from production problems. Thus, in recent years problems of planning of the national economy within limits of the country, economic region, branch of industry or separate enterprise have been studied. Systems of physiological processes have been constructed. Specifically by this method many problems of modern physics and mechanics have been calculated out. We should like to note that in the book just mentioned (Leningrad, 5) a calculation system is given for an atomic reactor.

Statistical Problems of Reliability of Control Systems

In connection with the fact that exceptionally important problems are being entrusted now to control systems and that the going out of commission of such a system can lead to essential material losses, to a breakdown in the accomplishment of an assignment and, perhaps, to casualties among people [the rest of this sentence has been omitted from the original text]. Actually, if an electronic machine is entrusted with control of a blast furnace process and it fails to operate the blast furnace may be damaged. If the control apparatus is designed for regulating the operation of an atomic reactor, failure of this apparatus can cause serious accidents. In exactly the same way faulty operation of an apparatus controlling a large energy system entails not only very great losses of a production nature but also a stoppage of power supply to hydraulic pumping stations, hospitals, operating rooms, and others. This is why at the present time every year the assurance of continuity of action of control systems, reliability of them during the operation process are acquiring progressively greater importance. From this the exceptional importance of learning those factors which lead to imperfections in the manufactured objects as well as to unreliability of function of control systems can be seen. Then, problems arise of assuring the reliability of action of systems for setting up which unreliable components are utilized. Thus, the need appears for a complete study of problems of reliability and, therefore, construction of a theory of reliability.

A disturbance in the function of a control system can occur for many reasons, of which we shall point out only a few: failure of various components because of "death" of them, temporary deviation of the parameters from the normal state, distortion of the incoming information about the functioning of the system being controlled and, by the same token, an erroneous reaction of the control system.

For the purpose of assuring greater reliability of the information being transmitted several methods are used: excess information is transmitted; information transmission is duplicated; special noise-proof coding systems are worked out. To be sure, thereby one runs into certain additional difficulties, among which we shall point out only one for the present -- an increase in the load on the communication channels and, by the same token, complexification of their arrangements. Construction of noise-proof codes has now grown into a large research field and already constitutes a large division of the information theory.

Problems of constructing reliable systems from unreliable components have given rise to a large literature in recent years. To a considerable degree the solutions proposed proceed along the line of complexification of the apparatuses, introduction of additional components which go into operation either simultaneously with the main components or immediately after the main component goes out of commission. After the first studies by J. Neumann (Leningrad, 6) and C. Shannon (Leningrad, 7), these problems have been studied by many authors. Here we have the entire literature on reserve theory.

Numerous observations have shown convincingly that the time of perfect operation of an element is random. The elements taken from the same group, kept under the same conditions and operating literally under identical circumstances work for different periods of time. This spread in the "longevity" of an element is exceptionally great and varies from several hours to tens of thousands of hours. The tremendous statistical material accumulated in the practice of production of manufactured articles of different kinds as well as from special tests has shown convincingly that this picture of going out of commission is characteristic of almost all elements: during the initial period of operation of the elements the probability of going out of commission is comparatively great, then it falls off rapidly, and for a long period an almost constant value is maintained which later again begins to increase. We should like to note that this picture is strikingly similar in its structure to the mortality rate curve known from demography. There, an increased child mortality rate is also observed; then, there is an almost constant probability of mortality during maturity, and a new increase in the probability of death for persons of advanced age. This fact can bring to mind the utilization of the well-developed mathematical concept of the mortality rate theoryin problems of reliability of elements.

Frequently, a system consists of a large number of components each of which, independently of the others, can go out of commission. In recent years some general regularities of going out of commission have been elucidated at least for one component during a given period. It has been shown that under very general conditions the probability of a given system's having exactly k breakdowns of its component elements during the period T is subordinate to the so-called "Poisson Law":

$$P_k(T) = \frac{(\lambda T)^k e^{-\lambda T}}{k!}.$$

Here λ greater than zero is a constant which characterizes the quality of the given system; specifically, λ is equal to the average number of breakdowns per unit time.

The period of perfect work of the system (that is, the period of time between two successive breakdowns) is thereby found to be quite definitely distributed; specifically, the probability of the fact that the system will operate continuously for more than the time T is equal to $e^{-\lambda T}$.

Naturally, the problem arises of checking on how well this general theoretical system depicts the true picture. For the premises made the basis of the theory somewhat distort the actual conditions under which the system has to operate. Thus, for example, when an electronic computer operates all of its components are immediately under conditions of elevated temperature. Even this fact shows that there is no complete independence of the components in actual practice. However, there is every reason for the believe that the general result mentioned will preserve its significance also for components operating under conditions where dependency is not too great. Incidentally, the empirical material which we have at our disposal shows quite convincingly that breakdowns of systems consisting of a large number of components actually do fit well in the Poisson distribution.

In actual systems sometimes there is not too large a number of components. To what degree will the breakdown distribution deviate from Poisson's Law? For this purpose recently accurate estimates have been obtained. We shall not present the corresponding formulas here. We should like to point out only that for a small number of components the results found make it possible to approximate the factual data considerably to those which should be expected from theoretical considerations.

Let us now return to problems of reserve and let us analyze several formulations of the problems and general results. Suppose it is necessary for us to provide reserves for a controller. How is it more advantageous to do this: to provide reserves in small or large units? If the switching is absolutely reliable, it turns out, no matter what the law of probability distribution for the longevities of the components shows, that it is more advantageous to provide reserve units for the controller in small rather than large units. It would be even more advantageous, certainly, to provide reserves for each component. A different result may be obtained if the switches are unreliable. Then, a situation can arise in which small unit reserves will be less advantageous than large ones.

For many controllers it is important not only to provide for adequate reliability of the system but also to subordinate it to some additional conditions. Thus, for example, for aircraft apparatus it is important that it be as light as possible, occupy minimum space (no more than a certain given space) or that the total expenses for the apparatus do not exceed a certain limit. Under these additional conditions the question of the most advantageous provision of reserve units arises with consideration of the necessary limitations. We have come to the need for analyzing distinctive problems of variations.

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The theory of reliability is in a state of vigorous development. In it many new problems are occurring which require not only the application of standard investigation methods but also the development of new methods. Since problems of reliability are associated with random events statistical methods are basic for them. This idea at present is predominant. This does not at all exclude but rather presupposes the idea that the physical aspect of providing reliability for components and systems should not in any case be overlooked.

Problems of Optimal Planning

The last question which was analyzed cursorily in the previous section pertains to a group of problems which have been vigorously and completely developed after the studies of L. V. Kantorovich (1939). The totality of problems pertaining to these works and methods of solving them have been separated out into a significant and interesting mathematical discipline. It is called "linear and dynamic programming". L. V. Kantorovich himself suggested calling it the "theory of optimal planning". It seems to me that this latter name is closest to giving us the essence of the matter. We shall begin our general acquaintance with the group of problems under analysis with the theory of reliability problem already mentioned.

It is necessary to provide reserve units for a certain apparatus consisting of units of types B_1, B_2 ... B_n so that with a weight of the apparatus which does not exceed a set value P the reliability of the entire apparatus be maximal. If we designate the number of units of types B_1, B_2 ... B_n and their weights by P_1, P_2... P_n, respectively, the condition which is superposed on the solution sought has the following appearance:

$$P_1m_1+P_2m_2+\ldots+P_nm_n\leqslant P.$$

We are to select numbers m_1, m_2, ... m_n so that this condition be fulfilled and so that simultaneously the apparatus have the maximum operational reliability.

In this problem the condition is linear with respect to unknown numbers m_1, m_2, ... m_n, but the reliability, the maximum of which we are seeking, is expressed nonlinearly by these numbers. Therefore, our problem, being a typical problem of optimal planning, is a problem of nonlinear optimal planning.

Let us now analyze several problems of optimal linear planning for the solution of which more or less convenient general methods have been worked out. These problems, as follows from their very formulation, are encountered in literally every field of activity.

Suppose the problem has arisen before us of making up a diet for animals, whereby the aim which we are striving to achieve lies in arriving at food outlays which do not exceed a certain limit while providing the animals with the necessary quantity of fats, proteins and carbohydrates. Suppose each animal needs a minimum of some P_1 units of proteins, P_2, of fats; and P_3, of carbohydrates. Reduction of the norm of each of these components is inadmissible, whereas some increase over the norm is permissible. We have at our disposal four types of feed: A_1, A_2, A_3, and A_4, the cost per kilogram of which is respectively equal to c_1, c_2, c_3, c_4. It is well known that feed A_i will contain p_{1i}, p_{2i}, p_{3i} units of proteins, fats and carbohydrates per kilogram. We want to know the number of kilograms x_1, x_2, x_3 and x_4 of feed of types A_1, A_2, A_3 and A_4 which must be obtained so that the total outlay

$$c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$$

be at a minimum if the animal obtains a quantity of protein equal to

$$x_1p_{11} + x_2p_{12} + x_3p_{13} \geqslant P_1,$$

of fat equal to

$$x_1p_{21} + x_2p_{22} + x_3p_{23} \geqslant P_2$$

and of carbohydrate equal to $x_1p_{31} + x_2p_{32} + x_3p_{33} \geqslant P_3$.

There is no need to say that a problem formulated in this way has a direct relation to efficient control of resources or, in other words, to the attainment of the goals set by the optimum method. We have not confronted ourselves with the aim of analyzing the calculation of rations in all its details and, for this reason, we have overlooked a number of important factors. However, for the purpose of clarifying the nature of the problems which arise our schematic form of presentation is fully adequate.

The following manners of formulation of the questions are differentiated simply by the actual content in the example analyzed:

a) there are several production points for a certain product (for example, coal basins), and the points of consumption of it are given. The cost of transportation of the product to any point of consumption from any point of production is known. Also known are the requirements of each consumption point and the output of every production point. It is necessary to find a plan of transportation in which the total transportation expenditures will be minimal. At the present time, a large number of specific transportation problems have been solved by making out the optimum transportation plans. Specifically, plans for the most efficient coal supply of the main industrial centers of the country have been calculated. Several years ago an optimum plan was made up for supplying 216 large structures of Moscow with building sand from a few score quarries and piers. The daily saving on cargo transportation proved to be somewhat more than 2,000 rubles according to the new currency.

b) at each of the enterprises there is a large quantity of different equipment which is capable of doing different types of work.

The productivity of the various types of equipment is given for each of the types of work and the necessary assortment of products. The question is how the work should be distributed among the equipment in order to assure its maximum output in fulfillment of the assignment on assortment of products. There is no need to say that we often do not use calculation of the optimum plans for equipment load. It is sufficient to say only that each percentage of increase in output of the equipment for the Soviet Union signifies an increased production output of many hundreds of millions of rubles without introducing into operation additional equipment, so that it has become clear how important it is to make such calculations by rule rather than exception.

c) in mass production it is necessary to prepare products in a certain assortment from material of a certain shape and size (for example, sheets, strips). It is necessary to find a plan of patterning in which for the purpose of obtaining a single set of products the minimum quantity of material would be used and hence the minimum quantity of it would go to waste. At the present time, quite a large number of different specific problems have been solved and thereby it has been made clear that by comparison with existing plans of patterning of the materials it is possible to save three-10 percent. There is no need to emphasize the economic importance of methodical solution of problems of this kind in all types of production. For an initial acquaintance with problems already solved we can recommend the interesting book by L. V. Kantorovich and V. A. Zalgaller (Leningrad, 8).

If it is necessary to solve a problem of linear programming with a large number of unknowns, the need appears for considerable calculation work, for the purpose of doing which it is necessary to use electronic computers.

So far, we have assumed that all the parameters (cost, weight, size and others) were constants. On this supposition general methods of solving problems were worked out both for optimum linear planning (linear programming) and parabolic optimum planning, in which more than just linear relations are possible. The situation is complexified in those cases where the parameters under analysis are stochastic. In some problems of control, both in economic and production situations as well as in the study of problems of control in living organisms an analysis specifically of such complexified problems is essential. I do not know whether any general results have been obtained along this line.

Problems of the Mathematical Theory of Games

Frequently, in control processes situations arise which are similar to those which constantly are dealt with in games of various kinds: there are two or several groups, each of which tries to achieve the goals with which it is confronted. These goals may be contradictory to each other. In order to achieve the desired result, each of the groups strives under the given conditions to find the most reasonable line of behavior or, as they say, to choose the optimum strategy. At the present time, the mathematical theory of games with two participants has been well developed. There is nothing surprising about this, because even with the presence of three participants the situation can be complexified exceedingly: the third participant, in achieving his own goals, can form a coalition, variously, now with one of the other participants, now with another. The history of diplomacy gives us a multitude of examples, where depending on the course of the diplomatic game one of the governments vacillated between the main opponents and gave aid, now to one of them, now to the other. In some cases such behavior has given the main advantage specifically to this third power. Coalition games essentially presuppose the existence of at least three participants.

Under actual conditions one not uncommonly has to deal with a game of two players (sides). Games of this kind specifically are chess, soccer, volleyball and others. It goes without saying that in such a game as soccer a generalized concept is used for the participant of the game, the entire team. Such a situation is encountered also in cases where we are not dealing with games in the narrow sense of the word but rather with economic, production or military behavior.

As always, when a mathematical theory is being constructed for describing and studying one phenomenon or another it is necessary to abstract from many actually existing circumstances. We have to approximate reality, construct a schema of its occurrence and mathematically study this schema rather than actuality itself. It is entirely clear that the creation of a schema of a phenomenon to which the mathematical method will be applied represents the main part of construction of any theory. The more details we take into consideration during the course of development of the phenomenon the closer our schema will be to reality, generally speaking. Therefore, we obtain the possibility of studying the phenomenon itself more accurately. Simultaneously with this, bringing up too large a number of details for analysis complexifies our schema and makes it boundless in practice. This is why we always strive to distinguish only the main factors influencing the nature of development of the phenomenon in constructing a theory of a phenomenon and why we reject as inessential numerous other characteristics of it.

First of all, it is assumed that every game is characterized by a system of accurately formulated rules. Further, it is supposed that the players make their moves in a certain order. Every move is associated with a certain number, the advantage of one player; this number assumes different values depending on the move which has been selected by the player. If the number of possible selections in every move is finite for the player, the game is called "finite" . If at the end of the game the sum of the advantages of all the players is equal to zero, the game is called a game with a zero sum. If player A has the opportunity of selecting one of m possibilities every time and thereby obtaining a certain advantage depending on the position which player B is in, and player B can with every move select one of n possibilities, the game is called rectangular or matrix. Such a game is assigned a rectangular table of numbers, matrices.

$$\begin{matrix} p_{11}, & p_{12}, \ldots, & p_{1m} \\ p_{21}, & p_{22}, \ldots, & p_{2m} \\ \ldots & \ldots & \ldots \\ p_{n1}, & p_{n2}, \ldots, & p_{nm} \end{matrix}$$

The number p_{ij}, which is at the intersection of the j line and the i column indicates the degree of advantage of player A if at the time of his routine move he selected the j possibility and player B selected the i possibility. The strategy is called mixed if player A selects the j possibility with a certain probability x_j, and player B selects the i possibility with a probability of y_i.

$$\left(\sum_{j=1}^{m} x_j = \sum_{i=1}^{n} y_i = 1 \right).$$

It can be shown that if instead of using mixed strategy player A reacts with a certain movement to every move by player B, A has every possibility of coming out the loser with frequent repetition of the sets. The problem of player A is to select strategy $(x_1, x_2, \ldots x_m)$ so as to guarantee himself the maximum advantage even when his opponent's actions are most disadvantageous for him. The main fact in the theory of matrix games lies in the fact that the matrix game always assumes a solution, that is, there is always a pair of optimum strategies for both players.

Despite the fact that the conditions which are imposed on the nature of the execution of the game in the theory of games are exceptionally rigid, the game theory has made it possible to demonstrate a number of conclusions which are important for practice and other branches of science. Specifically, interesting and important formulations of problems have drifted into the mathematical statistics of the game theory. For example, the theory of statistical solutions is one which has found serious application particularly for statistical control of the quality and production.

At the present time, there is a large number of books and articles from which it is possible to become acquainted with the game theory. We should like to mention only some of them (Leningrad, 9-11).

Conclusion

In our outline we have not touched at all on a number of the most important divisions of mathematics which are of great importance for cybernetics. Thus, we have almost entirely passed over all of mathematical logic with such important divisions as the theory of algorithms, the theory of automata, and the theory of columns and nets. We have not at all touched on problems confronting the theory of programming for computing machines, including the attractive problem of automation of programming. We have passed over a description of problems confronting the theory of probability in connection with cybernetic problems and over mathematical statistics. While to a certain degree problems of mathematical logic and the solution of problems on machines have already been analysed in the Soviet literature designed for initial acquaintance with the subject (Leningrad, 12), there is no literature from which one can become acquainted with the ideas of modern statistics or the theory of stochastic processes to date. Fat textbooks and special monographs in existence have not at all been designed for this purpose.

In many automatic control apparatuses a tremendous influence is exerted by random effects on the signal being transmitted. Thus, the phenomena of atmospheric turbulence exert an influence upon the readings of the autopilot. In order that the autopilot carry out the job entrusted to it it is necessary to eliminate noise of this kind automatically. In precisely the same way, in transmission of a signal over the radio, an abundance of random interference makes its impression on it. Thus, during the transmission of films of the opposite side of the moon from a Soviet space rocket the effective signal constituted only a small part of the fortuitous interference accompanying it. For the purpose of using the signals as they were intended it is necessary to learn how to separate them from spurious influences. If we analyze strictly literally all the processes of control we see how during transmission of the control signal or information about the condition of the system being controlled they are distorted by fortuitous interference. In order to work out standard principles for taking into consideration the influence of interference and measures for controlling it it is essential to subject the actual stochastic processes to a methodical theoretical analysis. Only distinct knowledge gives us the basis and the confidence to work out efficient methods for working on nature.

A careful study of those actual processes which occur in numerous physical, chemical, geophysical, biological and technical processes has shown that with an adequately good approximation it is possible to represent them schematically in the form of two types. The first type has been given the name of "Markovian" processes; the second, stationary processes. Markovian processes are interesting because not only do they cover a tremendous number of problems important in a theoretical and applied respect but also because in a certain sense any process can be reduced to a Markovian process. Incidentally, the mathematical apparatus of the theory of Markovian processes has been well worked out, and in its analytical part amounts to a theory of differential and integro-differential equations, that is, to the ordinary mathematical apparatus of physics, engineering and chemistry.

Before giving a definition of Markovian and stationary processes we need to introduce certain concepts. Suppose we are interested in a certain system, S, the actual content of which does not interest us at the given moment. This can be an automatic line or a dial telephone, a control system of a rolling mill or an autopilot. It is important to us that at every moment this system is in a certain condition which we shall designate by the letter "x". The letter "x" can signify not only the value of some single parameter characterizing the condition of the system; it can include the values of many parameters. Thus, for example, if a system consists of a single particle which is under the influence of other particles, we shall describe its condition by means of six coordinate numbers and three speed components on the coordinate axes. Let us designate the multiplicity (conglomeration) of all possible conditions in which this system can be found by "X". If at any moment "t" the system "S" can be in any of these conditions only with a certain degree of probability, this means that the change in states of the system represents a stochastic process, x (t).

It is characteristic for Markovian processes that if the state of a system becomes known to us at a given moment t_0, the probability of its going into another state, "x", at a time t greater than t_0 depends only on $x_0=x(t_0)$ and does not change if the states of the system become known to us at any time previous to t_0. We should like to note that a tremendous number of problems of the theory of mass service, reliability, theory of diffusion, automatic control, multiplication, death and others have been reduced to processes of the Markovian type.

A characteristic feature of stationary processes is the invariability of their distributions with change in the time reading. We should like to clarify what has been stated: let x (t) be a stochastic process. Let us analyze a new stochastic process, $y(t)=x(t-\tau)$. If for any values of n and $t_1, t_2 \ldots, t_n$ the distributions of the groups of stochastic factors $x(t_1), x(t_2), \ldots, x(t_n)$ and $y(t_1), y(t_2), \ldots, y(t_n)$ are the same and do not depend on τ, the process x(t) is called stationary. The theory of stationary stochastic processes has acquired tremendous significance in modern radioengineering, statistical physics, meteorology. This is natural, because in the established working conditions of any system its characteristics at any moment maintain not only the probability distribution but also the connections between the states at equally spaced intervals.

There is no need to emphasize the fact that these two types of processes, however great their possibilities, are incapable of encompassing the entire variety of real situations. Specifically, for this reason in recent years there has had to be an introduction of new types of processes for analysis capable of describing real situations more accurately. These are processes with stationary increments, linear processes, enclosed Markovian chains and others. At the present time, there is quite an extensive literature on the theory of random [a synonym of "stochastic"] processes in its application to applied problems (Leningrad, 13-17). However, it should be stated that this literature makes quite great mathematical demands on the reader.

We have already mentioned that for all processes of control the problem of the information, which would be, on the one hand, exhaustive and minimal, on the other, is of exceptionally great importance. In order to answer this question it is necessary to work out appropriate general methods. These methods with respect to information concerning random events, magnitudes and processes are being worked out in mathematical statistics. How should the observations be organized and how many of them should there be so that an unknown value of a factor being measured can be estimated with a set degree of accuracy? What should we do in order to check the correctness of a hypothesis concerning the nature of a process which we have encountered: will it be Markovian, stationary or both? Can we consider that two groups of observations on certain random magnitudes give us the basis for concluding an invariability of distribution of these magnitudes? We have an opportunity to come across this latter problem in numerous important applied problems, for example, in the organization of current statistical control. Suppose we know that a random magnitude with which we have to deal is normally distributed. In other words, that the [omission in Russian text] in these observations is equivalent to only two magnitudes:

$$P\{\xi < x\} = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{(z-a)^2}{2\sigma^2}} dz,$$

where the magnitudes α and σ are constant. How are we to evaluate these parameters by the most economical method if we know the results of observations $x_1, x_2, \ldots, x_n$ of the factor ξ? It turns out that the entire information which is contained in these observations is equivalent to only two factors:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \text{ и } S^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2.$$

Thereby, the magnitude $\bar{x}$ (the arithmetic mean) serves as the estimate of α while S (the root-mean-square deviation) will serve as the estimate of σ. Through this example we become convinced of how much saving of time and effort is provided by the use of ready-made general principles of mathematical statistics. Thus, if we should like to know the average weight of a certain article, about which we know that it is normally distributed, there is no need to weigh every article separately. It is sufficient for us to find the total weight of all the articles and divide this value by the number of them. The making of separate measurements adds nothing to the simpler method of information obtained.

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From the very intention of the present article it is limited only to certain general concepts of mathematical disciplines related to statistics and the theory of probability which are of significance for cybernetics. In subsequent collections concerning their results and methods we hope to relate more details.

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R. L. Dobrushin and Ya. I. Khurgin

Problems of the Information Theory

(Part of this material has been borrowed from a book prepared for publication by R. L. Dobrushin in coauthorship with L. Ye. Filippova and B. S. Tsybakov, Teoriya Optimal'nogo Codirovaniya Informatsii (The Theory of Optimum Coding of Information)).

1. The concept of "information" is utilized extensively. An exact definition of it, like any of the other basic natural scientific concepts, is difficult, if only because such concepts evolve in the course of time, but everyone understands its objective content. Abstraction, transmission, reception, storage and transformation of information represent some of the essential phenomena of life around us. Because any controlling system deals with information, information is one of the basic concepts of cybernetics.

Study of information, its properties and characteristics offers considerable difficulties chiefly because of the subjectivity of the determination of the value or usefulness of the same information for different people. Thus, for example, information about the birth of a child is received absolutely differently by the father of this child, the relatives, neighbors, friends and strangers.

The development of communication engineering (telephone, telegraph, radio, television and others), engineering designed for the transmission of information, has led to distinguishing one aspect of this concept which has proved to be well adapted for study on mathematical and technical levels. This study has produced abundant fruit at the present time. Therefore, we shall give our main attention to the theory of transmission of information and shall only briefly touch on the other aspects of this theory.

2. Attention should be directed to the fact that information is not a physical characteristic of objects. In the transmission, storage, transformation of information energy is expended, but this energy in no way characterizes the information itself. Thus, for example, information about the weather can be transmitted over the telephone, radio, telegraph, can be published in the newspaper, and some energy is everywhere expended for this transmission. However, the quantity