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Agent-based models review for option pricing

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Introduction

Options are one of the most used derivative contracts in financial markets. They can be used by speculators to generate profits or by hedgers to protect their positions. Trying to find the real value of these derivative contracts it's a hard task.

Since the introduction of the Black-Scholes formula, things become easier. With a mathematical formula it was possible to link option's value to the parameters it depends on. Unfortunately, the assumptions on which the formula is based are very strong and it put doubts on its accuracy to find the real value of options. In the original framework the volatility is assumed to be constant. Concerns arise around this assumptions. Empirical observation shows that volatility during option's life changes especially for options with long expiration. Thus, researchers started studying models, still based on Black-Scholes formula, to account for non constant volatility. This produced the construction of new models that were used by practioners in the valuation of options.

The natural question that arises is how wrong is the original Black-Scholes formula from its further developments? Zhang et al. (2009) proposed in their work an original way to make this comparison. They valuated first the option with the original Black-Scholes formula (to compute the theoretical price) and then they used the ABMs (Agent-based modelings) methodology to compute the formula with stochastic volatility. This is an original idea to use ABMs methodology in finance and so far is the first work that apply it to option pricing. To study a complex phenomenon such as the generation of option prices requires, indeed, tools that can handle complexity more efficiently. ABMs serve to this scope.

The aim of this work is to review the ABMs methodology and see an application of it in option pricing. In chapter 1 the ABMs methodology is presented with a comparison with other models. Moreover, a description of tools and their comparison is presented as well. ABMs is a very powerful tool but it is very difficult to explain the model clearly once it is constructed. Therefore, a standard protocol for model description is needed. The most

natural and easy way to describe ABMs is the ODD. In chapter 2 is presented the ODD: description, advantages and disadvantages. Chapter 3 explains how the Black-Scholes formula is obtained. In this chapter it is also explained the uncertain volatility framework. In Chapter 4 is presented the model of option pricing with uncertain volatility using ABMs. An ODD prospect of the model is presented as well.

1 Agent-based models

1.1 Social Science, Complexity, Models, and ABMs

Social science is the field that mainly study human nature. It covers many areas such as demography, sociology, economics, anthropology, politics and so on. It is considered to be a complex science. We refer to it as a social complexity that is the study of social phenomena in a complex system environment. What is a complex system? Flake (1998) describe it as:

> "A collection of many simple units that operate in parallel and interact locally with each other so as to produce emergent behavior."

Complex system, therefore, must have the following two characteristics:

- It is composed of interacting units that operate in parallel.
- The system has emergent proprieties. It is not possible to deduce system features by summing up the characteristics of the units that compose the system.

Therefore, when studying social sciences we cannot decompose it into separate subprocess, that the aggregate analysis will give us insights about the process as a whole. In other words, to understand it we cannot just have a simple look at the single individuals that constitute the system and pretend to understand the characteristics of the whole system. We need to analyze the interactions among individuals and try to explain how the output is possible. We can distinguish between *reductionism* that allows us to start from a system as a whole and find the simple micro rules that make the system possible and *constructivism,* that is not applicable to social science as it starts from simple rules and tell about the proprieties of the system as a whole.

Experiments in social science are also very difficult to be implemented. It is difficult to test hypothesis concerning the individual's behavior related to macro regularities. Moreover, social scientists have to deal with a perfectly

informed individual that has infinite computing capacity who maximizes a fixed exogenous utility function. Unfortunately, this is not what in the real world happens. Individuals do not have all the available information and their computing capacity is limited.

Regarding social science, equilibrium is one of the biggest issue that they have to face. Equilibrium is studied only in a static way as there is no other approach to see the effects of the time on it. To overcome these major issues a new way of treating social science is needed.

Models can be considered as an interesting and very powerful tool for representing real systems. They are constructed to answer questions about real systems. In this way, model purpose depends on the question we want to answer. The question serves as a filter for selecting the criteria in what way the model will be constructed. If we want to represent a certain part of a real system then different parameters will be considered and other will be ignored. Thus, model parameters will depend on the purpose of the model. The parameter choice is not easy. At the beginning a simple model will be constructed. This simple model should represent a simplified version of observed patterns in real systems. Simplified model can be constructed starting from theory, previous models, empirical evidence, or imagination. Then, to say if a parameter is important or not, researchers have to test them and compare the results with the observed patterns. Thus, starting from simple model researchers can add more details to obtain what they originally had in mind.

"Agent Based Models" are a new way of treating social science. It consists of population set of agent implemented on computers. Running an ABMs consists in letting the agents population interact with each other and observe the results. This new way of doing science has been considered favorably in the last years.

The use of ABM can be tracked in the late 1940 to the Von Neumann machine capable of reproduction. The idea was later improved thanks the suggestion made by Stanislaw Ulam to construct the famous cellular automata. Improvements to the model were made by John Conway to build the well-known "Game of life."

One of the first agent based model was the Thomas Schellings's segregation model. In his model agents with simple rules interact with each other and emergent behavior is observed.

Later in the early 1980, Robert Axelord contributed to ABM trying to solve the Prisoners Dilemma using ABM. Axelord developed many other models in political science. In the late 1980, Christopher Langton coined the term artificial life meaning the field that study the systems related to life and its processes and its evolution using computer models and simulations. Langton can be considered one major author that contributed in the field of ABM (his famous model is called Langton's ant).

The first scientists to use the term "agent" were John Holland and John H. Miller in their famous paper "Artificial Adaptive Agents in Economic Theory." Holland and Miller contributed as well in developing the field.

Epstein and Axtell (1996) developed the first large scale ABM the "Sugarscape" to model the role of social phenomena.

With the appearance of programs such as StarLogo, NetLogo, SWARM, and RePast, ABM becomes a very used tool to do science and its applicability spread among many fields.

The main feature of artificial society model (ABM applied to social science) is that:

> "fundamental social structures and group behaviors emerge from the interaction of individual agents operating on artificial environment under rules that place only bounded demands on each agent's information and computational capacity" Epstein and Axtell (1996) .

This allows the social scientist to recreate social phenomena and study it on computers. Simply creating agents with simple rules and an artificial environment where they interact, social scientist can study very complex phenomena. Complex phenomena emerge by simply letting agents interacting with each other.

Agents are the "peoples" of artificial societies. Each agent understands his situation and based on his internal rules makes decisions. Agents may

execute various behaviors appropriate for the system they represent for example: producing, consuming, selling and so on. Agents may be capable of evolving, allowing unanticipated behaviors to emerge. They have internal state or behavioral rules and these may be static or changing through interaction with other agents or with the environment.

The environment is the space where the agents interact. It can be a physical or non-physical place. We can think about markets, forests, roads, cities and so on. The space plays an important role in the final results. It has to be considered as a separate entity from the agents. Sometime, the way the environment is designed may have the same role as the way agents were designed. While constructing an ABM we have to ask if the space where agents will operate will influence the final results or not.

Rules are the instructions that guide agents' decision. They define the way agents interact with each other or with the environment. In Agent Based Modeling Object-Oriented Programming (OOP) is applied. It consists in encapsuling the agents' rules, environment, and internal variables in separate objects. The OOP is a straight way to build ABMs.

The easy way to transform an idea, that social scientists have in mind, to an executable code is to follow ERA (Environment-Rules-Agents) scheme developed by Gilbert and Terna (2000). The main feature of this scheme is that it keeps the environment and the agents at a different level. Thus, simplifying the code, agents should not communicate with each other but through the environment. It might be a little bit abstract but it's easier implemented. Agents' "brain" or their cognition is represented by another level. Rule Master decides the agents' behavior. At the same conceptual level Rule Maker changes the way Rule Master decides the agents' behavior thus, acting like a Meta ruler. Rule Master can obtain the necessary information to change rules from special agents that have the task to collect and exchange data from the model. The scheme is presented in Fig. 1. Even though, the scheme can appear to be rigid it is a source of better comprehension.

Concluding this section we can say that whenever a system presents the two main features of complex system ABMs becomes the most practicable method of analysis.

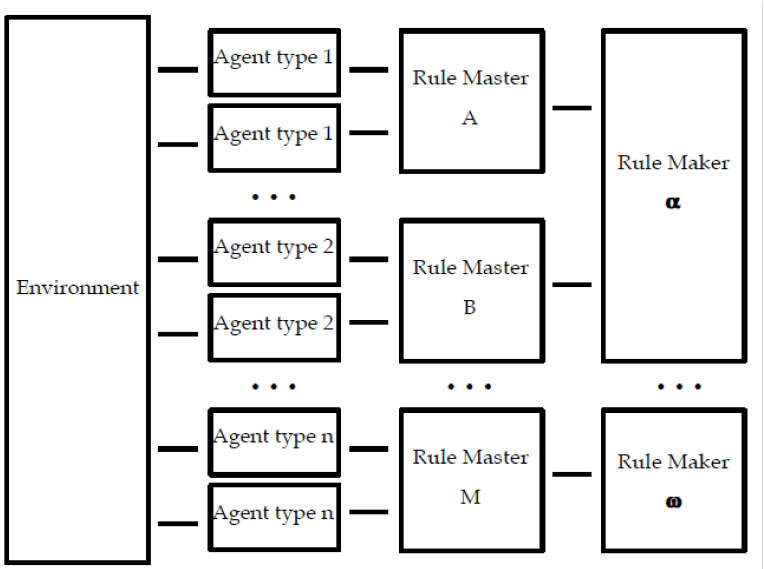

Figure 1:

1.2 ABMs: a new way of doing science

In this section a brief explanation of why ABMs have become important is given. In building models we typically use two ways:

- Verbal argumentation
- Mathematical equations

The first one is very flexible and allows for a major description of the model but unfortunately it is not testable and verification of hypothesis is not possible. With mathematical equations we can achieve what with verbal argumentation could not, but it suffers in flexibility and adapting especially when accounting for agents' heterogeneity and interactions. The third way of describing models takes advantages of the previous two. In particular, computer simulation with agent based combines the flexibility of verbal argumentation with the rigorous of mathematical equations. We can generate data series and apply statistical methods to verify its exactness. In this sense, we can say that ABMs are models that better adapt to reality compared to the previous two models Terna (2010).

Now let see the role of ABMs compared to the traditional deduction and induction. This is best explained by Axelrod and Tesfatsion in the following statement:

> "Simulation in general, and ABMs in particular, is a third way of doing science in addition to deduction and induction. Scientists use deduction to derive theorems from assumptions, and induction to find patterns in empirical data. Simulation, like deduction, starts with a set of explicit assumptions. However, unlike deduction, simulation does not prove theorems with generality. Instead, simulation generates data suitable for analysis by induction. Nevertheless, unlike typical induction, the simulated data come from a rigorously specified set of assumptions regarding an actual or proposed system of interest rather than direct measurements of the real world. Consequently, simulation differs from standard deduction and induction in both its implementation and its goals. Simulation permits increased understanding of systems through controlled computational experiments." Axelrod and Tesfatsion (2005)

Traditionally with deduction we mean the process of deriving b from a where b is a formal consequence of a. With induction we are allowed to infer b from a where b does not follow necessarily from a. The quote cited above can be conducted to abduction. Abduction is a method of reasoning where one chooses the hypothesis that if realized give the best explanation of the actual evidence.

Let's see the position that ABMs has compared to the classical and constructive mathematics. Generally classical mathematics is ruled by the Law of Excluded Middle (LEM). It says that for any proposition P, either P is true or its negation is true. Thus, classical mathematicians accept existence proofs based on proof by contradiction. In contrast, constructive mathematicians require a direct proof that P is true in as a computational procedure to rule out both the falseness and the undecidability of P. Constructive proofs can, in principle, be realized with computer programs.

ABMs combine both constructive and classical approaches. Agents can acquire new data constructively through interactions and this is the same for real people. Moreover, like real people, agents can have *"uncomputable beliefs"*. It means that they can have unexpected behavior and this might be possible due to interactions with other agents or inborn rules.

The choice between a more classical or constructive approach in ABMs depend on the purpose of the model. For descriptive purposes, it permits human behavior to be captured with greater fidelity than simple algorithmic representations. For optimization purposes, it permits a deeper and more creative exploration of large domains, a melding of experience-tempered guesswork with step-by-step computation that could vastly extend the power of traditional finite search methods. In conclusion ABMs can be expressed as finite systems of discrete time recursive equations over finite state domains. Nevertheless, they can be data-driven dynamic applications systems. Thus, it is true that ABMs can be considered as a new form of mathematics. Borrill and Tesfatsion (2010)

1.3 On the motivations for using agent computing in social science.

Many models rely on strong assumptions such as ideal condition or ideal agents. The main goal of these models is to understand the relationship between key variables and it can turn out to be useful as a good approximation of the reality. However, in reality departure from ideal world is the rule more than the exception. To better understand the real world we need to adopt models that can try to explain such departures.

The reasons for using Agent Based Modeling (ABMs) in social science can be enveloped in three main blocks depending on the solubility of the underlying mathematical equations. The first one derives when the equations is completely solvable analytically or numerically. ABMs can be used to present results or as a new method of Monte Carlo Simulation. The other motivation derives when the underlying equations cannot be solved. In this case, ABMs can help understating the proprieties of the model structure, test

results on parameters and assumptions, and illustrate dynamical proprieties. The third reason derives when writing equations is not useful. In this case ABMs can help in understanding better the structure of the model.

Several are the advantages that arise from using the ABMs in social science. In particular, it is easy to limit agents' rationality and even if one would use complete rational agents that would result in a tricky operation. In most contest both social and spatial matters and this is difficult to account in a mathematical framework while in ABMs it is easier. There is a disadvantage in using ABMs. In particular, in order to obtain robust results one needs to compute several runs. Thus, the single run itself cannot say much about the general results and explain the dynamic. Axtell (2000)

An important point in favor of ABMs comes from the Nobel price winner E. Ostrom. In her nobel lecture work Ostrom (2010) she stated:

> "To explain the world of interactions and outcomes occurring at multiple levels, we also have to be willing to deal with complexity instead of rejecting it. Some mathematical models are very useful for explaining outcomes in particular settings. We should continue to use simple models where they capture enough of the core underlying structure and incentives that they usefully predict outcomes. When the world we are trying to explain and improve, however, is not well described by a simple model, we must continue to improve our frameworks and theories so as to be able to understand complexity and not simply reject it."

Even though, it is not stated explicitly the need of ABMs by accepting the complexity paradigm it is accepted the need of models that can handle it properly.

1.3.1 ABMs construction in financial economic system

In this section, I will explain what are the positive aspects of using ABMs in a particular field of social science: financial economics. ABMs can be useful to explain the connection between micro-simulation properties of financial

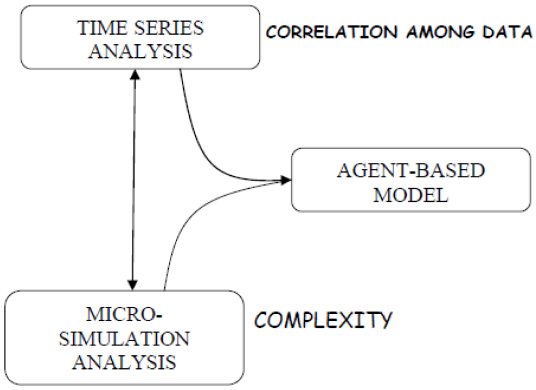

Figure 2:

system with its macro or statistical properties Fig. 2. Explaining macro-level things in micro-simulation is the aim of using ABMs in social science.

In financial markets events such as fat tails, jumps, volatility grouping, etc., have become an important issue that need to find a solution. The use of ABMs can spread light through such a puzzles. Several important models have achieved important results so far. The Artificial Stock Market developed by Santa Fe Institute is one of those. Why is that possible? This is possible mainly because the ABMs is bottom-up approach. We can model investors the way we want to operate. This can generate unexpected results and perhaps explain results that were not explained with other models. For example in their work, Situngkir and Surya show how to explain phenomena such as:

- Volatility grouping
- Excess kurtosis
- Multifractality character

present in Indonesian market using ABMs Situngkir and Surya (2005) .

1.4 System dynamics, Discrete Events and ABMs

Modeling is a way of solving problems that occur in real life. Through modeling it is possible to test systems before it is implemented. In general,

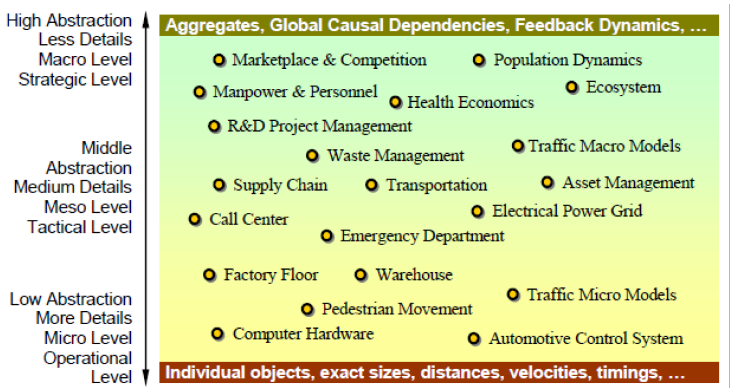

Figure 3:

for complex problems where dynamics is important, modeling is probably the best solution. Before proceeding in the comparison of the approaches in simulating models we need first a classification of models based on their level of abstraction.

Depending on the level of abstraction we can classify models from high to low level of abstraction. At the low level we have the so called "physical" modeling. Individual objects with exact size, distances, velocities, and timing characterize it. To this level fields such as mechatronics, control systems, micro-level traffic systems are located.

At the middle level models that deal with factories, warehouses storage, transportation, and macro-level traffic are included. Typically all the models that require a little level of abstraction.

At the top of the chart problem are considered in terms of aggregate rather than analyzing the single element. In this part, models have a high level of abstraction and are considered more complex to analyze. In Fig. 3, we can notice some of the fields and the classification based on their level of abstraction.

The major approaches in simulating model are:

- Discrete Events (DE)
- System Dynamics (SD)
- Dynamic System (DS)

- Agent Based Modeling (ABMs)

Technically, SD and DS work mainly in continuous process whereas DE and ABMs work in discrete time. If we consider the level of abstraction used in the model we can notice that: System Dynamic is at the bottom level while Dynamic Systems dealing with aggregates is located at the top level. Discrete Events is located at the low to middle level and Agent Based Model, due to its flexibility, can be located across all the three levels. ABMs is a relatively new subject. It started being used as an increasing demand for platforms that could combine the main characteristics of the three models together.

System Dynamics developed by electrical engineer Jay W. Forester is

> "The study of information-feedback characteristics of industrial activity to show how organizational structure, amplification (in policies), and time delays (in decisions and actions) interact to influence the success of the enterprise".

The range of SD applications includes urban, social, ecological types of systems. In SD, the real-world processes are represented in terms of stocks, flows between these stocks, and information that determines the values of the flows. To approach the problem in SD style one has to describe the system behavior as a number of interacting feedback. Mathematically is a system of differential equations.

Dynamic Systems modeling may be considered as the ancestor of System Dynamics. It is used in mechanical, electrical, chemical, and other technical engineering disciplines. The underlying mathematical model of a dynamic system would consist of a number of state variables and algebraic differential equations of various forms over these variables. In contrast with the SD, variables here have direct meaning: location, velocity, acceleration, pressure, concentration, etc., they are inherently continuous, and are not aggregates of any entities. The mathematical diversity and complexity in dynamic systems domain can be much higher than in system dynamics. The tools used for dynamic system simulation could easily solve any SD problem with even much better accuracy than SD tools.

Discrete Event is the modeling approach based on the concept of entities, resources, and block charts describing entity flow and resource sharing. This approach is dated to 1960s developed by Geoffrey Gordon. Entities are passive objects that represent people, parts, documents, tasks, messages, etc. They travel through the blocks of the flowchart where they stay in queues, are delayed, processed, seize and release resources, split, combined, etc. DE modeling may be considered as definition of a global entity-processing algorithm, typically with stochastic elements.

Agent Based Modeling has its roots in different disciplines like artificial intelligence, complexity science, game theory, etc. There is no a universally accepted definition in this area, and people still discuss what kind of properties an object should have to "deserve" to be called an "agent." Compared to SD or DE models, the modeler defines the behavior at individual level, and the global behavior emerges as a result of many individuals, each following its own behavior rules, living together in some environment and communicating with each other and with the environment. That is why AB modeling is also called bottom-up modeling. Borshchev and Filippov (2004)

1.4.1 Differences between System Dynamics and ABMs

The major differences between the two modeling techniques represent also their relative strengths and weaknesses. Agent-based modeling focuses on individuals who interact based on of simple rules. The resulting emergent behavior of such agents as a complex system is the basic unit of analysis. The researcher may modify rules and environmental parameters and then try to understand what the outcomes are with regard to the emergent behavior of the overall system. As long as rules are known or can be discovered by some sort of observation, the modeling and testing of such emergent structures is a relatively straightforward process. However, once the reverse direction of study is employed, that is, a complex aggregate behavior of a system has been observed, and now its agents and the rules by which they interact shall be identified, the process can be very complicated. Discovering agents and rules and then building a model, which is capable of mimicking the previously

observed dynamic behavior, is a very complicated task.

In SD, modeling the feedback loop is the unit of analysis. Individual agents or events do not matter much in SD models, since the dynamics of the underlying structures are seen as dominant. Feedback structures, for example in social-science fields of study, can become subject to controversy since perspectives on a problem and perceptions may differ widely. Constructing models is a process in which expert consensus regarding the feedback structure is essential to the credibility of any given model. If the feedback structure of a model captures the structure of a system insufficiently, the resulting insights may be faulty. On the other hand, if the model does represent the systemic problem sufficiently, leverage points for intervention can be identified effectively. This, however, is not possible at an individual but at an aggregate level.

Both techniques aim at discovering leverage points in complex aggregate systems, modelers of agent-based models seek them in rules and agents, while SD modelers do so in the feedback structure of a system. Jochen (2009)

1.4.2 Which approach to use?

In general, using AB approach allows capturing more real life phenomena than with SD or DE approach. However, AB is not always a replacement for SD or DE modeling. There are a lot of applications where SD or DE model can efficiently solve the problems and agent based modeling will result in a less efficient approach, harder to develop, or simply not matching the nature of the problem. Whenever this is the case, traditional approaches should be used.

Agent based modeling is for those who wish to go beyond the limits of SD and DE approaches, especially in the case the system being modeled contains active objects (people, business units, animals, vehicles, or projects, stocks, products, etc.) with timing, event ordering or other kind of individual behavior. We should also consider using different modeling paradigms for different parts of the simulation model.

1.5 Tools for ABMs

The number of products that can be used now days for doing ABMs has increased considerably. Each of these products has potentially and limitations depending on the work that the research has in mind. Three main programs are mostly used in the simulation field Terna et al. (2006):

- Swarm
- JAS
- NetLogo (StarLogo)

1.5.1 Swarm

Swarm is one of the most consolidated project and environment for agent simulation. It was originally developed by the Santa Fe Institute as a universal simulation language to be applied not only in economic simulation but also in every field.

Swarm has been developed as a library written in Objective C, which is a similar language to C but object oriented one. Many years after a version that reads Java was available. Summarizing the characteristics and the advantages in using Swarm:

- Swarm can use both Objective C and Java. The first one is more suitable to optimize models on computer and the other one let the user to more freedom in use and the possibility to use the major knowledge of Java.
- The tool is comprehensive of software for scheduling, GUI with basic instruments for numerical and graphical analysis.
- The simulation run with Swarm allows saving the results in file or in a graphic mode where it is possible to access the single agent and modify his internal parameters.
- It is very easy to add on more tools especially if we are using Swarm with Java.

- It has a powerful community where doubts are solved altogether and updates are issued frequently.

1.5.2 JAS

JAS (Java Agent-based Simulation) was developed my Michele Sonnessa. It consists in a library of object-oriented functions that has as objective the creation of a simulation model in discrete time using a standard programing language. Other than the library it has a standard protocol that allows the users to create their own code. This allow other users to easily understand the code and thus to reproduce it. Below a description of the main features of JAS is provided.

- JAS is an easily tools for discrete event simulation.
- It has an easily and highly intuitive graphical representation.
- It has a library for artificial neural network and genetic algorithm.
- The network library can be very easily implemented and allows the users to implement complicated simulation on networks.
- An object oriented model very flexible that allows the users to have different statistics during the simulation and the possibility to save the results in a database.

1.5.3 NetLogo

NetLogo offers a simplified language for the realization of simulating model. Initially was introduced as StarLogo. It is based on Logo language introduced in the 1970s. In NetLogo, the main features are the turtles. Turtles can become agents, they can communicate with each other and with the environment surrounding them. The turtles moves around an environment called patches that can have different states each described by internal variables. The interactions between turtles and patches allow the writing of complicated and enriched programs in a short time.

1.5.4 A tool comparison

NetLogo has the advantage of being user friendly. It has very short learning time compared to Swarm or JAS that requires a previous programing knowledge. Obviously, the things that we can do with NetLogo are less powerful compared to the other two programs. It is useful at the beginning to first build the model in NetLogo and then proceed in a further implementation of the model in more complicated program such as Swarm or JAS.

2 On the protocol to describe ABMs

The agent-based simulation has become a very popular tool in many fields. However, the great potential of ABMs comes at a cost. ABMs are more complex in structure than analytical models. They have to be implemented and run on computers. Thus, they are more difficult to analyze, understand, and communicate than traditional analytical models Grimm et al. (1999). The results obtained from ABMs are not easily reproduced. A standard protocol for the description of ABMs would make reading and understanding them easier because readers would be guided through their expectations Gopen and Swan (1990).

Grimm et al. proposed a standard protocol for describing ABMs called ODD. This is the acronym for Overview, Design, and Details. The ODD was primary designed to give ABMs a standard description structure. In this way, the reading and the comprehension of these models will be easier. The protocol other than being widely used by researchers combines two fundamental features that are necessary to explain a model Grim et al. (2006, 2009) :

- A general structure for describing the model
- Mathematical language necessary to explain the agent rules, schedules, etc.

Having a standard structure models will result very easy to replicate and thus, being more scientific. Since it was introduced in 2006, the ODD protocol has seen an increase in the number of researchers that use the protocol. This makes the ODD the primary source to refer when describing a model. The protocol has been updated and some aspects of it have been improved based on the experience and suggestions of hundreds of researchers. The ODD is defined by its seven elements, its identifiers, and its sequences. Using ODD means using all those elements described by the protocol in the order they have been described.

When the ODD is used it should be referred to it as:

Figure 4:

Overview	Purpose
	ENTITIES, state variables, and scales
	Process overview and scheduling
Design Concepts	Design concepts - Emergence - Adaptation - OBJECTIVES - LEARNING - Prediction - Sensing - Interaction - Stochasticity - Collectives - Observation
Details	Initialization
	Input
	Submodels

> "The model description follows the ODD (Overview, Design concepts, Details) protocol for describing individual- and agent-based models (Grimm et al. 2006).

The logic of the ODD is to provide a first general information about the model (Overview) followed by more strategic consideration (Design concepts) and finally more technical details (Details).

In the first block a general idea is given about why a complex model is needed and what is supposed to be done with the model.

2.1 Overview

Purpose It is the description of the overall objectives for which the model was developed It is important to start a model description with the description of the model's purpose. It is necessary to understand the models objective in the first part of the description without having to read all the rest. Moreover, it is important to find the same kind of information at the same place. This also helps reducing redundancy by describing model pur-

pose in few sentences, which also forces the modeler to clearly specify the model purpose.

Entities, state variables, and scales By entity is meant the actors, objects, or things that interact with other actors or with the environment factors. Its current state is characterized by state variables. State variables characterize individuals from others. They records values that have changed through interactions, movements and other actions. State variables should intended as "low level" or "elementary" one. In this sense, state variables cannot be calculated by other variables and thus, they represent the basic information that we can have looking at them. Typically, ABMs includes the following entities:

- Agents or individuals. Models can include different type of agent and in the same model different types of sub-agents. Each of these agents usually has state variables that are helpful to understand the evolution of the simulation.
- Spatial units. These are usually state variables that indicate environmental conditions that vary over space.
- Environment. These are variables that influence the whole space where the agent behave.
- Collectives. It is useful sometime to distinguish between group of agent that have the same behavior but are different from the rest of the population. It is represented by a list of agent that altogether performs the same action. It is important to specify what model's spatial and temporal units represent in reality.

Process overview and scheduling In this subsection only a list of "things" that agent do should be written. The detailed description of it should later be done in the "Submodels" section. Moreover, we should explain both the order in which process happen and the way they are executed by agents. The problem of when variables are updated depends on whether

the new value is stored as it is calculated (asynchronous updating) or when the new value is stored until all the agents have executed the process and then all update at once (synchronous updating). Defining a model's schedule includes stating how time is modeled if it not done in the Entities, State Variables, and Scales element. Verbal description is not very well suited for schedule description. To decide whether a schedule description is good or not we should ask ourselves if from the description we could rebuild all the schedule process.

2.2 Design Concepts

In this section, a list of common framework concepts is provided. By describing each of these it may be helpful for the reader to understand the general concepts underlying the design of the model. The purpose of this element is to link model design to general concepts identified in the field of Complex Adaptive Systems.

1. *Emergence.* Explain the general outcome that emerges from the interaction of individuals. It also can be explained by changes in an unpredictable way of the general outcome due to changes in individuals' rules.

2. *Adaption.* Rules that individuals have to make decision as a response to changes in environment conditions or other individuals behavior. It should be cited whether or not these particular traits are made to increase individuals success compared to other individuals.

3. *Objectives.* Once explained the adaptive characteristics of the agents it is therefore, necessary to explain what the goal of this individuals is. The criteria that agents use to make decision when choosing between alternatives should be explained in this subsection.

4. *Learning.* Here all the rules that allow individuals to change their adaptive rules as consequence of learning should be stated in this subsection.

5. *Prediction.* Prediction is important in successful decision-making. In this subsection all the rules that allow the agents to evaluate their current situation and thus, to make predictions should be listed here.

6. *Sensing.* These are the rules that allow agents to perceive signals, situations and other parameters that allow them to make decision. It should be explained if these rules are modeled explicitly or are agents supposed to know by themselves.

7. *Interactions.* Here the interactions between agents and agents and environment should be stated. It should also, be stated whether these interactions are direct or indirect. Moreover, if interactions are represented by communications how this is done.

8. *Stochasticity.* If there is any process that is random then, this process should be listed here. Moreover, it should be cited here the effect of the stochasticity on the general result.

9. *Collectives.* Agents may belong to groups or form groups during the simulation. If this is the case then, the rules that allow it should be stated here.

10. *Observation.* In this subsection all the data that come as output should be listed here. Moreover, if this data are used for tests then it should be notified what kind of test are run on these data.

These ODD elements do not describe the model entirely as this is not their goal. It is important to note that through these elements important concepts are not left unsaid. These elements make particular the ODD as it only requires a verbal explanation and does not need equation of flow charts. Most models include the major part of these elements but many others may not have all of them included.

This part of the ODD is also the most critiqued one. In particular, critiques focus on the redundancy of the information provided here. Moreover, as it does not describe the model *per se* this element should not be put into the protocol.

2.3 Details

In the third block details omitted in the previous blocks are presented. A description of the initial conditions is presented in Initialization. In the subsection Input a description of the variables is given. Readers need to know what input data are used, how they are used, how they can be generated and the influence that these variables have on the model. In the last part of the ODD the subsection Submodels presents the skeleton of the underlying model. In particular, both a mathematical formulation of the decision making of agents is presented and a full verbal model description is presented as well.

Initialization In this section, the initial value of the model should be cited. In particular the number of initial agents, their initial state values, time and spatial values. It should be reported whether the initialization changes with simulation or is always the same. Initial conditions is important as it may affect the results. Thus, in order to replicate the model it is important that all the initial conditions values and how they change should be reported.

Input Data In order to obtain realistic results real data, in case they are used should be reported. In order to replicate the model the data and the model should be provided.

Submodels The submodels are presented in detail and completely. Equation and algorithms, should come first and be separated from additional information. However, presenting only the factual description of submodels, separate from their justification and explanation, creates the risk of making the model design appear arbitrary. Many ABMs include submodels that are algorithms rather than equations. Verbal descriptions are a limited medium for describing schedules and algorithms exactly.

2.4 Complaints and benefits from using ODD

2.4.1 Complaints about ODD

ODD can be redundant The three elements of the ODD are seen as being redundant:

- Purpose: can be presented in a paper's introduction
- Design Concept: can be included, more or less explicitly, in submodels' descriptions.
- Submodels: submodels are also listed in Process Overview and Scheduling.

Indeed some redundancy exists but it's necessary if we want to have and ordered hierarchical protocol. The protocol makes descriptions easier to understand by providing an overview before all the details. The Purpose redundancy can be reduced by keeping the description of the model's purpose very short. The Design Concepts redundancy often does not exist. The descriptions of the submodels typically do not explicitly refer to design concepts. Any details needed to describe design concepts can be left out of the Submodels element. Finally, the minor redundancy introduced by first providing the Process Overview and Schedule before all the submodel details is in fact needed if we want to know and understand the context of each submodel. It is also particularly appropriate if submodel details are published in an appendix or separately.

ODD is overdone for simple models Some ABMs are extremely simple, and describing them in ODD could use considerably more space than a complete description not in ODD. The format of ODD can be shortened, when appropriate, such as by using continuous text instead of separate document subsections for each ODD element.

ODD is too technical ODD has been criticized for making model descriptions too technical. Providing technical details is the primary goal of

the ODD protocol. In this way model can be described completely and can be replicated by a reader. It is obvious that writing down the entire story and the ultimate purpose of a model, for example motivations and goals of human actors, functions, and emergent phenomena of ecosystems, or self-organization in tissues and cells, to a skeleton of attributes can be long.

Modeling is all about finding a simplified representation of things and processes that captures key characteristics of the real systems.

The Overview includes the framework and purpose of the model, a brief description of the model entities, their properties, and what they do in the model world. Using self-explanatory names for variables and processes is essential for communicating the scope of the model at once. The Process Overview and Scheduling element should be intended as a summary of the model while also providing a technical description of its schedule.

ODD separates units of object-oriented implementations In object-oriented programming (OOP), model entities and their behaviors is one unit. However, the ODD requires the properties and methods to be presented separately. OOP certainly is natural for implementing ABMs, but ODD was designed to be independent of software platforms. Presenting entities first and then what these entities can do has the advantage that we get a complete overview of what the model world is.

The principle of encapsulation in OOP is designed to promote source code that is easier to maintain through collecting the data and methods that operate on them in one place.

2.4.2 Benefits of using ODD

The ODD protocol was originally designed just to promote an efficient and complete communication and, thereby, replication, of ABMs. However, the protocol has turned out to be much more interesting. There are benefits that were forecasted by the ODD developers and which are very important.

ODD promotes rigorous model formulation When it was introduced, the ODD was seen as a new rigorous way to write ABMs. It was a response

to the need of more rigor in describing models and made possible model replication. Therefore, after it was introduced, people developed experience in describing models and consequently new ABMs were formulated.

ODD provides what is called "design patterns." This means that the ODD has all the necessary elements to provide a full and comprehensible description.

ODD spotlights incomplete or poor model descriptions When using ODD frequently it becomes easy later when reading the description of other models to recognize the missing parts (if any!). This is one of the goals of the protocol, to easily understand models. Moreover, when you use the ODD frequently it becomes natural way of thinking. This could be on the success factor of the ODD.

ODD facilitates reviews and comparisons of ABMs When comparing two or more models for similarities the ODD turns to be very useful. Since the ODD is divided in parts it turns to be quite an easy task to do comparison between similar models.

3 Black and Scholes model and parameter uncertainty

3.1 Black and Scholes model

In the early 1970s Fisher Black, Myron Scholes, and Robert Merton achieved a major breakthrough in the pricing of stock options. They introduced the famous Black and Scholes (or Black, Scholes and Merton) formula. Black and Scholes (1973). The model was a huge change in the way options were priced and put the basis for the development of financial engineering.

Clearly, when evaluating the characteristics of an option we need to have in mind what are the characteristics of the underlying asset itself.

Thus, we know that the value of an option will depend on:

- S and t: the current stock price and current time.
- σ and μ: parameters associated with the stock indicating the volatility and the drift parameter
- E and T: parameters associated with the particular contract indicating the strike price and the expiration date
- r: the risk-free interest rate.

We will start by constructing a portfolio of financial securities and from that we will derive the famous Black and Scholes formula to price options. The derivation will be shown in the case of a European Call option but similarly it can be shown for a European Put option. Suppose we have a portfolio which its value is given by:

$$\Pi = V(S,t) - \Delta S$$

This portfolio consists in a long position in the derivative contract and in Δ short position in the underlying asset. For now, we set the quantity Δ equal to some constant. Now we assume that the underlying asset has the following form and follows a lognormal random walk:

$$dS = \mu S dt + \sigma S dX$$

At this point we ask what the change in value of the portfolio is if we change time from t to $t{+}dt$. The change will be given partially by changes in the value of the underlying asset and partially in changes in the value of the derivative contract. In particular:

$$d\Pi = dV - d\Delta S$$

We know that the parameters that influence the derivative contract are the time and the underlying asset. Thus applying the Ito's lemma, we get a function for the derivative contract:

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt$$

The second order term for the time is omitted since it is very small and will have no influence if we consider it. In this way, changes in portfolio substituting are given by:

$$d\Pi = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} dt - \Delta dS$$

By observing the equation we can notice that there are deterministic terms and stochastic ones. Deterministic terms are those with the dt while random terms are those with the dS. Our goal is to eliminate the risk associated to the random term. We can do it by choosing the right quantity of Δ. In particular if we choose Δ to be:

$$\Delta = \frac{\partial V}{\partial S}$$

then the random term is reduced to zero. This operation is called delta hedging. It consists in eliminating the risk connected to any derivative contract by opportunely selecting the right amount of underlying to hold. By selecting the right Δ we now hold a portfolio that has the following form:

$$d\Pi = (\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2})dt$$

As we can see in this equation, there are no stochastic terms. Thus, our portfolio is a completely risk-free one. By holding a risk-free portfolio we are expected that changes in time over our portfolio should be equivalent to risk-free interest-bearing account:

$$d\Pi = r\Pi dt$$

This is an application of the no arbitrage principle. Substituting the previous equation we obtain:

$$(\frac{\partial V}{\partial t}dt + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2})dt = r(V - S\frac{\partial V}{\partial S})dt$$

At this point dividing by dt and rearranging the terms we get the famous Black and Scholes equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} = 0$$

With final condition in case of a European Call option:

$$V(S,T) = max(S - E, 0)$$

By solving this equation (which the solution requires the use of complicated technique) we finally get the famous Black and Scholes formula to price European Call Options:

$$V = SN(d_1) - Eexp^{-rt}N(d_2)$$

Where

$$d_1 = \frac{log\frac{S}{E} + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

and

$$d_2 = d_1 - \sigma\sqrt{t}$$

Assumptions the underlying assets follows a lognormal random walk the volatility is considered constant during the life of the option the interest rate is known and function of time there are no dividends on the underlying delta hedging is done continuously there are no transaction costs there are no arbitrage opportunities.

3.2 Option pricing under unknown volatility

According to APT (arbitrage pricing theory), if the market presents no arbitrage opportunities, there exists a probability measure on future scenarios such that the price of a securities is the expectation of its discounted cash flows. Such a probability is known as a martingale measure. Determining the appropriate martingale measure permits to determine the value of any contingent claim based on these securities. Pricing measures are difficult to calculate precisely and there may exist more than one measure consistent with the incompleteness of markets. This can be one of the explanations for the presence of different prices on the same security. The fair option price and hedging strategy cannot be calculated precisely and the "volatility risk" is a concrete manifestation of market incompleteness.

For this reason, Avellanda et al. propose a work for solving the issue of taking into consideration the uncertainness of volatility in option pricing.

First thing when valuating option with uncertain parameters is to acknowledge that we can no do better than a given range of future values.

For volatility, this can be a range of historical volatility, implied volatilities or something that take into consideration both of them. The range we choose represents our estimates for the upper and lower band for the estimates of the value for the parameter for the entire life of the option. Thus, when calculating the option price this technique leads to range values. It becomes natural to think of the highest and lowest option value. If we are long an option, we can call the highest value the best and the lowest the worst.

Now let see in detail how the pricing works under parameter uncertainty.

We will follow the Black–Scholes proof as far as we need it. Suppose we do not know the true volatility. What we know is that it might lay within two bounder values:

$$\sigma^- < \sigma < \sigma^+$$

We proceed with the construction of a risk-free portfolio with V(S, t) options and hedged with - Δ of underlying asset:

$$\Pi = V - \Delta S$$

We have the same equation for price movements that is given by:

$$dS = \mu S dt + \sigma S dX$$

Changes in the value of this portfolio are given by:

$$d\Pi = (\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2})dt + (\frac{\partial V}{\partial S} - \Delta)dS$$

Thus choosing Δ=∂V/∂S we obtain:

$$d\Pi = (\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2})dt$$

In this way the risk is eliminated. So far, we have used the famous Black-Scholes proof to derive a formula for the pricing of a European call option. Now we will set our portfolio in a way that it increases by the least amount. To do so what we need is to set the right volatility. For example if we have a long call position, we will choose the volatility σ^- such that the value of the portfolio will increase by the least amount the opposite in case we have a short position. After choosing the minimum value for the volatility the return over this portfolio is computed and thus equal to the risk free return.

$$min_{\sigma^- < \sigma < \sigma^+} d\Pi = r\Pi dt$$

or

$$min_{\sigma^- < \sigma < \sigma^+}(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2})dt = r(V - S\frac{\partial V}{\partial t})dt$$

From the above equation, we observe that the minimum value depends on the value of gamma. When gamma is positive, we choose σ to be σ^- . When gamma is negative then we choose σ to be equal to σ^+.

We find that the worst case V^- satisfies:

$$\frac{\partial V^-}{\partial t} + \frac{1}{2}\sigma(\Gamma)^2 S^2 \frac{\partial^2 V^-}{\partial S^2} + rS\frac{\partial V^-}{\partial S} - rV^- = 0$$

Where

$$\Gamma = \frac{\partial^2 V^-}{\partial S^2}$$

and

$$\sigma(\Gamma) = \begin{cases} \sigma^+ \ if & \Gamma < 0 \\ \sigma_- \ if & \Gamma > 0 \end{cases}$$

At the same way we can find V^+ by solving:

$$\frac{\partial V^+}{\partial t} + \frac{1}{2}\sigma(\Gamma)^2 S^2 \frac{\partial^2 V^+}{\partial S^2} + rS\frac{\partial V^+}{\partial S} - rV^+ = 0$$

Where

$$\Gamma = \frac{\partial^2 V^+}{\partial S^2}$$

and

$$\sigma(\Gamma) = \begin{cases} \sigma^+ \ if & \Gamma > 0 \\ \sigma_- \ if & \Gamma < 0 \end{cases}$$

This is the equation with which Avellanda et al. (1993) came up. Unfortunately it is impossible to have a close form solution for this equation so the only way to have a solution is to solve it numerically. Wilmott (2003)

3.3 Is the Black and Scholes formula used in practice? Criticism and appreciation.

In this paragraph I will describe the point of view of two of the major practitioners and researchers in the field. Nicolas Nasrim Taleb and Paul Wilmott. Taleb argues that the Black and Scholes formula is not an important argument nowadays while Wilmott gives some arguments in defense of the formula.

The critique by Taleb and Haug (2007) goes beyond the well known issues about the formula and focuses mainly on three points:

- It's not used (even by those who think that they use it).
- It wasn't needed.
- It wasn't original.

Most textbook assumes that Black and Scholes enjoys a huge success and is widely used by practitioners. There are two main reasons why Black-Scholes is not used. First, option prices (at least for liquid contracts) may be simply the result of supply and demand interaction, with no model involved at all. Options often are priced via other options, through the well known put-call parity, which allows a trader or investor to derive the price of a call from a put and vice versa. No mathematical modeling and no partial differential equations is required.

When Black and Scholes is used, it's not really Black-Scholes. The Black-Scholes formula is like a black box: certain inputs must be inserted in order to obtain some output, only one is not directly available: the underlying asset's volatility. Traders must estimate such a number before putting it into the Black-Scholes.

The basic version of Black-Scholes assumes that volatility should be constant. Valuating options of different strikes one is expecting to obtain the same volatility. Instead, what we get is something similar to a "smile" the so called volatility smile. At-the-money options tend to have lower implied volatilities than in- or out-of-the-money options.

Assuming that traders are using Black-Scholes, this reflects the fact that the volatility input is being manipulated in order to obtain more realistic option prices, essentially to correct for the formula's unrealistic assumptions and to allow traders to freely express their opinions.

The insight from Taleb is to point out that once you manipulate the volatility, you are no longer using Black-Scholes, even if such manipulation happens to take place within a Black-Scholes framework.

The model may have not been needed at all. Taleb and Haug show how before 1973 traders had very sophisticated knowledge about how options should be traded, priced and risk-managed, even as far back as the 1800s. Taleb and Haug produce a chronological list of technical and academic work on option pricing pre-dating Black-Scholes, including basically identical formulas.

According to Taleb and Haug, the acceptance of this tool is really the result of an "academic marketing exercise" rather than the appearance of a innovative piece of work.

We already knew that Black-Scholes was unrealistic. What we didn't know is that it's not really used, it wasn't truly needed and it wasn't entirely original.

This is the view of Paul Wilmott which goes in the oposite direction of that of Taleb and Haug. Black-Scholes is a robust model that behaves very well even when its underlying assumptions are violated, as they inevitably are in practice. It just needed a minor adjustments. Sometimes it is needed to work with something that while not perfect is good enough and is understandable enough that you don't do more harm than good. And that's Black-Scholes.

It is well accepted that the Black-Scholes formulae were around well before 1973. Ed Thorp plays a large role in that history. Ed wrote a series of articles "What I Knew and When I Knew it" to clarify his role in the discovery, including his argument for what is now called risk-neutral pricing.

They say traders don't use Black-Scholes because traders use an implied volatility skew and smile that is inconsistent with the model. Sometimes traders use the model in ways not originally intended but they are still using

a model that is far simpler than modern-day improvements. Black-Scholes performs better compared with many of these improvements. For example, the deterministic volatility model is an attempt by quants to make Black-Scholes consistent with the volatility smile. But the complexity of the calibration of this model, its sensitivity to initial data and ultimately its lack of stability make this far more dangerous in practice.

Transaction costs may be large or small, depending on which market you are in and who you are, but Black-Scholes doesn't need much modification to accommodate them. The Black-Scholes equation can often be treated as the foundation to which you add new terms to incorporate corrections to allow for dropped assumptions.

Discrete hedging is a good example of robustness. It's easy to show that hedging errors can be very large. But even with hedging errors Black-Scholes is correct on average. If you only trade one option per year then, yes, worry about this. But if you are trading thousands then don't. It also turns out that you can get many of the benefits of (impossible) continuous dynamic hedging by using static hedging with other options. Even continuous hedging is not as necessary as people think.

As for volatility modelling, the average profit you make from an option is very insensitive to what volatility you actually use for hedging. That alone is enough of a reason to stick with the uncomplicated Black-Scholes model, it shows just how robust the model is to changes in volatility. You cannot say that a calibrated stochastic volatility model is similarly robust.

When it comes to fat tails, it would be nice to have a theory to accommodate them but why use a far more complicated model that is harder to understand and that takes much longer to compute just to accommodate an event that probably won't happen during the life of the option? Keeping it simple and pricing quickly and often, using a simpler model and focusing more on diversification and risk management.

The many improvements on Black-Scholes are rarely improvements, the best that can be said for many of them is that they are just better at hiding their faults. Black-Scholes also has its faults, but at least you can see them. Wilmott (2008)

4 Option Pricing under unknown volatility: an agent based model

In Black-Scholes world the full knowledge of the asset price model rules out the model risk. It means that if we are able to define a model for the underlying asset there will be no uncertainty about the pricing model of the derivative contract. The goal of this paper is to understand how the price of the option depends on the unknown parameter μ, how to model traders' behavior reasonably with unknown volatility and how these parameters can be integrated in option pricing. Therefore, we are questioning whether there is a considerable difference between market prices and theoretical prices given by the Black-Scholes formula. With agent based model and simulation it is possible to shed light on the proprieties of this parameters. Zhang et al. (2009).

4.1 Model assumptions

We start assuming that option price is determined ultimately by supply and demand that results from trader's behavior. Moreover, everyone knows and uses the Black-Scholes model. We put ourselves in an observing contest as we know the true volatility but the market participant doesn't know it. The underlying stock follows a Geometric Brownian Motion (GBM) with drift μ and volatility σ:

$$\frac{dS_t}{S_t} = \mu_o dt + \sigma_0 dW_t$$

$\mu_o$and $\sigma_0$are the true ones but they are unknown to market participants. Therefore, they might agree or not but basically everyone has his personal view on the parameters:

$$\frac{dS_t}{S_t} = \mu_i dt + \sigma_i dW_t$$

The subscript i indicates that the equation is valid for each investor from i = 1...10000. According to his personal view of the parameters each

investor calculates his own valuation of the option according to Black and Scholes formula:

$$V = S_t N(d_1) - Kexp(-rt)N(d_2)$$

with

$$d_1 = \frac{log\frac{S}{E} + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

and

$$d_2 = d_1 - \sigma\sqrt{t}$$

In *d1* and *d2* we can notice that σ has the subscript i to indicate that this equation is valid for each single investor according to his personal view of the volatility.

Investors' personal view of the volatility could become relevant to option pricing and so can the implied volatility influence the price of the option. Therefore, our goal is to model investors view on volatility and include them in the option pricing formula.

Following the approach proposed by Avellanda et al. one way to deal with uncertainty is to specify a band. Thus, each investor has its own view of the volatility band as:

$$\sigma_i^{min} < \sigma_i < \sigma_i^{max}$$

Traders are modeled as heterogeneous and autonomous agents characterized by:

- Patience: is the minimum length of days to calculate historical volatility, given by a uniform random integer between 5 and 45 days
- Judgments: is traders' personal adjustment of his experience in historical volatility band. It is given by a random uniform number between 0.5 and 1.5.

The volatility band thus, is calculated for example as: trader i has patience equal to 30, he calculate the volatility band of the underlying asset with 30 days of historical prices and then he multiplies it with his judgment. In this way, traders obtain a range of options evaluations with their volatility band.

Our final goal is to verify if the price that will emerge from the market is the same as predicted theoretically by Black and Scholes. Once the investors have their own subjective volatility valuation they proceed to calculate the option price with Black and Scholes. They obtain a range of prices since they use a volatility band. Thus, they have to adopt a trading strategy. Their strategy is buying low and selling high. This means that traders will sell the option with the highest value and will buy the one with lowest value. They will put both buy and sell orders in the market. The market will record the orders submitted by traders and will match them with the orders of other traders.

We need to model the market for the underlying asset. In particular, we can use simulation to provide a series of prices. Using the following formula to generate price we have:

$$S^i_{t+\Delta t} = S^i_t exp^{(\mu_0 - \sigma_0^2/2)\Delta t + \sigma_0 Z_i (\Delta t)^{1/2}}$$

The simulation is organized in two stages. In the first one the stock market and the simulation of the stock prices start. The simulation will run for 250 days so we will generate each simulation 250 stock prices. After the stock market is open the traders start collecting data. Based on their personal settings they start calculate the volatility and thus consequently the option price. After 50 days of stock prices the option market starts. In this way, all the traders have the chance to calculate their volatilities and update them every day. Once the valuation is done, they enter the market and start trading. Every day option price is calculated as the average between the minimum ask quote and maximum bid quote. After calculating the price of the option it is possible to compute the implied volatility.

The simulation is run with 10000 agents for 1000 runs for each condition.

4.2 ODD model description

Purpose. Option pricing is one of the biggest issues in modern finance. Since the introduction of the Black and Scholes formula, new models have been constructed. Related to option pricing the constant volatility is one such a major problem. Should volatility be considered constant through the life of the option? Many practitioners use new formula with stochastic volatility and many others uses the formula with constant volatility (way faster and easy to understand). The purpose of this paper is to understand whether there is a significant difference in prices using the Black and Scholes formula with certain and uncertain volatility. Moreover, it is important to verify the role of the uncertain parameter μ. The particularity of this work is that it is implemented in an ABMs contest. Theoretical price through Black and Scholes formula will be computed and the market price will be calculated as the interactions of the agents.

Entities, state variables, and scales. In this simulation, agents are represented by investors and the market is the place where they operate. Agents will have different state variables. Judgment and patience are parameters that will characterize all the agents. In particular, patience represents the minimum length of day for them to calculate the volatility. It will be calculated as a random integer between 4 and 45 days. Judgment represents the adjustments that investors do the historical volatility based on their experience. It will be calculated as a uniform random variable between 0.5 and 1.5. The market is represented by "the book." In this book bid and ask offers will be recorded and operations with the same sign will be matched. The market is a virtual place so no concrete interaction through agents will occur. The spatial dimension thus, will be insignificant. Regarding time, each simulation will represent 1 day. After 250 days the simulation will be concluded.

Process overview and scheduling The investors do the main activity in this model. They calculate the volatility and then based on their valuation

proceed to put an order in the option marker. Chronological order of the operations goes like this:

1. Simulation begins.
2. At day 1 the stock market starts and run until the end of the simulation.
3. At day 50 agents can calculate their estimations of the volatility and then after their option prices.
4. Option market opens the same day. Agents are allowed to put their bid and ask offers.
5. The market book records all the operations and matches the operation with the same sign.
6. At the end of the day, the daily price is computed and compared with the theoretical price.
7. Investors can compute the implied volatility and adjust the volatility estimations.
8. After 250 runs the simulation is over and a new one can be started.
9. The results are presented.

The order in which agents make their offer in the market is random.

Design concepts

Emergence Through simple interactions of agents, buy and sell options, we should be able to find complex results. We do not know whether the result will be the same according to Black and Scholes or not. The unexpected element is thus, given by the uncertainty in the result.

Adaption Agents do not have any adaptive rules. The model proposed by the authors is quite simple and the only rules that govern the investors are investment rules.

Objectives Investor's goal is to make a sure profit driven by their option valuation. They will present a bid and an ask offer to the market and when their offer is matched then the transaction will be concluded. This simple rule is represented by the option valuation formula.

Learning In the original work of the authors, agents do not have any complex rule to learn from past situation. It would be very interesting to develop kind of agent capable of creating strategies based on their experience.

Prediction This is the same as the previous rule. Inserting some rules that will make agents capable of forecasts will make the model more realistic and probably will have a different impact on the results.

Interactions Interactions through agents occur in indirect way. They do not see each other but they just place orders in the market. Is the market itself that will match the orders.

Stochasticity The randomness is presented in the choice of the numbers of days to calculate the volatility and on judgment the investors do, based on their past experience, of the historical volatility. The stochasticity is present also in the simulation of the path of the underlying asset.

Initialization & Input data The initial inputs are the option parameters. These parameters are choose by the user to make test on them. In particular:

- The variance σ.
- The drift parameter μ.

Moreover, the user can choose the number of simulation and the number of agents. In a more complex work, the user can also have the possibility to modify investors' strategy to verify the effects on the result.

4.3 Model replication

Our final goal will be that of recreating the model proposed by the author and further complicating the agents' behavior. We want to verify if the results, found by the author, hold in a more complicated agents' behavior. We suspect that the work proposed by the author is not an ABMs in the whole sense of the word.

In particular, we think that agents need more complicated rules such as: having their own strategy, the possibility to make forecasts and so on. This may be more realistic and perhaps brings some unexpected results.

To do so we think that the model should follow the author's idea and then have a different development as regarding the agents' behavior.

At the beginning we should develop the stock market. We will use the underlying asset price model as the one proposed by the authors to simulate the stock path.

The second step consists in creating agents. To have an easy task the use of OOP (Object-Oriented-Programming) is a natural way to model agents. Agents will be encapsulated as object with their state variables. These particular investors will be able to estimate the variance, will have scheduled action, and will be capable of learning from past history.

After, the option market will be created. In the option market all the offers will be recorded and the option market price will be calculated daily.

In conclusion, we think that the work of the original authors is a very nice attempt to evaluate options under uncertain volatility but to have a realistic and more precise ABMs a further extension is needed.

Conclusions

In this review, I analyzed the ABMs methodology in option pricing view. From this, we can understand that this methodology is the most suitable for social sciences. This is due to the fact the ABMs can well capture the emergent behavior that emerge from individuals interaction. Moreover, comparing the ABMs to other paradigms (System Dynamics, discrete Events and Dynamic Systems) we notice that ABMs generally better perform in all the level of abstractions of the models under analyze. This can be explained by the fact that ABMs better combine the main features of the other paradigms.

ABMs can also represent a new form of mathematic. In the traditional comparison between classical and constructive mathematics ABMs place in the between of the two approaches. In fact we can present ABMs as finite systems of discrete time recursive equations over finite state domains or as data-driven dynamic applications systems.

ABMs, as said, have great potentials but it comes at a cost. ABMs are very hard to read when they are not written in a standard way. To solve this problem a standard protocol was introduced by Grim. The main features of this protocol were shown in Chapter 2. ODD is standard protocol that makes things easier to understand and makes the model way easy to be implemented again.

A mathematical proof of the Black-Scholes formula was introduced and as well the unknown volatility framework. This serves to us as a background for developing our work.

Packed with all the necessary tools, finally, the option pricing under unknown volatility studied with agent-based methodology was introduced. Analyzing the paper under the ODD protocol it emerges that the work doesn't look like an ABM model. This leads to a further implementation of this model taking inspiration from the original work. Especially, the further implementation should go in the way of modifying agents behavior. We are expected to reach more realistic results.

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