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Master of Science in Economics

Master Thesis

Brexit, Labor and Migration: an Agent-Based Model

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"I think the next century will be the century of complexity".
— Stephen Hawking

Contents

1 Introduction

This work follows up on two discussed and uncertain debates nowadays: Brexit and immigration. In particular, we want to shed the lights on the likely economic impacts due to cutting immigration in a given country.

To address this question, one has to understand the mechanisms that take place in the labor market and how they affect macroeconomics variables.

Cutting immigration from EU was one of the main reasons that brought Britons to vote for leave in 2016 and therefore there is plenty of literature that analyze the economic impact of immigration with a particular focus on the United Kingdom. However, these works approach the topic with orthodox Equation-Based Models that have several assumptions which hide the complexity inherent to markets.

To study market mechanism, we instead create an Agent-Based Model (ABM), a new kind of modeling approach, that is not tied to analytical tractability.

In Chapter 2, we describe the real world phenomena that have inspired the research.

In Chapter 3, we analyze the literature related to the economic impact of immigration. After a general overview we focus on its impact on the labor market and public budget.

In Chapter 4, we present ABMs. We dedicated the first section to ABM description. In the second one we describe the differences with respect to Equation-Based Models. Finally, in the third section we explain when ABMs are most beneficial.

In Chapter 5, we describe the first version of our model. It represents a simplified economic cycle where firms and households interact following basic economic rules.

In Chapter 6, we discuss the second version of the model. This version adds to the previous one the ability of agents to adapt to the market conditions. Moreover, we include heterogeneity among agents.

In Chapter 7, we describe the third version of the model. This version represents a dual labor market with high and low skill households. Each skill level has different characteristics.

In Chapter 8, we analyze the results of the third version of the model and its sensitivity to parameters.

In Chapter 9, we report the final version of the model. In this version we add a third class of agents: immigrants. Moreover, in order to incorporate Brexit we add a phase of the run where unemployed immigrants and firms can abandon the market.

In Chapter 10, we discuss the results of the Model IV with seven different scenarios.

Finally, in the appendix, we add the entire code of each model.

2 Brexit

2.1 Overview

According to the estimates provided by Ipsos MORI, despite the final 52% versus 48%, there are huge differences in voting intentions if we divide them by age, class, education level and ethnicity. In particular, we notice that:

- on average, younger, high-middle class, black and minority ethnic (BME) and more educated people voted to remain while older, working-class, white and less educated chose to leave;
- the vote was more homogeneous within age and class. Indeed, considering all the classes, the majority of those aged 18-34 voted for remain while the majority of 55+ years old voted to leave. In details, in both age groups the middles-classes were in majority for remaining while the working classes more likely to vote for leaving;
- if we split by gender, a narrow majority of woman voted to remain, while man voted to leave;
- people in work (full or part time, public sector or private sector), students, mortgage holders and private renters voted to remain. Those who own their home outright, social renters, the retired and those looking after homes all voted to leave;
- there were big differences along party lines, but a majority of those who did not vote in the last general election chose to leave. 2015 Conservative voters voted to leave by roughly the same margin as in 2015 Labour and Liberal Democrat voters voted to remain; 95% of UKIP's 2015 support voted to leave. Among those who did not vote in 2015 (but who were not too young to do so), there was a 16-point lead for leave.

		Remain	Leave
2015 vote	Conservatives	39	61
	Labour	65	35
	Liberal Democrat	68	32
	UKIP	5	95
	Green	80	20
Age	18-24	71	29
	25-49	54	46
	50-64	40	60
	65+	36	64
Education	GCSE or lower	30	70
	A level	50	50
	Higher below degree	48	52
	Degree	68	32

Figure 1: *Voting structure from YouGov.com*

Since the beginning of Brexit process we did not have a strong majority in none of the outcomes and nowadays, after more than 2 years, still it is not clear what it will be the final result.

Indeed, having suffered the largest parliamentary defeat of the modern history, the UK Prime Minister Theresa May, has proposed her Brexit "Plan B" to the House of Commons on January 21st.

Her strategy remains the one strongly threw out the week before. Indeed, Mrs. May is reluctant to compromise and she still hopes that European Union enables to last-minutes concessions, which could allow her to reverse the 230-vote majority against her deal. Yet with the clock ticking, this increases the probability for UK to crash out of the EU without any kind of agreement since Britain holds a weak negotiating position and even a lack of parliamentary majority.

It is not easy to reach a majority decision on the Brexit process even because, according to a study on the public opinion published this week by the *UK in a Changing Europe academic network* shows, voters are as divided as MPs on what sort of Brexit they want. In failing to find a majority for anything, the Commons exactly reflects those divisions. Moreover, the study suggests that, if the 2016 referendum rerun now, Remain would win, albeit narrowly.

Most Brexiters argue that if there is no deal able to pass the Parliament majority before March 29th, Britain should leave without one. Anyway, many MPs and members of the current government believe it such a high-risk outcome so that they would like to avoid it.

In particular, the amendment proposed by labourist Yvette Cooper and voted by MPs on Tuesday 29th of January said that if the current deal does not get the majority

by February 26, the Parliament can vote for an extension of Article 50 up to 9 months. This significantly decreases the possibility of a hard Brexit on March 29th.

In other words, MPs are trying to change, temporarily, a system that has been in place for around 100 years. They want to suspend the parliamentary rule that gives precedence to the government's business, allowing them to take control over the Brexit process.

Their lately goal is to force the government to ask an extension of Article 50 and thus of the negotiation period, removing the risk of Britain crashing out of the EU without a deal. The reason behind this is that whatever will be the next step for UK -negotiate a new deal, ask for a new referendum or run general elections- it will take more than the available days before March 29th.

Even if the MPs succeed in their plan, the EU governments may disagree. Indeed, in Brussels not everyone agrees on giving UK more time to decide what it wants and, above all, to approve the article 50 extension which requires unanimous vote. However, since the no-deal scenario may damage Europe too, it might end up with the extension approval.

Mrs May is against the Article 50 extension because she wants to keep open the no-deal option in order to get her deal more valuable. Normally, after a defeat of 230 votes on the main project of its mandate, a Prime Minister resigns but there are three circumstances keeping the government in charge:

- the Tory Party already tried in December to change the leadership and failed, so they have one's hands tied for 12 months as their party's rules state;
- the government is likely to survive to any confidence vote since otherwise the result will be the rise of a radically left-wing labour government, and this worries both Conservatory Party and the Northern Irish Democratic Unionist;
- to find a new Prime Minister at this moment might not be an optimal solution since the Brexit is already in an advanced phase.

The Parliament sees this option as an emergency solution to preserve the country from a chaotic no-deal outcome that nobody had in mind when they voted to leave in 2016. The irony behind stopping the no-deal option is that even the strongest Brexiters have to face that the only alternatives is no Brexit at all or the May's deal, since Europe looks reluctant to reopen the negotiate. Moreover, this scenario will give to the current Prime Minister the best chance to see her deal approved by the parliament. This is particularly true if we take into account the rebellion that is happening in the Labour Party, where some MPs are now against the strategy pursued by Jeremy Corbyn and they might give an unexpected support to the Mrs May's deal.

In other words, Theresa May is battling to keep her Brexit deal alive, barely a month before Britain's scheduled March 29 exit date.

But the UK prime minister faces obstacles on several fronts. Eurosceptics in her Conservative party's ranks reject a central provision of the treaty she negotiated with the EU: the so-called backstop to prevent a hard border on the island of Ireland.

Pro-EU Tory MPs want to force Mrs May's hand to prevent a no-deal Brexit, if necessary by delaying the date of Britain's departure beyond March 29. Meanwhile, the rest of the EU has almost unanimously insisted it is not going to reopen the 585-page withdrawal treaty.

The issue is coming to a crunch, with the House of Commons due to vote on a revised Brexit deal — and possibly a second EU referendum — by no later than March 12. There could then be votes in subsequent days on a no-deal exit and on delaying Britain's departure from the bloc.

After having briefly analysed the current situation, we are trying now to find out, what are the most likely future steps of the Brexit process:

- *week of March 4th: negotiations with Brussels*
  Geoffrey Cox, attorney-general, is leading British efforts to make it clear that the backstop — which Eurosceptics say could "trap" UK in a customs union with the EU — cannot be permanent. His talks with Michel Barnier, the EU's top Brexit negotiator, are certain to intensify as the UK tries to find a revised deal that can command the support of the House of Commons by March 12th. EU diplomats say London is pressing for a date to show the backstop will come to an end. But they add that such a date can only be "soft" or aspirational rather than a clear time when the backstop will no longer exist. One possible approach could set a target date for putting in place alternative arrangements to replace the backstop while committing to maintain an open border between Northern Ireland and the Republic of Ireland;
- *March 12th of before: second meaningful vote*
  under Mrs May's current plan, MPs would be given a "meaningful vote" on a revised exit deal by no later than March 12th, just 17 days before the UK is scheduled to leave the EU. This would seek to overturn the House of Commons' record 230-vote rejection of the Brexit package in January. The prime minister says another vote could take place before EU leaders give final formal approval to a revised deal (the EU has long insisted that Mrs May needs to demonstrate what deal can really get through parliament). The Labour party now says that when the meaningful vote takes place, it will put forward an amendment to call a second referendum. This is a big shift, which follows Commons defeats for Labour proposals for a softer Brexit — previously the party's preferred alternative to Mrs May's plans — and the defection of eight pro-EU Labour MPs to the new Independent Group in parliament. Labour leader Jeremy Corbyn wants to prevent large scale further defections. But the numbers suggest that a second referendum

does not command sufficient support in Westminster to become reality — in large part because many Labour MPs from Brexit-supporting constituencies reject a revote;

- *March 13th of before: Parliament consulted on no-deal*
  if the House of Commons rejects the renegotiated deal by March 12th, Mrs May has said she will then give MPs another vote on whether to press ahead with a no-deal Brexit. This vote would take place by March 13. "The UK will only leave without a deal on 29th March if there is explicit consensus in the House for that outcome," the prime minister has told MPs. Previous votes indicate that only a minority of MPs in the Commons would back a no-deal Brexit. But Mrs May's strategy risks a split with the pro-Brexit European Research Group of Tory MPs as well as Northern Ireland's Democratic Unionist party, which provides her government with a majority in parliament;
- *March 14th or before: decision on delay*
  Mrs May has told MPs that, if parliament rejects no-deal, MPs would then be asked by March 14 whether they wanted a "short limited extension to Article 50". If they did, the government would subsequently seek the EU's agreement for a delay and bring forward the necessary legislation. She warned that no extension should last beyond the end of June, since otherwise the UK would need to participate in elections to the new European Parliament, which will take its seats at the beginning of July. But by this point MPs may also have more success in imposing alternative agendas on the government — such as a softer Brexit, a longer delay, or a second referendum. Mrs May will be hoping that such a prospect will deter the Brexiter MPs of the European Research Group and that they will rally to her deal beforehand to avoid such choices;
- *March 21-22: EU Summit*
  EU officials increasingly believe that any deal will only be formally approved by the bloc's leaders at their summit on March 21-22. Mrs May has agreed with Jean-Claude Juncker, European Commission president, to wind up EU-UK negotiations by this date. The EU27 prime ministers and presidents have little inclination to hold an emergency summit before then, given that they convened in Brussels for a special meeting on a Sunday in November — only to bear witness to delay and paralysis in Westminster over the succeeding months. But the fact that the March 2019 summit is taking place just a week before the scheduled date of Brexit could leave very little room for any last-minute slips;
- *March or later: deal passed into UK law*
  if the Commons finally approves a Brexit treaty in a meaningful vote, the government will put forward a new piece of legislation: the European Union (withdrawal agreement) bill. This would pass into law some of Brexit's biggest issues, such as the agreement on citizens' rights, the financial settlement and the

details of the transition. It will be a hugely consequential piece of legislation. Many battles could take place on individual details. This has led some MPs to warn that approving the legislation could become a prolonged "guerrilla war" that would be difficult to conclude by Britain's scheduled exit date. It may also mean that Mrs May seeks a limited Article 50 of a couple of months after the meaningful vote is passed, so that the necessary related legislation can go through;

- *until March 29th: EU ratification*
  before any Brexit deal can take effect, it must also be approved by the European Parliament in a plenary vote. Any legally questionable elements of the withdrawal treaty could also be referred to the European Court of Justice by MEPs. EU member states must also give the deal final approval in a ministerial meeting;
- *March 29th: Brexit day*
  there will be plenty of political declarations on this historic day. But whether there will be discernible changes to everyday life depends on the negotiations of the preceding months — and on the battles in Westminster. They could produce a largely seamless transition or, if they fail to yield any deal, a much more chaotic, cliff-edge Brexit. The events of recent days have also increased the likelihood of Brexit being delayed beyond March 29. The UK government is committed to passing several other bills before exit day and putting forward almost 600 statutory instruments, which require less scrutiny. Mrs May herself could therefore ask the EU to extend the Article 50 divorce timetable for a few months in order to pass legislation before exit day, whether or not the UK is on course to leave with a deal. Or if Britain has not agreed on a form of Brexit, a longer extension — perhaps to the end of the year — may be necessary to hold a general election or a referendum. The rest of the EU can grant an Article 50 extension request by unanimity. One last alternative is that the UK could revoke its notification under the Article 50 divorce process with the EU and opt to remain in the bloc for good. The European Court of Justice ruled in December that Britain could make such a unilateral volte-face. At present, however, Downing Street rules out both requesting an extension and revoking Article 50;
- *after March 30th : trade talks and transition*
  assuming that Brexit has taken place on schedule, fully fledged trade talks can begin between the UK and the EU. While Britain remained a member state, such talks were not permitted under EU law. Under the deal reached in principle in 2018, this is when the 21-month transition period would begin. During this time most aspects of UK membership of the EU will remain in place, including free movement across borders and membership of the customs union and single

market. But Britain will no longer have a vote;

- *April 2019: no-deal negotiations*
  if, by contrast, Britain leaves the EU without a deal on March 29th, Europe's Brexit negotiators will not end talks but reset their clocks to a new cliff-edge date: April 18th. After 20 days of likely disorder at ports, supermarkets and borders, the deadline will be Britain's chance to avoid a more lasting rupture with its biggest trading partner — if it can stomach the price. By April 18th, according to European Commission contingency plans, Britain must confirm whether to make about seven billions of euro net contributions to the EU's budget for 2019. The first payments, which require House of Commons approval, are scheduled for April 30th; EU negotiators say missing them will "ruin" relations. Driving the EU side will be a new aim: making Britain meet its withdrawal treaty obligations, including the 45bn budget bill and backstop arrangements to avoid a hard border on the island of Ireland, even though the treaty itself will have perished. As a result, a no-deal exit would kick off the most fast-paced and consequential period of negotiations since the EU referendum in 2016;
- *December 31 2020: an end to transition?*
  the transition period is scheduled to end on this date, although that could be changed, according to the draft withdrawal treaty. The problem is that some EU negotiators doubt that a full UK-EU trade deal will be agreed by this point, or for some time to come, given the protracted nature of such talks. So the treaty also establishes that the transition could be extended up to December 31 2022;
- *December 31 2022?: entering the backstop*
  the provisional withdrawal treaty makes clear that if, at the end of the transition, no deal is in place to avoid a hard border with Northern Ireland, the so-called backstop will automatically kick in. This will keep the entire UK in a "temporary" customs union with the EU, although Northern Ireland will be more deeply integrated into the bloc. Brexiters are deeply opposed to the backstop plan, arguing that it imperils the integrity of the UK, increases Brussels' powers over Britain on issues such as tax, state aid, and labour and environmental regulation, and provides no guaranteed date for exit. As a result, Mrs May is seeking to agree changes to the backstop, to avoid the UK remaining under the measure indefinitely and against its will. Her talks with the EU will determine whether the backstop remains part of the deal, whether any of its provisions are changed — or whether there is any deal at all;
- *mid-2020s: journey's end?*
  many business leaders suggest that the "maximum facilitation" plan favoured by some Brexiters that would rely on advanced technology to speed up customs clearances — and so exit the backstop — would need years to put in place, delaying "full Brexit" until deep into the 2020s. A less ambitious "fast-track"

system on the US-Canada border took decades to develop and billions of dollars in investment. FT Brexit and Beyond Summit 2019

2.2 Economic scenarios

Chancellor Philip Hammond in 2016 stated " It is clear to me, people did not vote to become poorer", now with Brexit getting closer and closer and with a withdrawal agreement negotiated by Prime Minister Theresa May and refused by MPs in January, the official estimates from government economists and bank of England look in a different direction.

Based on their analysis, indeed the agreement reached by Theresa May, will avoid the worst scenario, i.e , the UK crashing out of the EU without any deal, but they all agree that Britain would ultimately be better if it remains in the EU. What clearly emerge from these analyses is that more UK will be able to stay in closer relationship with EU lower will be the negative impact on the economy.

The outcomes predicted by government economist are slightly different based on the different possibilities of the Brexit process. The closest one to the May's deal, that include a strong control of immigration and small trade frictions, seems to contract the growth of UK GDP of "only" 3.9 per cent in the following 15 years with respect to stay in the EU, bringing just to a small reduction of living standards. This reduction is estimated by the Treasury and it is 1,100 pounds less per year/person and it is mainly due to the assumption that cut to zero the net immigration from the EU is cut to zero.

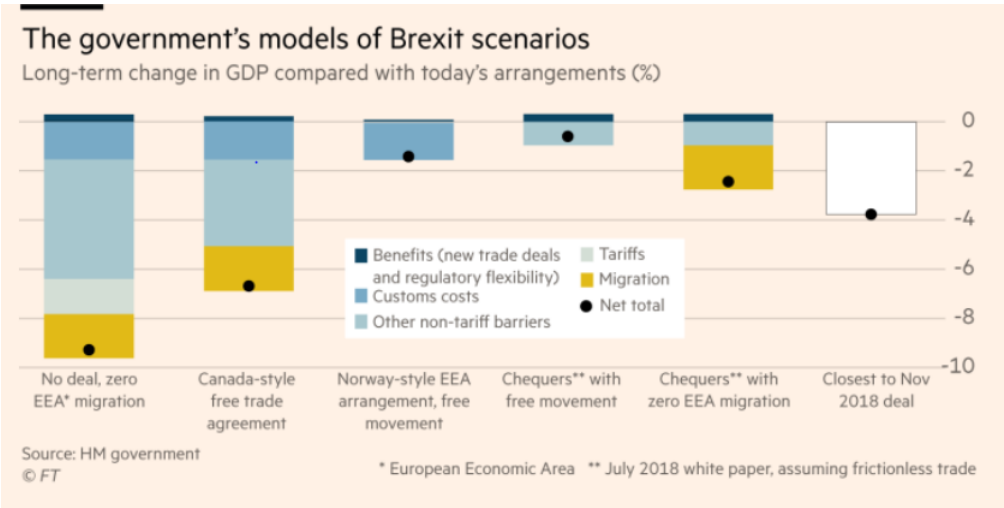

Figure 2

This solution is much better than to end up with a no-deal that, for BoE analysis, may bring to a worse recession than the last financial crisis. Indeed, the no-deal

scenario may drop national income to its lowest level from the Second World War; the government economists foreseen a GDP decrease up to 10.5% and house prices dropping of 30% over a five-year period compared to their pre-referendum analyses. This is even worst with respect to the 2007 financial crisis where house prices tumble of 17% and GDP shrank of 6.25%.

Anyway, the Bank of England's Financial Policy Committee "judges that the UK banking system is strong enough to continue to serve households and business even in the event of a disorderly Brexit", it said. "The UK economic scenario in the 2018 stress test of major UK banks was sufficiently severe to encompass *worst-case* assumptions about the challenges the UK economy would face in the event of a cliff-edge Brexit".

This worst-case scenario set sharp drop in house prices of 48%, an increase of unemployment rate to 7.5% and a rate of inflation of 6.5%. They would be mainly due to the introduction of trade barriers and custom checks on the border creating severe disruption. Despite the resilience of the financial system, the BoE's Governor Mark Carney specified that the rest of the economy might not be ready for such a shock, indeed, only around 14% of the small business made a plan for a no-deal scenario, at the same time more than 250,000 firms did not even ever filled a customs form. The BoE governor, Mark Carney, said that in the case economics partnership with EU was signed in five years' time "GDP is between 1.25% and 3.75% lower than it would have been had it continued to grow at its May 2016 trend rate".

These results have been welcomed by independent entities and world economic community as reliable and impartial, for example Gemma Tetlow the chief economist in the Institute for Government, said that this report passed "very clearly" all the 9 tests that she had set up for it. Nevertheless, Garry Young, head of macroeconomics at the National Institute of Economic and Social Research defined the government analyses "very fair", despite the big uncertainty surrounding the Brexit process.

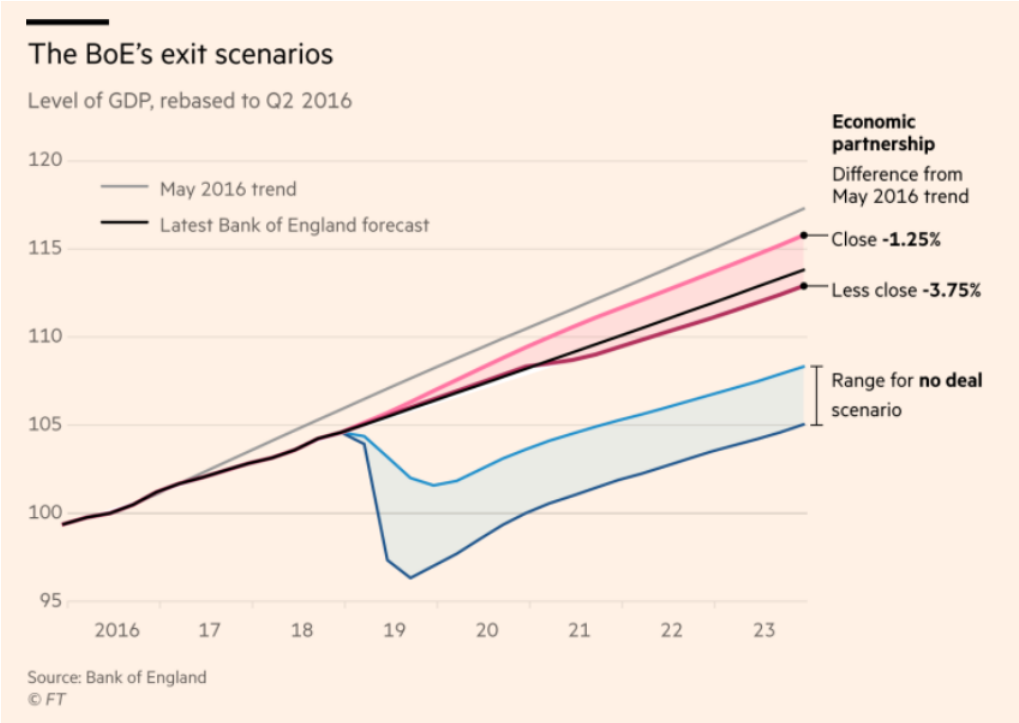

Figure 3

Anyway, the prediction of a negative impact of Brexit, has put BoE strongly in contrast with the government and especially with the Eurosceptic: according to BoE a no-deal will bring one of the ever worst financial crisis and any deal will be able to reach the potential growth as if UK keep staying inside EU.

2.3 Brexit and uncertainty

The referendum's outcome to leave The European Unions in the June 2016 has created an environment of large uncertainty for UK firms. This uncertainty covers several aspects of their business such as the future access to EU market, product regulation and availability of migrant labor. Nevertheless, there is also uncertainty around how the UK will go to that new end state, how the relationship will look at different points in time and what each of these will mean for the prospects of individual businesses.

In order to understand how firms might react and what will be the macroeconomic implications for UK, it has being crucial understanding the several aspects of uncertainty surrounding the Brexit process. There exist a wide literature and several empirical evidences that assess the effect of uncertainty on economic outcomes. First of all, firms delay investment because they prefer to wait for more clarity or information. Since a lot of investments have fixed costs not possible to be recovered once done,

the uncertainty increases the option value of a delay making firms more cautious and business conservative until they get more information. This is the so called 'chilling effect' [Baker *et al.*(2016)Baker, Bloom and Davis] and implies a reduction of investment and productivity and explain how uncertainty may affect Business cycle.

Furthermore, uncertainty increases the risks related to investment as there are less information on the pay-off required. As risks grow, the cost of borrowing money increases too since financial institutions require higher risk premia which especially affect firms that are financially constrained. The same happens to household that prefer increase their precautionary savings shrinking the aggregate demand in the short-term.

Regarding Brexit, the common indicators of uncertainty provide different results from the referendum in 2016. For example, as described in Figure 4, *The Policy Uncertainty Index* developed by [Bloom *et al.*(2018b)Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry] and based on media reports, rose to its highest level after the referendum. On the other hand, for example, the market volatility remained relatively low as did the principal-component-based index developed by the Bank of England and described by [Haddow *et al.*(2013)Haddow, Hare, Hooley and Shakir].

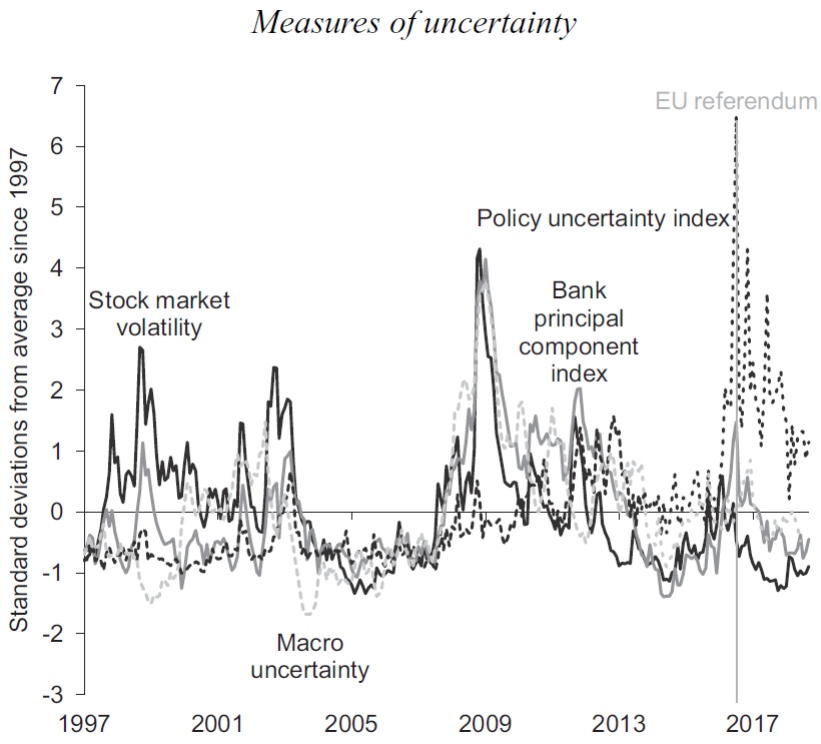

Figure 4: *[Bloom et al.(2018a)Bloom, Bunn, Chen, Mizen, Smietanka, Thwaites and Young]*

The conflicting results provided by the existing literature highlight the difficulty

to tackle the uncertainty related to Brexit. Indeed, traditional indicators diverge in results because this is an unique event which differs from any other uncertainty shock of the past since it is not associated to a recession but it relates to the UK's trading relations and by its longer duration. Perhaps, the uncertainty began immediately after the referendum but, since UK has not even left the EU yet, the main consequences of Brexit will affect the economy mainly in the long-term.

In August 2016, a collaboration between Bank of England, Stanford University and the University of Nottingham launched the Decision Market Panel (DMP). The DMP is a firm-level survey that collects information by asking questions to UK based firms about Brexit expectations on a regular monthly basis. These results are particularly effective on tackling uncertainty since it got information from firms with different characteristics that can change their sensitiveness on Brexit. Moreover, DMP is able to collect a large share of the business population all over the UK (around 15% of total private sector employee job).

One important finding pointed out by the DMP, as analysed by [Bloom *et al.*(2018a)Bloom, Bunn, Chen, Mizen, Smietanka, Thwaites and Young], is how firms rank Brexit among their other uncertainties. As explained in Figure 5, this uncertainty is getting bigger as Brexit is closer and closer, indeed in September 2016 almost 30% of firms did not consider Brexit as a source of important uncertainty, but only 13% of firms replied in the same way on October 2018. At the same time, the firms that consider Brexit the top source of uncertainty have more than doubled in these two years (from 9% in September 2016 to 19% last October).

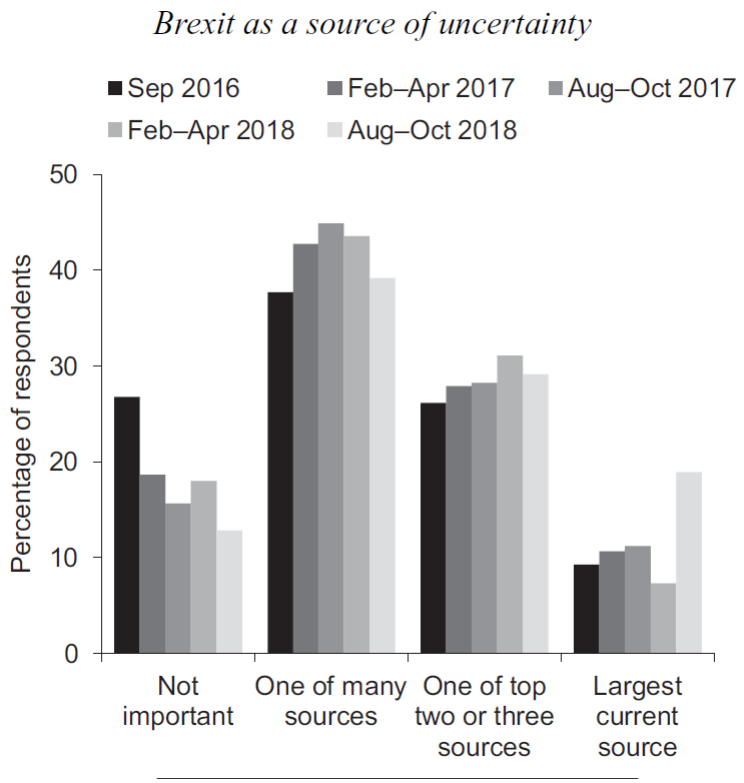

Figure 5: *[Bloom et al.(2018a)Bloom, Bunn, Chen, Mizen, Smietanka, Thwaites and Young]*

Since the sample of firms that respond to the survey is heterogeneous the DMP allows us to understand which kind of business results more concerned about Brexit uncertainty. Furthermore, we can see the share of firms that reported Brexit as top source of uncertainty grouped by industries. In October 2018 the most affected are wholesale retail and manufacturing while the lowest is in human health and social work (Figure 6). The industries more concerned are then the ones which are more dependent on EU through exports and the use of migrant labour.

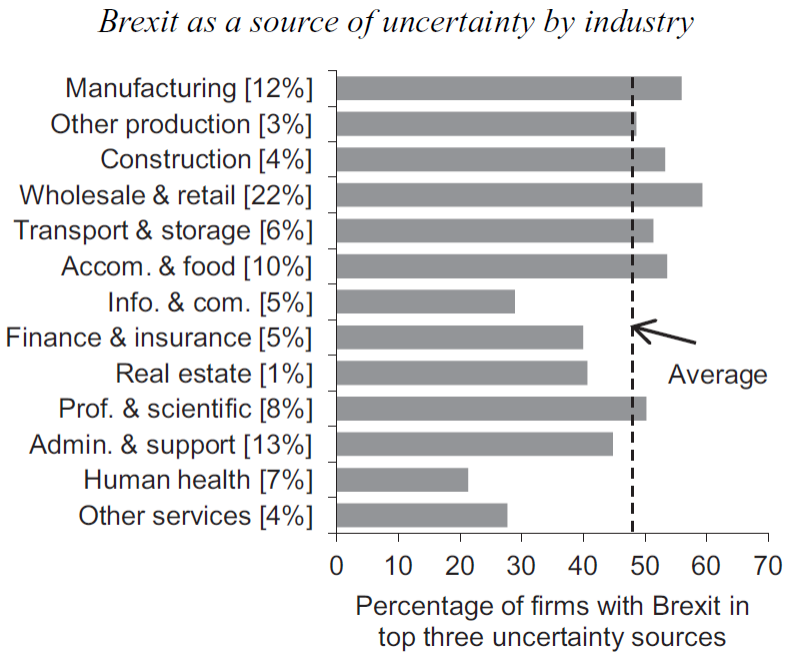

Figure 6: *[Bloom et al.(2018a)Bloom, Bunn, Chen, Mizen, Smietanka, Thwaites and Young]*

The importance of Brexit uncertainty varies across regions and firm size. Indeed, in the October 2018 survey large firms represent the highest percentage among those that put Brexit on their top three uncertainties (50%). This is probably due to their larger exposure on international market. Perhaps, large firms do not operate only in the region where are based, so it can be useful exclude them when grouping by region. Indeed, results change if they are included or not since they are located more in Northern Ireland than anywhere in UK (Figure 7).

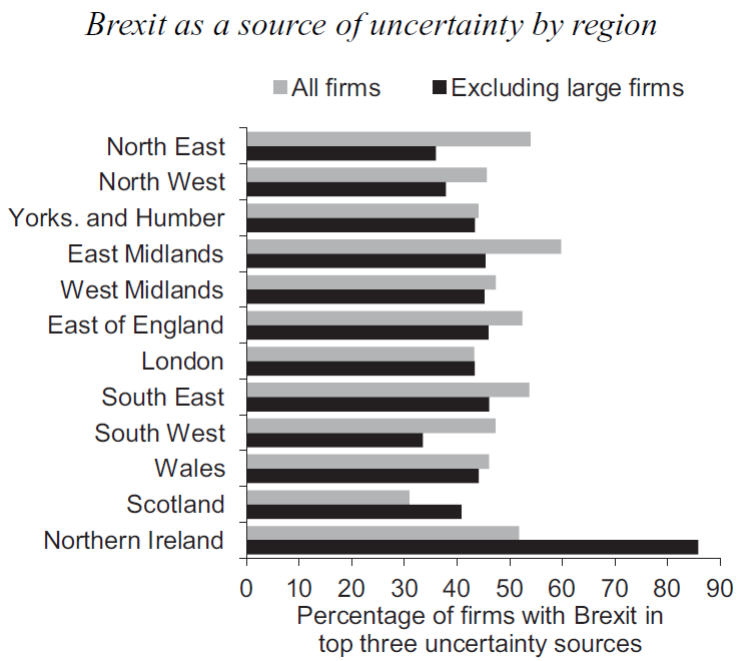

Figure 7: *[Bloom et al.(2018a)Bloom, Bunn, Chen, Mizen, Smietanka, Thwaites and Young]*

Brexit uncertainty is strongly related to the fact that we still do not know which economic model the UK will follow and how this affect firms' business and sales. To prepare their business, firms need to know what will be the Brexit plan and if it will happen or not in a disorderly way and nevertheless how it will affect the characteristics that determine firm performances such as the access to the Single Market, customs, freedom degree of labour movement and other regulations. For instance, as we have seen, firms more dependent on the EU market and migrant labour are more concerned on the uncertainty Brexit-related as their sales and labour availability will be more affected by the futures EU-UK relationship. The expected effect of Brexit on labour cost and export is one of the questions proposed by the DMP survey and as expected firms believe that, on average, Brexit will shrink exports and increase labour cost (Figure 8).

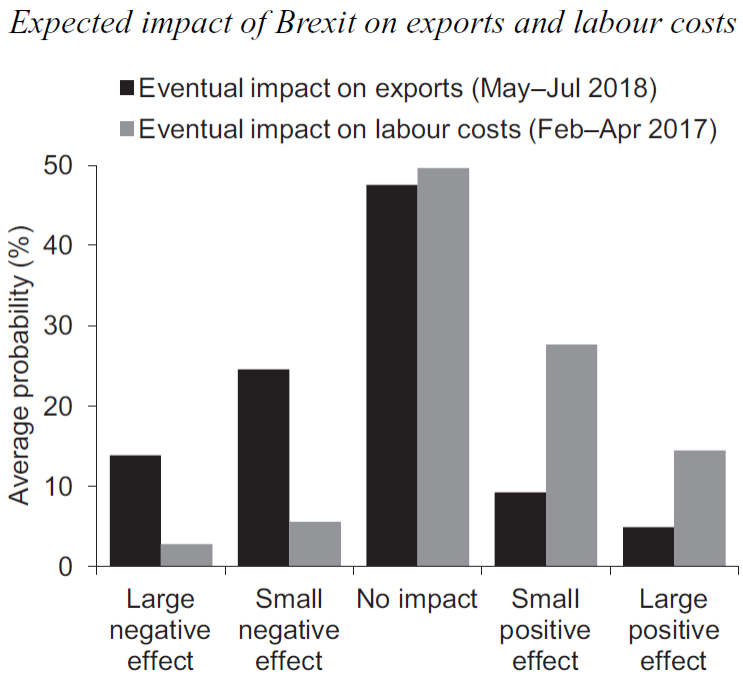

Figure 8: *[Bloom et al.(2018a)Bloom, Bunn, Chen, Mizen, Smietanka, Thwaites and Young]*

3 Economic impact of immigration

3.1 Overview

It is not easy to estimate migration flows, but according to the work of [Azose and Raftery(2019)] the numbers of individuals migrating internationally yearly is between 1.16% and 1.29% of the global population considering the period between 1990 and 2015. These numbers include emigration, transition and returns. Returns only represent around one quarter of total migration. Migration can be significative: it is able to influence the total population of several developing countries more than fertility or mortality. What affects the population is the net flow of migrants defined as the number of in-migrants from all origin locations minus the number of out-migrants to all destinations in a given period. On the other hand, the total number of individuals born elsewhere living in a given country at a precise time is called migrant stocks.

Worldwide, around 258 million of people live outside their native country. In 2016, during a ONU conference, the president Obama was the first asking for a treaty in order to guarantee a safe and regular migration. On 13th of December 2018, overcoming the opposition of nationalistic governments, 164 countries signed the global compact for migration that represents a global step ahead in the management and coordination of the migration flows. It is aiming to enhance the legal ways for migration and to fight the illegal and black market which control most of migrants putting their life at serious risk.

The global compact for migrations refers to 23 points whose objective is to enhance coordination and cooperation for the management in both origin and destination countries as well as at local, national and global level. Although this agreement does not restrict national power about 15 countries decided not to sign it because they believe that it will favour illegal migration destroying national borders. These countries are specially those which receive the highest number of migrants such as Australia, United States and several European countries.

During this debate, Europe has showed up all its internal disorder and lack of cohesion about the migration issue. Migration has seen as a problem especially for those countries which are still recovering from past economic crisis and are governed by populistic or far-right parties. This lack of unit is due to the massive migration that affected some European countries over the last years, changing equilibrium and political power.

Among them, United Kingdom has been deeply changed by migration, indeed, during the 2016 referendum decided to leave the European Union even to take back the control of its national borders.

UK has increased its immigrant population substantially over the last 17 years, indeed, it increased from about 4.8 million in 1995 to around 9 million in 2011, an

increase from 8.4% to 14.7% of the general population. Breaking these figures down into EEA versus non-EEA immigrants, in 1995, the former was less than 20% of the total UK immigrant population. EEA immigrant population more than tripled between 1995 and 2011, growing from 885 thousand to 2.8 million. Over the same period, the non-EEA population grew at a considerably lower rate, increasing from 3.9 million to 6.1 million, so that in 2011, 32% of the immigrant population was composed of EEA immigrants which are even more nowadays. As expected, migration has contributed strongly to the UK increase of population and jobs, indeed about 2.3 million out of 3.1 million of new jobs created during this period were covered by immigrants. In details the EEA migrants employed in UK grew of 400% from 1995 to 2011, while the number of non-EEA employed increased by almost 80%, from 1.5 to 2.6 million.

Moreover, the immigrants are more educated with an higher rate of graduate and are substantially younger than native ones, keeping down the average age of the whole United Kingdom.

Indeed, in 2011, the percentage of natives with a degree is around 21% doubling since the beginning of the period, while the percentage for immigrants rose even more, to 32% for EEA and 38% for non-EEA. At the same time, 50% of natives fall into low skilled job while the same happened just for 20% and 25% for EU and non-EU immigrants respectively.

Fiscal year	*Natives*	*EEA*	*Non EEA*
1995	52,172,016	885,367	3,920,502
1996	52,053,113	823,820	4,049,663
1997	52,024,832	953,449	4,178,270
1998	52,044,969	1,044,056	4,258,364
1999	52,198,811	1,065,211	4,294,403
2000	52,167,122	1,054,930	4,509,258
2001	52,254,626	1,124,239	4,577,880
2002	52,221,725	1,161,818	4,762,303
2003	52,346,927	1,229,381	4,819,508
2004	52,384,909	1,282,428	5,010,460
2005	51,580,064	1,411,814	5,216,225
2006	52,191,015	1,677,650	5,543,197
2007	52,054,165	2,271,159	5,436,642
2008	52,115,726	2,373,601	5,702,679
2009	52,331,186	2,432,699	5,800,989
2010	52,333,130	2,763,560	5,987,809
2011	52,360,031	2,847,289	6,146,430

Figure 9: *[Dustmann and Frattini(2014)]*

3.2 Wages and labour market

The perceived economic impact of migration is one of the main drivers to the growing hostility for immigrants. In particular, political ballots mirrored the widespread concern that in the host economy immigrants take out more than they actually contribute.

For example, one of the main advantages to leave the EU for Brexiters is to cut the strong immigrations coming from other member states. Indeed, since EU membership allows free movement of natural persons, a strong flow of job seekers has reached the UK looking for better opportunities and hitting significantly the local job market.

This happened mainly for high skilled European workers attracted by favourable market conditions with respect to their home countries, led especially by the strong growing of UK service sector. In addition, the number of EU migrants strongly increased after the EU's East enlargement in 2005 when the UK government, in contrast to Germany, immediately allowed access without limits for immigrants coming from new member states. The overall number of immigrants from EU countries living in UK increased from 0.9 million in 1995 to 3.3 in 2015 [Dhingra *et al.*(2016)Dhingra, Ottaviano, Sampson and Reenen]. Around 2.2 million out of 3.3 million people living in UK are employed with respect to a total working age of around 2.5 million. On the other hand, the strong migration from EU member states has lowered the number of immigrants from Non-Eu countries who are allowed through a limited number of working visas only. Such a treatment could be reserved even to all or some Eu countries after Brexit.

Has really migration affected the UK's labour market negatively over these years? And what will be the outcome of cutting migrations for the economy in the future?

Leave supporters' fear, of course, is that immigration crowds out domestic labour and strains public service but both theoretical and empirical literature seem to move in a different direction.

In theory, there is a strong variability in the impact of immigration on the host economy. it is strongly correlated with immigrant's skill-set with respect to existing workers and labour market characteristics. The impact can be totally different between short and long period due the market ability to evolve and increase labour supply. In the short-term Immigration affects the supply side mainly along two dimensions: wages are reduced and competition increased if newcomers have similar skills and lower minimum wage than existing workers. On the contrary the presence of migrants with complementary skills is likely to rise productivity and bring wages to an overall increase. Which one of the effects prevails depends on whether and to what extent migrants' skills are complements or substitutes to the skills of locals [Borjas(1995)]. Empirical evidences suggest that low skills workers face more competition from immigrants because their skills are easier to be replaced [Vargas-Silva(2015)]. For example, the work [Nickell and Saleheen(2015)] shows that inside the UK unskilled

and semi-skilled service sector there is a decrease in occupation around 0.2% for 1 percentage point increase in the share of migrants.

From the demand side immigrants boost consume and demand for certain goods and services, create jobs by changing the mix of goods and services produced in the economy as well as the occupational and industrial structure of the labour market.

For the enormous uncertainty around Brexit and for what it will actually mean to the UK economy it is necessary to make analyses on a wide variety of scenario whose effects can strongly differ due to quite different assumptions. According to MAC study published in September 2018 the impact of migration is not seen as one of big magnitude with respect to other changes. They asses that immigration has minimum or no effect on both employment and wages level and,in the case, it is mainly limited to certain group or areas. The dropping of the pound after Brexit has caused price decrease almost of 2%, shorting individual income even more than the effect of migration. Since they do not see any big overall impact, positive or negative, they recommend to move into a system without preferential access of migrants from EU ending perhaps the free movement of people between UK and EU. This is in line with the UK government Indeed, the control of migrations was one of the core point put in place by the conservative party in order to stop "uncontrolled immigration" from the EU " and as Theresa May, the prime minister, put it in a speech on September 20th,it appears to be the reddest of her red lines in the negotiations over Brexit. Any deal that keeps free movement of people, she said, would fail to meet the result of the Brexit referendum. The report also stated that immigration from EU countries in the last thirty years had reduced the employment rate for born Britons workforce around 2 percentage points and increased unemployment by 0.6 percentage points. However, it also noted that with employment rates at a historic high towards the end of this period, one should "be cautious in suggesting these outcomes could be much better than they already are".

Other studies seem to move in a different direction regarding the impact of migrations on UK labour market and wages. For example [Jafari and Britz(2018)] using a CGE framework assessed that repatriating EU citizens working in UK and controlling further immigration from Europe will reduce UK workforce of approximately 1 million out of a total working population of around 31 million.

This reduction of workforce and population has a threefold negative effect:

- small population means less demand for services;
- reducing labour stock shrinks the production possibility of the entire economy;
- reduction of tax income.

The authors, that estimate Brexit impact on trade, immigration and foreign direct investment, surprisingly state that workforce reduction solely has, among the three, the highest impact on UK economy considering a mid-range scenario. They predict a

welfare shrinking at around -402 USD per capita and real GDP dropping of almost double (-2.49%) with respect to the decrease of population (-1.24%). Furthermore, even income tax is decreased of 0.65% mostly reflecting from reduced savings on capital stocks and meaning a cut by about 7.56% on direct taxes to offset the losses.

These results are consistent with other researches, for example, a study of [PricewaterhouseCoopers(2016)] predict a net reduction of 600,000 immigrants which lower the GDP of 1.6%. Even [Portes and Forte(2017)] estimate a GDP reduction of -2.7% considering due to an annual immigration decrease of 91,000 over 12 years.

3.3 Public budget

Another important and possibly even larger concern in the public debate on migration is whether immigrants contribute positively or negatively in the public budget of host countries. Indeed, as showed by [Dustmann and Preston(2007)] there is a strong evidence that whatever the position of natives toward immigrants is, the cost for public finances appears as the main concern about migrations, rather than wages and employment. Such fears regarding the negative fiscal effects of immigration are also reflected in survey responses. For instance, according to *2014 European Social Survey*, only 52% of EU citizens agree to accept new immigrants from poorer countries. Among them, when asked whether immigrants receive more or less in social benefits than they contribute in taxes, 30% of European citizens responded that immigrants receive more than they contribute, while only 18% believe that immigrants in general take jobs away from native workers. The same answers have a proportion of 61% and 45% respectively among people who replied negatively to the first question, i.e. those who want few or no new immigrants at all.

The theoretical and empirical literature instead shows, on average, positive results of immigration on the host country public budget. This is especially true for those countries that are already running structurally public deficits and aging population as European countries are.

The present literature on migration effects over a country public budget can be summarized in 3 main groups. The first is the group based on the cost-benefit analysis approach, it follows the work of [Tienda and Jensen(1986)]. These studies are statics, in other words they calculate costs and benefits of immigrants in the host country public finances for a given year. For example, according to [Dustmann and Frattini(2014)] during the period 1995-2011, immigrants from EEA countries have had a positive impact on UK public finances, in particular they find a net fiscal contribution of about 8.8 billion GBP (in 2011 equivalency), that is a contribution to the fiscal system of 4% more than they received in transfers and benefits. On the other hand, natives made an overall negative net fiscal contribution of 604.5 billion GBP, that were just 93% of what they received. They also show that non-EEA countries have made a

negative fiscal contribution overall, when considering the same period. Since one year is an oversimplification in order to analyze the immigrants impact over the whole period of staying in the host country have being introduced dynamic accounting tools to include further generations [Auerbach and Oreopoulos(1999)]. Furthermore, the work of [Preston(2014)] assesses that as the period of staying increases, immigrants gradually acquire and adapt their skills to the ones needed in the host job market.

These studies, including only accounting effects, in general show relatively low impact of immigrants on the host country economy. This is because immigrants represent only a small share of the population in the host countries and so their weight is particularly small. In this group the role of migrants is even lower because are not take into account such important properties as externalities, complementarities and more broadly any general equilibrium effect.

The second group of studies is indeed the one referred to general equilibrium models. However, these models are infrequently used to make forecast since they are strongly dependent from parameters which are difficult to calibrate or not observable. For example, we have to make strong assumptions on the evolution of income and population or about the level of skills complementarity between locals and immigrants in the job market.

The third and more recent group of studies belongs to time-series analysis and use VAR modelling in order to quantify the impact of migration shock on public budget.

Particularly relevant for our analysis is the work of [d'Albis *et al.*(2018)d'Albis, Boubtane and Coulibaly], where authors use a structural VAR model for a panel of 19 OECD countries during the period 1980-2015 in order to identify the demographic dividend of international migrations. The idea behind is that the flow of immigrants as a shock increases the overall workforce with a positive effect on both employment rate and working-age on total population. International migration, by increasing the total working-age population, also reduces old-age public spending per capita and per capita transfers paid by government. Indeed, in their panel data the authors found a GDP per capita increase by 0.25% the year of the migration shock and by 0.31% the year after. Furthermore, even government purchases per capita and net taxes increase respectively of 0.22% and 0.85% the year of the shock and by 0.47% and 1.11% percent at their peak resulting in a fiscal balance increase of 0.12% points of GDP.

According to the latter results, we may find the same effects but with opposite signs after the Brexit process. As estimated by [Jafari and Britz(2018)], the UK workforce will be reduced of 1 million out of 31, this net negative flow of migrants will bring to a decrease of the demographic dividend. Considering the UK demographic structure that is aging and the volume of public transfers to their non-working citizens a decrease of demographic dividend may be particularly significant for the economy. Especially, by decreasing the share of the workforce and increasing per capita transfers we may see a decrease in both GDP per capita and fiscal balance.Furthermore, if the marginal

cost of providing fixed public goods to immigrants is (close to) zero, immigrants provide support by sharing the total provision costs among a larger pool of people, allowing a substantial implicit savings to the native population.

This is particularly true since EU migrants contribute, on average, on public pursue more than the average British-born. The European newcomers indeed give a contribute to the public pursue of approximately £78,000 (in 2017 prices) over their lifetime, earnings then around $3,000 (£2,370) more than the average born-Britons. In addition, most of the EEA migrants reach the UK after completing their education abroad, and thus at a point in their lifetime where the discounted net value of their future net fiscal payments is positive and they are less likely than natives to receive state benefits or tax credits.

4 Agent-Based Models

4.1 Overview

Agent-Based Model (ABM) is a formal tool that through computational modelling is able to observe interactions among entities over time, namely agents. As computational model, we mean a model that takes certain inputs and generates outputs through algorithmic way. The agent is the individual element of our computations and it has own properties, states and actions.

In the last decades ABM uses grew as the computer technology did; indeed, the introduction of powerful computation has changed dramatically many areas of our life as well as the practices and contents of science.

As the access to powerful computation increases (and its cost decreases), everybody can perform calculations and simulations that simply were not possible in the past. The same thing happened at data level, indeed it is now possible to build and analyse very large datasets that can collect information and map everything at very micro-level. Thanks to these technological improvements we can then extract more insight about individual's behaviour. We can, for example, exactly know people preferences, their interests and what mainly drive their decision-making process by using big data from internet. Agent-Based model became one important tools in modelling complex systems since we can describe most of phenomena through interactions between different kind of agents and between the agents and the environment that is the landscape where they interact. The environment can be built with real data, it can be otherwise geometric or network-based. These interactions among agents can be quite complex and we can see them as an exchange of information that allow individuals to change their future status and behaviour. In other words, Agent-based models allow us to describe the behaviour of agents internally defined by encoding their behaviour with simple rules and let them interact.

ABMs are then powerful because they allow more interaction between user and its representation since they are easier to modify, dynamic and executable. Moreover, they are easier to understand even for people without a strong computational background and this happens because they use simple rules of individual behaviour closer to natural language and our natural thinking. Agent-based models are perhaps complements to equation-based models that now represent the main domain in economic theory. Indeed, which kind of model is better depends on the particular phenomena we want to describe. Sometimes, equation can easier represent a situation, for example when agents are sufficiently homogeneous or equation are easily solvable, it is not necessary to run a model that can instead require a large number of agents. For example, if we need to measure the temperature in a room it is not necessary tracking the interaction of each molecule.

4.2 Agent-Based Models vs. Equation-Based Models

Here we discuss the main differences between Agent-based models (ABMs) and Equation-Based models (EBMs), which are:

- *heterogeneity vs. homogeneity*:
  one important differences between AMBs and EQMs is the assumption about heterogeneity that plays a key role in most of social science systems. Indeed, ABM allows us to model a heterogeneous population with agents that have different properties among themselves. On the contrary, EBMs usually assume homogeneity in which all agents are identical, with the so called "representative agent". Nowadays, there are even EBMs such us DSGE models that allow the presence of heterogeneity with rational expectations agents, those models are anyway, difficult to solve;
- *discrete vs. continuity*:
  EBMs usually treat the population as continuous. Continuous models do not always map well real-worlds situations since usually the resulting individual interactions are discrete and not continuous. Instead, ABMs allow us to treat a population of discrete individuals as it is in reality;
- *knowledge of aggregate variables*:
  one advantage of ABM with respect to EQM is that they do not require knowledge of the global pattern resulting from individual behaviour. Indeed, to create an ABM it is not necessary to make hypothesis about how aggregate variables will interact. It is just necessary have a good understanding about the behaviour of agents involved and observe the aggregate results running the model. For instance, in our ABM representation we do not know the impact of immigrants in the job market, but we can, anyway, build it and get some new information once it runs. On the contrary, to build an EBM you need to know the relationship between aggregate variables, often derived through difficult system such as stochastic equations;
- *"bottom-up" vs "top-down" approaches*:
  one powerful thing of ABMs is that they can provide details on both individual and aggregate level at the same time. Indeed, they model each individual and its decisions allowing to observe its single evolution in the model as well as the overall results by aggregating them. For example, in our model it is possible to map the history of each household or firm as well as their aggregate outcomes. ABMs use than a "bottom-up" approach that is typically in contrast with the "top-down" presents in many EBMs which does not tell you anything about individual but only how the aggregate system is behaving;
- *randomness vs. deterministic*:
  one of the main advantages of computational models like ABMs is that, in

general, it is easier to include randomness. Indeed, in ABMs each agents can take different decisions based on a probability. For example, in our model we set random differences among household's minimum wages as well as the salaries offered by the firms, in reality each household decides its own minimum wage based on personal characteristics which are different from each individual and it is impossible to include all of these in a model, they are thus replaced by randomness. Therefore, the randomness serves as an approximation of all the factors difficult to determinate that can modify the individual choices such as the differences in minimum wage among individuals. Moreover, this simplification speeds up models construction allowing anyway the further addition of details regarding agent's behaviour. In contrast, EBMs usually require that individuals made their decision deterministically.

4.3 When are Agent-Based Models most beneficial?

We can use ABMs for any natural phenomenon, but they are not necessarily more useful than other modelling techniques, it depends on the context we want to analyze. Indeed, sometimes build an ABM has more costs than benefits but it is not always the case. In general, it is not always easy to understand which type of model is better to implement; anyway, there are some characteristics where ABMs are particularly valuable.

ABMs are particularly useful for problems with heterogeneous agents, especially when this heterogeneity affects overall result of the model. For example, in our model, each household has different preferences about its minimum wage and each firm can decide its own salary that is different from the others. Moreover, we have not to keep track of each single agent characteristics once defined its properties. On the contrary, models with a large number of homogeneous agents usually do not need to bear the full power of ABMs, but detailed equations can better describe the interactions among individuals.

Furthermore, ABMs are more useful in the presence of adaptive agents since they allow individuals to keep a history of interactions and thus change their behaviour based on that. For example, we define in our model that households will decrease their minimum wage if they are not able to find a job for some periods, and at the same time firms change their offered wage based on the labour market conditions. Complex interactions are thus easier to model through and ABM.

The rich conception of time is another advantage of ABMs that allows a more detailed collection of information than equation-based models. This happens because in ABMs the interactions among agents follow a precise order helping the user to go beyond a static snapshot of the system and toward a dynamic evolution of the system's behaviour. In another words, we can get rich and detailed information of

the process step by step and not just the final state. For instance, we can see how many households change their working status each period and not just the final rate of unemployment.

Additionally, there are models with equations very difficult if not impossible to solve. In this case, ABMs are a key tool that allows to see results not reachable otherwise. Even build a ABM can be computationally intensive, for example simulate a system with thousands or millions of individuals can require great computing power. There is then a trade-off depending on the precise model we want to implement that affects our choice regarding which one is better to use.

One of the most powerful tools of ABMs is their ability to determine causality. Since the degree of freedom of ABMs is just limited by the available knowledge of the interest phenomenon and not by the formalism they adopt, it is possible to model a more granular representation of reality. The ability to model realistically a phenomenon plays a crucial role in understanding what are the main mechanisms behind the phenomenon itself, and in which manners they interact during the whole process. On the contrary, in EBMs it is difficult to investigate about endogeneity and how the forces interact inside a phenomenon making its causality determination difficult.

ABMs result particularly useful in economic analysis, where there is still a strong mismatch between theoretical and empirical literature. The problem is that the main theoretical contributions follow the EBMs approach but, due to the complexity of the topics underlined, they often require a strong formalism and they are thus less realistic. It goes the other way with the economical empirical analysis that since usually is not matching with the traditional theory, it is considered inadequate and thus not deployed. The data difference between expected theoretical results and the empirical ones is due to fact that mainstream economic research is still based on the "as-if" principle rendering EBMs incapable to capture the algorithmic nature of behavioural data, giving up a certain degree of realism.

Instead, ABMs, can overcome this trade-off in theoretical and empirical literature by allowing the use of empirical knowledge in theoretical analysis and by helping researches addressing the main questions asked by the contemporary world. Another reason of ABMs power in economic research is their ability to address causality, for example about the developing of policy analyze, it is possible to investigate polices weak spots, their causes and not only its final result. In other words, it is possible give support and suggestion in terms of intervention to decision-makers. Furthermore, the use of heterogeneity can be very helpful in economic analysis, for instance, the traditional economic theory makes a broad use of the representative agents, attributing different behaviour to firms and consumer but they do not allow heterogeneity among agents. In other words, a representative consumer will interact with a representative firm but in which reality all consumers or firms are identical?

In conclusion ABMs have all the characteristics to improve economic both theoretical and empirical analysis by allowing a more realistic representation of phenomena.

5 Model I

The model aim is to replicate some basic economic interactions through an ABM simulation, it is composed of two classes of agents:

- Households
- Firms

Each time t is composed of a cycle of 11 commands that once completed brings to a step ahead of the counter t. The households, if employed, receive a wage from firm and consumes 90% of it, otherwise, if unemployed, they receive a subsidy that is entirely spent. The sum of total households consumption creates the aggregate demand, which is used along with the quantity of goods produced to determine the market clearing price of each period. Only a unique good is produced by the firms and each worker has the same productivity, so the production is determined by the number of workers that each firm belongs. The latter can be adjusted each t, with the possibility to hire or fire based on the profits or losses of $t-1$.

5.1 Initial conditions

We create the initial conditions of this economy with a `setup`, its function set the initial state of the economy before that each cycle can start. Starting with `clear-all` each previous output, variable, plot of the model is reset, no agents are there and ticks are clear. Thereafter, the initial conditions of each agent are created with 100 households and 10 firms randomly located in the world and the values of the parameters are inserted. At the beginning, we set the value of the following parameters:

- `wageAmout = 1`
- `subsidyAmount = wageAmount * 0.5`
- `Productivity = 1`
- `Consume = wageAmount * 0.9`

The workers receive a wage equal to 1, while unemployed households receive a subsidy that is half of if.The productivity per worker has the same value of wageAmount in order to try to keep a clearing price close to 1, making easier the understanding and well-functioning of the model.

5.2 Cycle

The main parts of our economic cycle are:

- *employment*:
  Once `setup` is ready we need to implement the 11 steps which compose an entire cycle of economy. At the beginning there are 2 options:
  - if $t = 0$
    The first command launched by `go` is `firstEmployment`, it acts only during the first cycle of the world so when `ticks= 0`. Here, households are randomly assigned to a firm or left unemployed; this is done by assigning the variable belonging to households `myFirm` to one random firm with a probability of 70 percent or to `Nobody` that means worker is not assigned to any firm with 30 percent of probabilities.
  - if $t > 0$
    In the following cycles or when `ticks>0`, the method `hiringFiring` says to firms if hire a new worker or fire and existing one.

    ```
        to hiringFiring
    if ticks > 0 [
    ask firms [ if profits > 0 [
    if else count households with [myFirm = nobody ] = 0 [ ] [
    ask one-of households with [myFirm = nobody ] [ set myfirm myself ] ] ]
    if profits < 0 [
    ask one-of households with [myFirm = myself ] [ set myFirm nobody ] ] ] ]
    end
    ```

    What firms decide to do with their workers depends on their performance during the previous period. Each firm with profits hires one random unemployed households, if any, as explained before by setting its variable `myFirm` equal to the firm's number. On the other hand firms with losses fire one random worker changing `myFirm` from the firms number to `Nobody`. Firms can then adjust each period their labor force by increasing or decreasing it of 1;

- *wages*:
  the Firms pay wages in the next step by using `firmWages` which first need to understand how many employees they have:

  ```
  set nEmployee count households with [ myself = myFirm ]
  ```

  The Firms identify their workers by counting the number of households which have `myFirm` equal to their identification number `myself`. Thereafter each firm sets the total amount of wages to be paid, `myWages`, equal to its number of employees multiplied by the single wage amount set equal to 1 previously.

  ```
  myWages = nEmployee * wageAmount
  ```

- subsidies:
  in the next step we set the quantity of subsidies to be paid given the present market conditions, in this world firms bear the cost of subsidies for unemployed households by paying a fixed amount for each worker they have. To do that we use the following methods:
  - `generalSubsidy` that first calculates the total burden of subsidies needed by counting all unemployed households, so ones with myFirm = nobody, then multiplying it by the `subsidyAmount` set at the beginning.
  - `singleSubsidy` simply divides `generalSubsidy` by the total number of workers in order to get the subsidy per worker.

  Now that the subsidy per worker has been established, each firm sets its own total amount as:

  ```
  mySubsidies = singleSubsidy * nEmployee
  ```

- *Production*:
  after that firms pay their subsidies and wages, they can set their production. The production of each firm as specified before depends on the number of employee and the productivity, it is so set equal to:

  ```
  Production = nEmployee * productivity
  ```

- *households*:
  the previous part of the model was more about firms functioning, now we focus a bit on the households side to create the demand for goods and close the cycle. First of all, households need to recognize their income and this is done with the following command:

  ```
  to households Income
  ask households [ if myFirm != nobody [ set myWage wageAmount ]
  if myFirm = nobody set mySubsidy subsidyAmount ] ]
  ```

  Here we use the variable `myFirm` and the `NetLogo` primitive function `Nobody` to allow households to link their working status and their income that is `wageAmount` in case they are employed or `subsidyAmount` otherwise;
- *demand*:
  now that households have their own income, we can aggregate it and assemble the demands for good of our world.
  In this model workers consume 90% of their wage while unemployed households consume everything, the value of consume is a parameter set at the beginning during `setup`. The `householdsDemand` is then built by summing all the wages

of households multiplied by their consume plus the sum of all the subsidies of each perdiod t:

$$D_t = W_t * c + S_t \tag{1}$$

Where:

- S is total subsidies amount
- W is the wage aggregation
- c is consumption parameter

- *price*:
  The economy, at this point of t, has a supply which is the firms production, and a demand which is the total consume of households, for goods. We can match them and get the price that is a clearing price since are assumed no warehouses and stocks.

$$P_t = \frac{D_t}{Q_t} \tag{2}$$

  Where Q is the sum of all firms production;
- *profits*:
  we evaluate firms performance in order to see if they can close the current period with profits or losses. To increase the variability of profits we add an exogenous shock to each firms price in the following way:

```
ask firms  [set myPrice price + random-normal 0 0.2 ]
```

  So that each firm adds to its price a value belonging to a normal distribution with mean 0 and variance 0.2. Firms use the variable `myPrice` to calculate their revenues multiplying it with production:

```
set revenues myPrice  production
```

  To get profits the firms need to calculate even their costs as the sum of wages and subsidies and subtract them to revenues:

```
set costs myWages + mySubsidies
set profits revenues - costs
```

  The last step of cycle t is a threshold to firm losses, since a firm can't afford unlimited losses just firing one worker, in the model we set that if profits are negative over a certain value the firm fires all its workers and then close, `die` primite in `NetLogo`.

6 Model II

This model replicates some basic economic interactions happening in a labor market. In particular, with respect to the previous version of the model, we add some powerful tools of ABMs such as heterogeneity among agents and their ability to change their behavior based on the model evolution, our households and firms are then adaptive agents.

More in details, additionally to the previous economic cycle, we add that each firm pays a different wage to its workers. Furthermore, each household has a different expected minimum wage that must be lower than the one offered by the firms in order to accept the job. Agents are adaptive in the sense that they modify their expectations depending on the evolution of the model. For instance, each household that remains unemployed for a certain time period will decrease its minimum wage in order to increase the probabilities to find a job. On the other hand, if firms can easily find the number of workers they want for some consecutive periods they will decrease their wages, on the contrary if they are not able to find enough workers they will increase it.

The initial conditions of this model are quite similar to the ones of Model I, we only add the following variables that allow agents to record their past behavior and adapt it:

- `unemployedTime`: is the variable that households use to track the time period they remain unemployed;
- `notEnoughWorkers`: : each firm utilizes it to record the case that, in the current period, there are not enough workers available in the labor market to accept its conditions;
- `gotAllworkers`:on the contrary, each firm knows whether hires all the workers it wants.

6.1 First cycle

Initially, $t = 0$, workers are randomly assigned to firms or left unemployed, following the previous procedure `firstEmployment`.

Thereafter, firms and households set their wages and expectations respectively, more in details:

- `ask firms [ set wageAmount 0.8 + random-float 0.4 ]`:
  each firm will offer a wage that is 0.8 plus a random number between 0 and 0.4, the latter is different for each firm;
- `ask households [ set minWage  0.8 + random-float 0.4 ]`:

at the same time, households set their expectation on a minimum of 0.8 plus a random number included between 0 and 0.4.

We use similar numbers between firms and households in order to clearly see their adaptive behavior as the simulation progresses, since some of them will not able to achieve their objectives during the first part of the run.

As in the previous version of the model, each firm counts the number of its employees and it obtains the total amount of wages to be paid by multiplying it for its wage. In this version, the total amount will be different even in the case that there are some firms with the same numbers of employees.

The next step is the introduction of agents' adaptation, in particular, regarding households, we add the procedure `changeExpectations`:

```
ask households  [
if myFirm = nobody [
  set unemployedTime unemployedTime + 1 ]
```

During this procedure, every household, if unemployed, increases the value of its variable `unemployedTime`, which, by reaching certain value, will bring to a decrease of its minimum wage. For instance, when a household remains unemployed for four consecutive periods it will decrease its wages of 0.9, 0.8 in the case of five or more periods.

```
if unemployedTime = 4 [ set minWage 0.9 * minWage ]
if unemployedTime >= 5 [ set minWage 0.8 * minWage ]
```

Firms' adaptation works in a similar way, indeed based on the values of `gotAllWorkers` and `notEnoughWorkers` firms decrease or increase their wages. For example:

```
if gotAllWorkers = 3 [set wageAmount wageAmount * 0.9]
if notEnoughWorkers = 3 [set wageAmount wageAmount * 1.1 ]
```

When the variable `gotAllWorkers` reaches the value 3, i.e when the firm got all the workers it needed for three consecutive periods, the firms decreases its wage of 0.9. At the same time, if for 3 periods in a row the firms haven't found the number of workers wanted, it will increase its wage of 1.1.

After this point, the first cycle proceeds similarly to the model I. Indeed, we define the total amount of subsidy per worker to be paid and each firms pays that amount times the number of its workers. Moreover, firms set their production multiplying

number of employee and productivity. At the same time, households recognize their income which is the firm's wage if employed, the subsidy otherwise.

Furthermore, the total households' consumption (90% if employed and everything otherwise) creates the aggregate demand for goods. The aggregate demand for goods divided by the total production of firms generates the clearing price. The cycle finishes with the firms that calculate their profits by subtracting the costs from revenues, if the losses overcome a certain threshold the firm closes leaving its worker unemployed.

6.2 Normal cycle

Since our agents are adaptive, they take some choices based on what happened during the previous run of the model. In particular, from the first period, $t > 0$, firms decide how many workers they want to hire based on the value of their profits with respect to production. This is regulated by the procedure `wantedWorkers`:

```
if profits / production > 0.2 [ set k 3 ]
if profits / production > 0.4 [ set k 4 ]
```

For instance, each firm that has profits between 0.2 and 0.4 of its production, will hire 3 new workers, while if profits are more than 40% of production the firm will try to hire 4 new workers.

How firms try to hire new workers is the most important procedure of the normal cycle and it is regulated by the procedure `hiringFiring`:

```
ask firms
        [ let potentialWorkers count households with [
          myFirm = nobody and
          minWage <= [ wageAmount ] of myself ]
         let kk min list k potentialWorkers
          ifelse kk = k
          [ set gotAllWorkers gotAllworkers + 1 ]
          [ set gotAllWorkers 0 ]
         ifelse kk < k
          [ set notEnoughWorkers NotEnoughWorkers + 1 ]
          [ set NotEnoughWorkers 0 ]
         if profits > 0 and potentialWorkers > 0
          [ ask n-of kk households with
            [ minWage <= [ wageAmount ] of myself ]
            [ set myfirm  myself
              set myWage [ wageAmount ] of myself
```

```
    set unemployedTime 0 ]
  ]
```

During this procedure, each firm first looks into the job market how many households are unemployed and willing to accept its wage.

Furthermore, the firm sees if the number of workers it wants to hire (set before during `wantedWorkers`) is at least equal to the number of workers available in the market. In this case, the firms is able to hire entirely the workers it wants and tracks it through the variable `gotAllWorkers`.

On the contrary, if there are not enough workers available in the market, the firm hires all the workers it can, and tracks that it was not able to find all the workers needed using the variable `notEnoughWorkers`. Finally the firm and its new workers are linked, with workers that recognize their new wage and the status of employed.

The rest of the cycle continues as the first cycle.

7 Model III

In this model, there are two different type of agents:

- firms;
- households.

Furthermore, households are divided in high-skill and low-skill and they interact with firms following basic economic rules.

Households, when employed, receive a wage from the firms and consume a fraction of it; otherwise, they get an unemployment benefit entirely spent over each period. The total consumption of employed and unemployed households defines the aggregate demand for goods. This, divided by the total production of firms determines the market price.

Firms produce only one good and their amount depends on the number of workers and their skills. Indeed, high-skill workers have a higher productivity as well as higher wages than low-skill ones. Firms, for each period, looking at their previous performance contract or expand their workforce hiring new workers or firing existing ones. Nevertheless, if profits overcome a certain threshold a new firm is created, while, on the other hand, firms go bankrupt due to heavy losses. Moreover, for each period, firms adjust their wages based on the availability of unemployed households. At the same time, households' minimum wage decrease if they are not able to find a job for some periods.

7.1 Initial conditions

At first, the model works by creating a defined number of firms and households and a certain number of them is immediately connected. Each firm can recognize its workers and each worker knows which firms is working for. In this section, we examine the initial agents' characteristics and first interactions, which are:

- households:
  households' main attributes initialization are the following:

```
  to setup-households                                    ;
create-households nHouseholds
[
  set myFirm nobody
  ifelse random-float 100 > highSkilledHouseholdOnPopulation
  [ set householdType 1
    set color green
    set minWage random-normal avMinWageLowSkill stDevMinWageLowSkill ]
```

```
    [ set householdType 0
      set color red
      set minWage random-normal avMinWageHighSkill stDevMinWageHighSkill ]

  ]
end
```

We can set the number of households living in our world through `nHouseholds`, once settled all households start as unemployed. The most important attribute of this agent class is its skill level that can be high or low. The average percentage of high skilled households among the total population depends on the value of `highSkilledHouseholdOnPopulation` set before running the model. Once households are assigned to their skill level, they set their minimum wage. The minimum wage is different for each agent; it belongs to a normal distribution with mean `avMinWageLowSkill` and standard deviation `stDevMinWageLowSkill` for low-skill, and mean `avMinWageHighSkill` and standard deviation `stDevMinWageHighSkill` for high-skill, all these value can be set in a different way to examine the model's evolution.

- *firms*: the next step is to create firms, through the following procedure:

```
    to setup-firms
  create-firms nFirms
  [
    set highWageAmount random-normal avHighWageAmount stDevHighWageAmount
    set lowWageAmount  random-normal avLowWageAmount stDevLowWageAmount
    set hireTot round random-normal averageWorkers stDevAverageWorkers
    set hireHigh  round ( hireTot * highSkilledWorkersRatio / 100 )
    set hireLow hireTot - hireHigh
  ]
end
```

As for households, we can decide the number of firms initially operating in this economy through `nFirms`. The main features for firms' initialization are the wage setting, how many workers they want to hire and how they split them between high and low skill workers. More in details:

  - `highWageAmount` and `lowWageAmount` are the wages offered by the firm respectively to its high-skill and low-skill workers as well as to the unemployed households available in the labor market and belonging to the same skill level. Each firm offers different wages with respect to each

other, the values value are set following a normal distribution with mean `avHighWageAmount` and standard deviation `stDevHighWageAmount` for high skilled workers, the same happens for low skilled ones;
  - the number of workers that each firm wants to hire during the current period is `hireTot`. Initially, the amount of workers is set as previous variables following a normal distribution, so that each firm needs a different number of workers. Thereafter, `hireTot` will be set based on the firm's profits with respect to its production;
  - each round of assumption firms need to decide how divide the whole number of workers they want between high and low skill. In order to do that the variable `highSkilledWorkersRatio` sets the percentage of high-skill workers out of the total, the residual amount will be the number of low-skill workers. The value of these variables is flexible, we can then change it before or during each run and it might significantly change model's results;

- *hiring*:
  Once that we have created firms and households we need to connect the firms to their workers. This happens through the procedure `hiring`, we now analyse how it works for high skill household but it is specular for low skill ones:

```
if profits > 0 [
  let potentialHighWorkers count households with [
   myFirm = nobody and
   minWage <= [ highWageAmount ] of myself
   and householdType = 0
   ]
  let HH min list hireHigh potentialHighWorkers
  ifelse HH = hireHigh
  [ set gotAllHighWorkers gotAllHighworkers + 1 ]
  [ set gotAllHighWorkers 0 ]
  ifelse HH < hireHigh
  [ set notEnoughHighWorkers NotEnoughHighWorkers + 1 ]
  [ set NotEnoughHighWorkers 0 ]
  if profits > 0 and potentialHighWorkers > 0 [
    ask n-of HH households with [
     minWage <= [ highWageAmount ] of myself and
     householdType = 0 and
     myFirm = nobody]
    [ set myfirm  myself
      set myWage [ highWageAmount ] of myself
```

```
      set unemployedTime 0
    ]
   ]
  ]
```

This procedure works whenever a firm had positive profits in the previous period and wants to hire new workers in the current one. Since the model has not started yet we assume that firms start with a small positive profits when created, such as small initial investment, in order to join the cycle. Furthermore, the variable `potentialHighWorkers` identifies all the firm's potential high skill workers through the fulfillment of three requirements:

- they must be unemployed;
- they have to belong to the skill level required;
- they must have a minimum wage less than the one offered by the firm.

Now the firm knows how many workers would accept its offer, then it chooses the minimum number between the number of workers it wants to hire, (previously defined `hireHigh`), and the ones available in the market. If the workers available to accept firm's offer in the labor market are less than the number wanted by the firm, it will hire as much as it can. The variable `notEnoughHighWorkers` tracks that for the current period the firm is not able to find entirely all workers it needs. On the contrary, if the workers available in the market are more or equal the number searched by the firm, it can hire the entire amount `hireHigh`. In this case, the variable `gotAllWorkers` records that for this period the firm found all the workers needed. The variables `notEnoughHighWorkers` and `gotAllHighWorkers` record wheter the firm is able or not to find all the workers it wants during the evolution of the model. This tracking allows each firm to adapt itself to the market conditions increasing or decreasing the wage offered to its workers. The last step is to link firms and households, each firm hires the final number of unemployed households, in this case `LL`, by setting their variables `myFirm` equal to its number and `myWage` equal to its wage. The workers are then able to recognize the firm they are working for. Firms follow exactly the same procedure for low-skill workers (see appendix A).

7.2 First cycle

Once that households and firms are set and linked to each other, the first cycle of the model can begin. Its main parts are:

- *agents' adaptation*:

firms and households are adaptive agents, this means that they change their behaviour based on the evolution of the model. For example, as we showed before in the procedure, firms use the variable `gotAllLowWorkers` and `notEnoughLowWorkers` to record whether they hire all the low-skill workers they need or not, in the event they are not enough in the labor market, the same happens for high-skill workers. Furthermore, firms look at the value of these variables and modify their wages. For instance:

```
if gotAllHighWorkers = 4 [ set highWageAmount highWageAmount * 0.9 ]
```

Means that a firm will offer only the 0.9 of the previous wage to its high-skill employees and potential workers, if for three periods in a row it is able to hire all the high-skill workers it needs from the market. On the contrary, a firm will increase its wages if it is not able to find the whole amounts of workers it wants for some consecutive periods:

```
if notEnoughLowWorkers = 3 [ set lowWageAmount lowWageAmount * 1.1 ]
```

Moreover, the wage adaptation to market conditions can follow a different or even inverse path for high and low-skill workers. Indeed, it can happen that just one skill type is scarce in the market while the other is abundant, bringing to a wage increase for one skill and a decrease for the other one. The same do households, they change their minimum wage if they are not able to find a job for some periods using `changeExpectations` :

```
    to changeExpectations

  ask households  [
    if myFirm = nobody [ set unemployedTime unemployedTime + 1 ]
    if minWage > subsidyAmount [
     if unemployedTime = 3 [ set minWage 0.9 * minWage ]
     if unemployedTime = 4 [ set minWage 0.9 * minWage ]
     if unemployedTime >= 5 [ set minWage 0.8 * minWage ]
     ]
    ]
  end
```

When a household is unemployed it increases by one its variable `unemployedTime`. For instance, when `unemployedTime` is equal to 3, it means the household was unemployed for three periods in a row and its minimum wage starts to decrease. For both households and firms, we set some thresholds to their minimum wage and wage offered. For instance, households never decrease their minimum wage

below or equal to the subsidy amount and firms will never offer less. This in order to stay as closer as possible to reality, indeed a person will never accept a job if it can take the same of money without working. We set even an upper threshold to firms' wages because having fixed productivity for each skill type a wage amount too high is not consistent.
By this representation agents are able to adapt their behaviour to the market conditions and model evolution, getting a better representation of how they behave in the real world;

- *wages*:
  the Firms identify their workers by counting the number of households for each skill level which have `myFirm` equal to their identification number `myself`:

```
set nHighEmployee count households with
[ myself = myFirm and  householdType = 0 ]
set nLowEmployee  count households with
[ myself = myFirm and  householdType = 1 ]
```

  Thereafter each firm sets the total amount of wages to be paid, `myWages`, equal to the total amount of its employees multiplied by its wages:

```
set myWages ( lowWageAmount * nLowEmployee ) +
            ( highWageAmount * nHighEmployee )
```

  Moreover, we set several colours depending on the firm's size, its total number of employees, in order to get a better representation in the interface and help the user;

- *subsidies*:
  in the next step we set the quantity of subsidies to be paid given the present market conditions, in this world firms bear the cost of subsidies for unemployed households by paying a fixed amount for each worker they have. To do that we use the following methods:
  - by counting all unemployed households, so ones with `myFirm = nobody`, then multiplying it by the `subsidyAmount` set at the beginning;
  - `single subsidy` simply divides `generalSubsidy` by the total number of workers in order to get the subsidy per worker;
  - now that the subsidy per worker has been established, each firm sets its own total amount as:
    ```
    set mySubsidies singleSubsidy *
    ( nHighEmployee + nLowEmployee )
    ```

- *production*:
  after firms paying their subsidies and wages, they can set their production. The production of each firm depends on the number of employees and their productivity, which depends on skill level and can be set at the beginning or changed during the run, it is then equal to:

```
set production   ( nHighEmployee * productivityHigh +
                   nLowEmployee * productivityLow )
```

- *households*:
  the previous part of the model was more about firms functioning; now we are focusing a bit on the households' side to create the demand for goods and close the cycle.
  First of all, households need to recognize if they are unemployed and this is done with the following command:

```
ifelse myFirm = nobody  [ set mySubsidy subsidyAmount ]
                        [ set mySubsidy 0 ]
```

  In this model workers consume a fraction of their wage while unemployed households consume the subsidy entirely. The value of consume is a parameter set in the interface and can be modified during the model's evolution. We mainly assumed that employed households consume 90% of their wage. The `householdsDemand` is then built by summing all the wages of households multiplied by their consume plus the sum of all the subsidies of each perdiod t:

$$D_t = (W_{h\,t} * c) + (W_{l\,t} * c) + S_t \tag{3}$$

  Where:

  - S is total subsidies amount
  - W_h is the wage aggregation of high skill workers
  - W_l is the wage aggregation of low- skill workers
  - c is consumption parameter

- *price*:
  the economy, at this point of t, has a supply which is the firms production, and a demand which is the total consume of households, for goods.
  We can match them and get the price that is a clearing price since no warehouses and stocks are assumed.

$$P_t = \frac{D_t}{Q_t} \tag{4}$$

  Where Q is the sum of all firms production.

- *profits*:
  we evaluate firms' performance in order to see if they can close the current period with profits or losses. To have heterogeneity among firms' prices we add an exogenous shock to each firms price in the following way:

  ```
  ask firms  [set myPrice price + random-normal 0 0.1 ]
  ```

  So that each firm adds to its price a value belonging to a normal distribution with mean 0 and variance 0.1. Firms use the variable `myPrice` to calculate their revenues multiplying it with production:

  ```
  set revenues myPrice * production
  ```

  To get profits the firms need to calculate even their costs as the sum of wages and subsidies and subtract them to revenues:

  ```
  set costs myWages + mySubsidies
  set profits revenues - costs
  ```

7.3 Normal cycle

During the normal cycle, `ticks > 0`, firms and households change their behaviour and take decision based on what happened in the previous periods. In particular, they decide how many workers they want to hire or fire, if close the firm or open a new one. In particular:

- *workers turnover*:
  in this model, a firm hires new workers if in the previous period it had positive profits. The procedure `wantedWorkers` sets how many workers it wants to hire and in which ratio between high and low skill. Moreover, we decided to set the number of workers on the ratio between profits and production, for instance:

  ```
  if profits / production > 0.2 [ set hireTot  3 ]
  if profits / production > 0.3 [ set hireTot  4 ]
  ```

  Means that if firm's profits are more than 20% of the production it will try to hire 3 additional workers during this period while if profits are more than 30% of its production, the firm will seek for 4 workers.
  As we showed at the beginning `hireTot` represents the total amount of new workers a firm wants to hire, and it is split in high and low skill through `highSkilledWorkersRatio`.
  On the contrary, if a firm recorded losses in the previous period, it needs to fire some workers. The procedure `cuttingWorkers` serves this purpose and it works in a specular way with respect to `wantedWorkers`, for example:

```
if profits / production < -0.1 [ set fireTot  2 ]
```

Means that if losses are at least 10% of firm's production, the firms will fire a total amount of 2 workers. Furthermore, again `highSkilledWorkersRatio` divide the number of fired workers, `fireTot`, between high and low skilled;

- *opening and closing*:
  if firm's profits of the previous period overcome a certain threshold it will open a new firm. Moreover, we set this threshold to half of the production, changing this parameter we can get different results from the model.
  The new firms follow the same procedure as the firms did at the beginning when they were established. Thus, the firm sets its wage amount for high skill workers and for low skill ones as well as the number of workers it wants to hire and how split them according to their skill levels. Furthermore, the firm will check if in the market there are enough workers available that satisfy its conditions, if so the firm hires them and enter in the world otherwise can not open and immediately `die` (see Appendix A).
  At the same time, if a firm has unaffordable losses it has to fire all its workers and close, we set that this happens is a firm has more losses than its production:

```
if  ( profits / production ) < -1 [
 ask households with [myFirm = myself] [set myFirm nobody]
 die
]
```

- *hiring and firing*:
  the most important procedure of this model is how the firms hire and fire their workers. We already showed under the initial conditions how firms hire their worker, during the cycle they follow the same procedure `hiring` but using `hireHigh` and `hireLow` provided by `workersAmount` instead of `averageWorkers`.
  On the other hand, the procedure `firing` allows firms to fire some of their workers in case of losses. `firing` works like `hiring` and it does the same process twice, one for high skill workers and one for low skill ones. The procedure works through the following steps:
  - if a firm has losses, < 0, it counts the number of workers for skill level it currently has;

```
let myLowWorkers count households with
 [ myFirm = myself and  householdType = 1 ]
```

- afterwards, the firm takes the minimum number between `fireLow`, recorded during the procedure `cuttingWorkers`, and `myLowWorkers`, got in in previous row;

  ```
  let FL min list fireLow myLowWorkers
  ```

- to conclude, the firm fires the number of workers it would like if they are more than its actual employees, otherwise it fires all its workers remaining with only one. We set this threshold to avoiding firms remaining without workers, we decided that a firm closes only if its losses go over a certain value with respect to its production.

  ```
  ifelse  FL = myLowWorkers and FL > 0
  [  ask n-of (FL - 1) households with
   [myFirm =  myself and householdType = 1 ]
   [ set myFirm nobody ]
  ]
  [ ask n-of FL households with
   [myFirm =  myself and householdType = 1 ]
   [ set myFirm nobody ]
  ```

After this part the cycle continue as a normal first cycle, starting from the agent's adaptation to the market conditions.

8 Results Model III

In this section we evaluate the effectiveness of the model and present some results. The section is divided in two main parts: the first one regards the outcomes of the simulations through a well dened set of parameters, which can be found in table (1); the second one presents the sensitivity analysis of the model to the above-mentioned parameters.

8.1 Baseline model

The following results arise from simulations with `nHouseholds`= 1000, `Firms`= 100 and over a fixed number of 200 cycles. The latter choice deals with the interpretation of a time step: given that firms' behaviour involves hiring, firing and production plans, it seems valid to assume a cycle to be equivalent to month. This interpretation does not actually affect the households, which in the model perceive a certain wage and consume a fraction of it: as a matter of fact, one could take this behaviour as an intensive property of a household.

We compose our population with 40% of high skill workers, OECD average of graduate total population is 37%. Moreover, firms hire 55% high skill workers and 45% low skill ones, this is because nowadays the labour demand for high skill workers is higher. On the contrary the ratio of fired worker is split equally between high and low skill. At the beginning, in order to keep the model balanced we set parameters in a equilibrate way (See Figure 1). Indeed, the minimum wages of households are on average 1.1 and 0.8 for high and low skill respectively, both with a standard deviation of 0.1. At the same time, firms offer on average a wage of 0.8 and 1.1, even in this case with a standard deviation of 0.1. Moreover, to keep the price close to 1 we set a productivity for high skill workers of 1.1 while it is 0.8 for low skill ones. When employed households produce, on average, as much as they get and consume 90% of their wage, otherwise they consume 100% and the clearing price starts really close to one. Only the shock on the single firm's price can affect it at the beginning while hereafter even the wage adaptation of firms to the market conditions.

We now summarize the main results of the model on macroeconomics variables:

- *unemployment rate*:
  the average unemployment rate is about 5%, but it decreases over time, its average is 6% in the first half ($t < 100$) of the run while it is 4% in the second half; this decrease is due to the adaptation of firms and households to the labour market conditions . Moreover, if we split the unemployment rate by skill type we see that high skill workers average unemployment rate (red line Figure 2) is 4% while for low skill households it is 6%, anyway they are both decreasing over time. This is consistent with the parameters of the model; indeed, firms hire

nHousehlds	1000
nFirms	100
consumption	0.9
highSkilledHouseholdOnPopulation	50%
highSkilledWorkersRatio	55%
fireHighWorkersRatio	50%
averageWorkers	10 ± 2
productivityHigh	1.1
productivityLow	0.8
wageAmountLowSkill	0.8 ±0. 1
wageAmoutHighSkill	1.1 ± 0.1
minWageLowSkill	0.8 ± 0.1
minWageHighSkill	0.8 ± 0.1
Number of cycle	200

Figure 10: *Baseline parameters*

more high skill households (55% vs. 45%) but fire equally (50%) and they are even a minority in the population (40%);

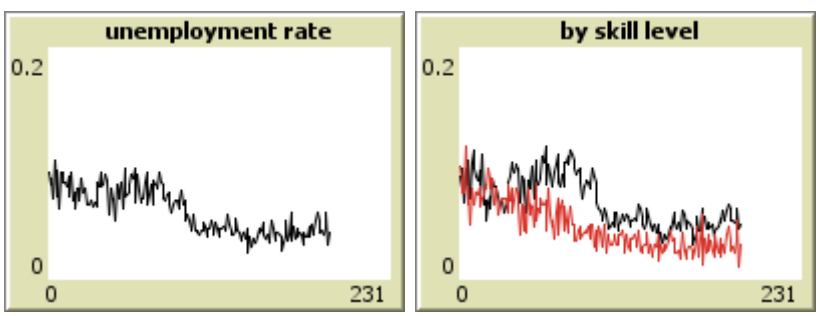

Figure 11: *Unemployment rate*

- *wages*:
  as we specified before, firms must adapt their wages to the market conditions. This adaptation is consistent with the unemployment rate evolution. Indeed, since there is a shortage of high skill workers, firms have to increase their wages to hire them (from an average of 1.1 to an average of 1.3). On the contrary, low skill workers must decrease slightly (from an average of 0.8 to 0.7) their minimum wage since they face more competition to get a job;

Figure 12: *Wages evolution of low (black) and high (red) skill*

- *firms*:
  the turnover of firms is slightly negative, they decrease of 3%. Firms are 97 at the end of the run, because employed households do not consume entirely their wage but only 90% of it. For the same reason profits have bigger negative peaks but, on average, they are almost null since the shock that affects firm's price has mean 0 and standard deviation 0.1. As a matter of fact, firms decrease on number, but they increase their size. At the beginning each firm had an average of 10 workers while at the end of the run there are 5 firms with more than 50 employees, 8 with more than 25, 63 between 25 and 10 while only 21 firms have 10 employees or less;

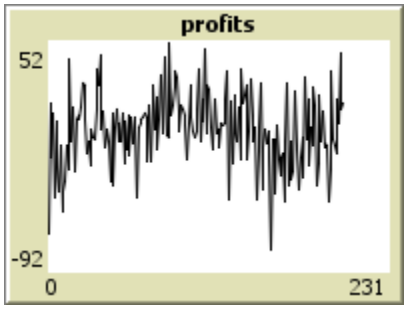

Figure 13: *profits of firms*

- *global price and GDP*:
  despite the small decrease of firms, the GDP, that we defined as the total production of firms, increase of 7% during the entire run. This is correlated with the decrease of unemployment rate, since firms have more workers and produce more. The real GDP (GDP / price) increases even more (+10%) since the market price decreases about 15%. Indeed, it starts from an average of 1.01 at $t = 1$ and it slightly but constantly decreases up the final average of 0.85. We have a decrease of price because of the employment rate increase. Indeed, there are more households which does not consume entirely their wage (while,

when unemployed, they consume entirely the subsidy) so the aggregate demand increases less proportionately than the quantity, bringing the price to a final lower level;

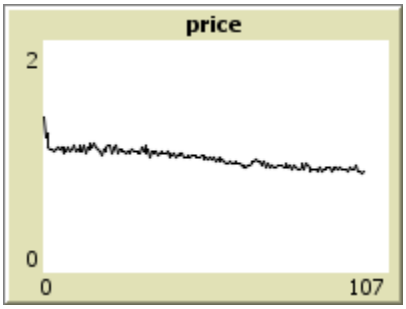

Figure 14: *price*

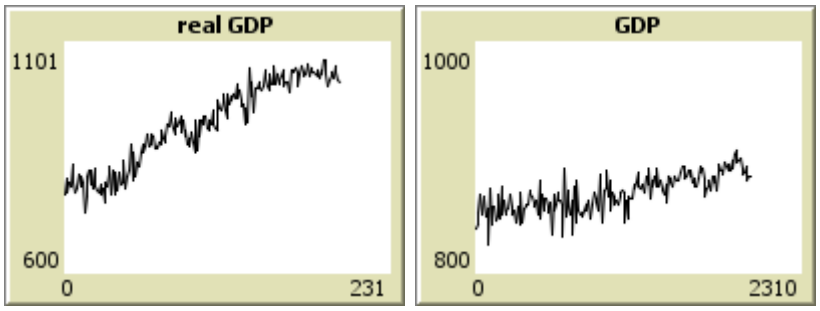

Figure 15: *GDP*

8.2 Sensitivity analysis

The model turns out to be robust to the variation of the initial parameters in figure 1 giving reliable expected outcomes.

8.2.1 Job creation and population

First, increasing ratio of high skilled workers hired by firms to 75% of the total (`highSkilledWorkersRatio`) we get catastrophic results in our economy. Indeed, there is a constant increase of unemployment rate over time (average 30%); when we split it by skill type we see that almost 50% of low skill households are unemployed while only 1% of high skill are.

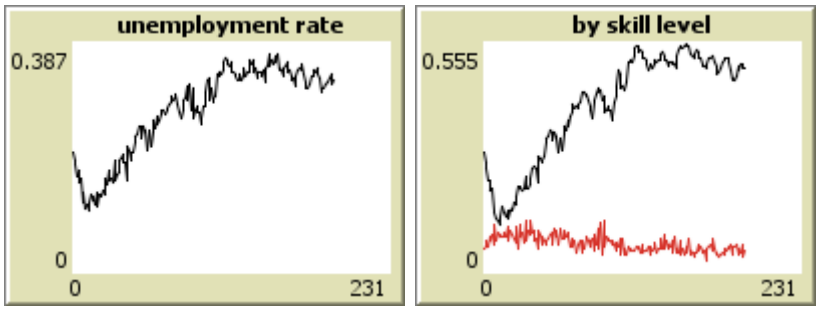

Figure 16: *Unemployment rate*

The increase of unemployment rate is related to the decrease of new job opportunities for low skill unemployed households (only 25% of new job creation) keeping constant the other variables such as their share out of the total population and initial minimum wages. Firms are then able to decrease their wage for low skill households, while they have to strongly increase the ones for high skill that now have more job opportunities.

Figure 17: *Wages of low (black) and high (red) skill*

At the same time, the increase of unemployment rate brings GDP to tumble over time, indeed firms production is proportional to their workers, that are constantly decreasing. On the other hand, the price increases proportionally to the decrease of production.

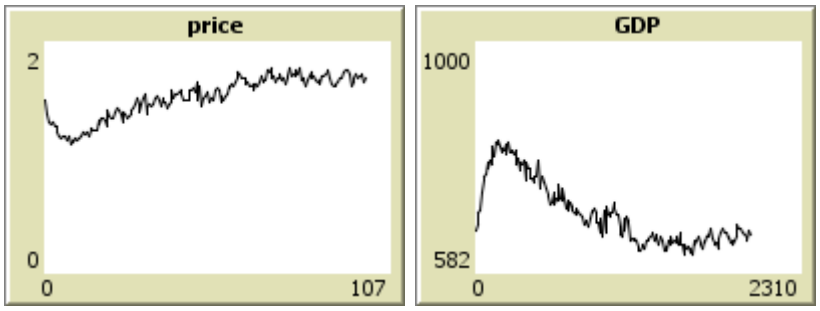

Figure 18: *Price and GDP*

The decrease of job opportunities for low-skill workers keep other variables constant and it mirrors nowadays scenario. Indeed, the raise of artificial intelligence and robotic is bringing the same results in many countries that are now facing high rates of unemployment among their low qualified population. On the contrary, if we increase even the ratio of high skill workers on total population (`highSkilledHouseholdOnPopulation`) to 75% we partially offset the negative results previously obtained. Indeed, we have an employment rate of 10% and a positive economic growth. The latter results show how much is important the match between population's skill and the ones required by the job market.

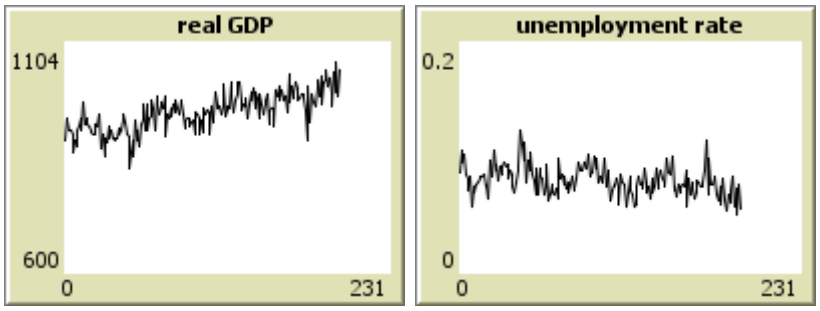

Figure 19: *Price and GDP increasing high skill households*

8.2.2 Wages

Furthermore, after restoring initial settings, we can analyze model sensitivity with respect to wages. First, we increase the average minimum wage of both high and low skill households up to 1.5 and 1.2. The results are consistent with our expectations, indeed there is a high employment rate during the first part (70% of unemployed households at $t = 10$) of the run since households have minimum wages too high compared to the ones offered by firms. Anyway, during the run, households that do not find a job decrease their minimum wage and this brings to market to its normal conditions in the long run.

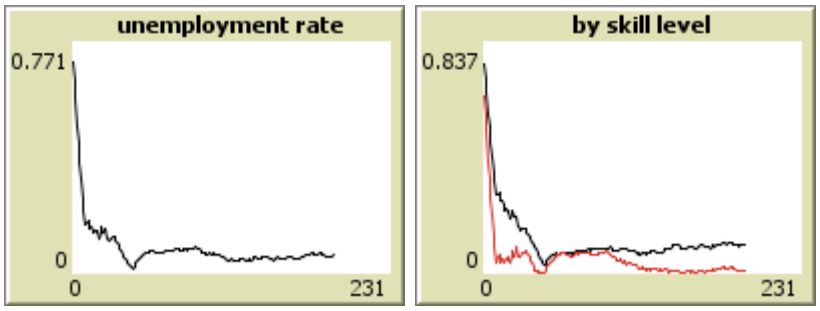

Figure 20: *Unemployment rate with the variation of wages*

As before, the initial high unemployment rate brings to a low production of firms and perhaps a low GDP level as well as an higher price since the demand, driven by the subsidies of unemployed households, is a way higher that production. Anyway, all these variables come back to their normal level once households have adjusted their wage to market conditions.

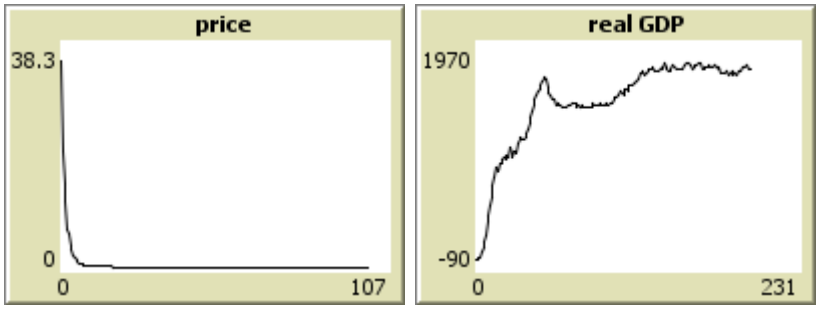

Figure 21: *Price and GDP with the variation of wages*

8.2.3 Productivity

Moreover, if we increase workers' productivity up to 1.5 and 1.1 for high and low skill workers respectively, keeping other settings constant we see the average production of firms increases (GDP). The boost of productivity has even a positive impact on profits which before were zero, on average, because workers productivity was set equal to their costs. In this model, workers productivity is no linked to their wages so that firms raise their production, keeping costs fixed with a positive effect on total GDP.

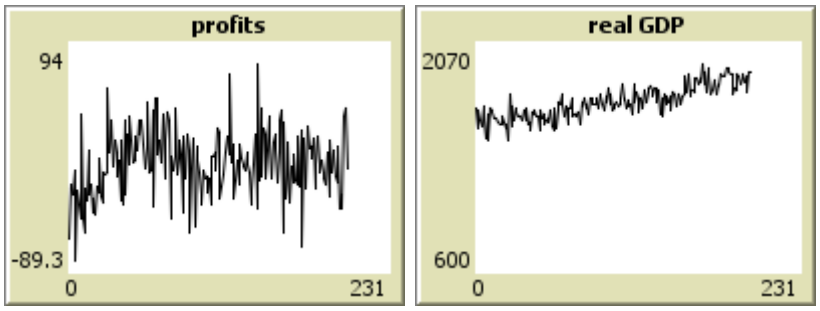

Figure 22: *Real GDP and profits increasing productivity*

8.3 Conclusions

The main aim of the work was to present an economic model able to reproduce a dual labour market of a country with reliable characteristics. This has been accomplished through an agent based approach, with the agents being households and rms. The main achievement of this model is that it represents a closed economic system which depends entirely on the (bounded) rationality of the agents. The simulation experiments carry out the importance of skill matching between demand and supply in the labour market and how some categories are likely to gain and some to loose by eventual differences. Moreover, it shows a stability in production and pricing . Though, the model is a baseline for further improvements: as a matter of fact, it could be expanded with the involvement of other types of agents such as banks and, at the same time, the public administration. In particular a mechanism able to capture the fraction of consume saved by households would be included.

9 Model IV

9.1 The Model

In this last version of the model we have three different type of agents:

- firms;
- natives;
- immigrants.

Furthermore, both immigrants and natives are divided in high-skilled and low-skilled but with different percentages. Moreover, they are merged together as households and they interact with firms following basic economic rules.

During the economic cycle, each household, when employed, receives a wage from firm and consume a fraction of it. Otherwise, it gets an unemployment benefit, always bear by firms that is entirely spent over each period.

The total consumption of employed and unemployed households plus an amount equal to average household savings (10% of their wage) defines the aggregate demand for goods.

This number, divided by the total production of firms determines the market-clearing price.

Firms produce only one goods and their amount depends on the number of workers and their skills. Indeed, high-skilled workers have a higher productivity as well as higher wages than low-skilled ones. Inside the same skill level, immigrants have a smaller minimum wage with respect to their natives counterparty. For instance, a high skilled native has, on average, a higher minimum wage than a high skilled immigrant but both of them have, on average, a minimum wage higher than low skilled workers.

Our agents are adaptive, firms check their previous performance contract every period or expand their workforce hiring new workers or firing existing ones.

Nevertheless, if profits overcome a certain threshold they create a new firm, while, on the other hand, firms go bankrupt due to heavy losses. Moreover, for each period, firms adjust their wages based on the availability of unemployed households. At the same time, households' minimum wage decrease if they are not able to find a job for some periods.

In order to simulate the effect of cutting immigration on the labour market we add to this last version of the model the probability that between the period 50 and 80 unemployed immigrants and firms can leave the market.

9.2 Initial conditions

At first, the model works by creating a defined number of firms, immigrants and natives. Furthermore, we merge natives and immigrants as households then a certain number

of them is immediately connected with firms. Each firm can recognize its workers and each worker knows which firms he is working for. In this section, we examine the initial agents' characteristics and first interactions.

Regarding natives, their main features are:

```
    to setup-natives
  create-natives nNatives
  [
    set myFirm nobody
    ifelse random-float 100 > highSkillnatives
    [ set householdType 1
      set color green
      set minWage random-normal avMinWageLowSkill stDevMinWageLowSkill ]
    [ set householdType 0
      set color red
      set minWage random-normal avMinWageHighSkill stDevMinWageHighSkill ]


  ]
End
```

We can set the number of natives living in our world through `nNatives`, once settled all natives start as unemployed. The most important attribute of this agent class is its skill level that can be high or low. The average percentage of high skilled natives among the total population depends on the value of `highSkilledNativesOnPopulation` set before running the model. Once we assign natives to their skill level, they set their minimum wage. The minimum wage is different for each agent; it belongs to a normal distribution with mean `avMinWageLowSkill` and standard deviation `stDevMinWageLowSkill` for low-skill, and mean `avMinWageHighSkill` and standard deviation `stDevMinWageHighSkill` for high-skill, all these values can be set in a different way to examine the model's evolution.

The setup of immigrants is quite similar to natives, indeed:

```
 to setup-immigrants
    create-immigrants nImmigrants
  [
    set myFirm nobody
    ifelse random-float 100 > highSkillImmigrants
    [ set householdType 1
      set color yellow
```

```
      set minWage random-normal avMinWageLowSkillImm stDevMinWageLowSkillImm ]
    [ set householdType 0
      set color white
      set minWage random-normal avMinWageHighSkillImm stDevMinWageHighSkillImm ]


  ]

  set households (turtle-set natives immigrants)
end
```

In this case, we set the number of immigrants with `nImmigrants`, this procedure works in the same way as for natives, the only difference is that we can set a different ratio of high and low skill on our immigrants' population, using `highSkillImmigrants`. Moreover, both immigrant's skill levels have a different average minimum wage and standard deviation with respect to their native counterparties. In particular, those are set with `avMinWageLowSkillImm` `stDevMinWageLowSkillImm` for low skill and `avMinWageHighSkillImm` `stDevMinWageHighSkillImm` for high skill.

Furthermore, we merge natives and immigrants as households. This is a fundamental part of our process because we can treat immigrants and natives as a unique population (households) during the evolution of the model.

Firms' initial conditions and the way they hire their workers is equal as we did in the previous version of the model (see Model III for explanation and Appendix D for full code).

9.3 Run

The economic cycle works in a quite similar way than the one presented in the previous version of the model (see Model III). The main difference is that households were one of our agents while now households are two classes of agents, natives and immigrants, merged together (i.e. before households were only our actual natives).

In other words, this version differs from the previous ones only when natives and immigrants are treated in a separate way during the evolution of the model. In particular, this happens during:

- *wages*: Since immigrants and natives can have a different minimum wage and productivity, each firm needs to know how many of them for each skill type are working for it. The `firmWages` procedure then works as following:

  ```
  set nHighNatEmployee count natives with
      [ myself = myFirm and  householdType = 0 ]
  ```

```
set nLowNatEmployee  count natives with
[ myself = myFirm and  householdType = 1 ]
set nHighImmEmployee count immigrants with
[ myself = myFirm and  householdType = 0 ]
set nLowImmEmployee  count immigrants with
[ myself = myFirm and  householdType = 1 ]

 set myWages  ( ( lowWageAmount * nLowNatEmployee ) +
                (lowWageAmount * nLowImmEmployee ) +
                ( highWageAmount * nHighNatEmployee ) +
                (highWageAmount * nHighImmEmployee)  )
```

Anyway the firm does not make any distinction in the wages paid to immigrants and natives, they will be always `highWageAmount` for high skilled households and `lowWageAmount` for low skilled ones;

- *subsidies*: Since in this model firms bear the cost of unemployment subsidies by paying a fraction for each worker they have, we need to count every employed households, both immigrants and natives. Indeed :

```
 set singleSubsidy totalSubsidies /
 (sum [ nHighNatEmployee ] of firms +
  sum [ nLowNatEmployee ] of firms +
  sum [ nHighImmEmployee ] of firms +
  sum [ nLowImmEmployee ] of firms )
```

Then, each firm sets its own amount of subsidies to be paid by multiplying the subsidy per worker for its number of employee (both immigrants and households):

```
set mySubsidies singleSubsidy * (
  nHighNatEmployee + nLowNatEmployee +
  nHighImmEmployee + nLowImmEmployee )
```

There are no distinctions of subsidy to be paid between a native and immigrant worker;

- *production*: The distinction between natives and immigrants results fundamental when firms set their production. Indeed, each group of households can have a different productivity for each skill level, the productivity values are `productivityHigh` and `productivityLow` for natives and `productivityLowImm` and `productivityHighImm` for low and high skilled immigrants respectively.

Furthermore, when firms set their production they need to count how many workers they have for each skill type and for each group of households:

```
ask firms [
  set production   ( ( nHighNatEmployee * productivityHigh ) +
                     ( nLowNatEmployee * productivityLow  )+
                     ( nLowImmEmployee * productivityLowImm ) +
                     ( nHighImmEmployee * productivityHighImm ) )
```

Moreover, we add the following procedures to our models:

- *Demand*: since the previous versions of the model were not taking into account the fraction of wage saved by households we add an exogenous variable quantity to the demand for goods. This amount represents households' savings they do not spend while they get their wage but in the future. Since employed households consume 90% of their wage each period we set this quantity, on average, of 10% of the total amount of wages received by workers. We define the amount with a random variable with mean 10 and standard deviation 2, that simulates a random investment outcome that can be slightly positive or negative. More in details:

```
set demand  (
 ( sum [ myWage ] of households * consumption ) +
 ( sum [ mySubsidy ] of households ) +
 ( sum [ myWage ] of households / 100 * random-normal 10 2 )
```

- *leaving*: in order to simulate the effect of Brexit in the UK labour market we add that for 30 cycles of our run (2,5 years) there is the possibility that each unemployed immigrant can leave the market as well as some firms can abandon it by relocating their production abroad. In particular, between the period $80 > t > 50$ the procedure `leaving` works in the following way:

```
to leaving

  ask immigrants with [ myFirm = nobody ] [
    if random-float 100 > immigrantStaying [ die ]
  ]
  ask firms [
    if random-float 100 > firmStaying [
```

```
      ask households with [myFirm = myself] [set myFirm nobody]
       die
     ]
   ]
End
```

The probabilities of both unemployed immigrants and firms to stay in the market for each of these 30 cycle are set with the variables `immigrantSaying` and `firmStaying` respectively. This procedure allows us to simulate the shock due to Brexit that is likely to see several immigrants labour force and related firms leave the market.

10 Results Model IV

In this chapter we evaluate the results of the model trying to measure its effectiveness over a baseline scenario and some possible variations. Since there is a lot of uncertainty surrounding the Brexit process we try to assess some possible economic impacts by varying one or more parameters for each scenario presented and test results' sensitiveness. We choose the scenarios based on the literature results we got and on the most recent forecasts of UK's decisions.

First we present a brief description of the scenarios we chose and then we focus on the model's results of each of them.

Each scenario's results arise from simulations with `nHouseholds`= 10000, `Firms`= 1000, `nImmigrants` = 2000 and over a fixed number of 200 cycles. The latter choice deals with the interpretation of a time step: given that firms' behaviour involves hiring, firing and production plans, it seems valid to assume a cycle to be equivalent to month. This interpretation does not actually affect the households, which, in the model, perceive a certain wage and consume a fraction of it (90% in our case): as a matter of fact, one could take this behaviour as an intensive property of a household. Moreover, we choose to set a population of 200 immigrants over a total population of 1200 (16%) because according to [Emmerson *et al.*(2016)Emmerson, Johnson, Mitchell and Phillips] the share of foreign-born people in the UK's total population increased from 8.9% in 2004 to 14.4% in 2017, and it should be still increased in 2019. The scenarios we analyze are the following:

- *baseline scenario*: the first scenario is a "smooth" Brexit. We imagine this possibility as the closest one to the May's deal, that includes a strong control of immigration and small trade frictions. Nevertheless, Mrs May is facing some difficulties to approve her deal in the Parliament, this is still the most likely outcome of the entire process. Here, immigrants have a higher share of high skilled workers with respect to native population but lower minimum wage. Immigrants have higher productivity with respect to natives if they are high skill, lower if low skilled. Firms offer, on average, wage that are in the middle between natives and immigrants minimum wage;
- *australian type*: the second scenario is UK reaches a deal that include the "Australian" model of immigration. In this case, UK allows only immigration for high skill workers, mainly needed by UK growing service sector and scarce in the internal labour market. This, according to [Committee *et al.*(2018)], is the best policy for the UK to offset the negative impact of immigration due to Brexit. In this model the parameters of the baseline scenario are constant with the exception that only low skill immigrants leave the market keeping constant the number of high skill ones;

- *hard-Brexit*: as third option we propose a "Hard-Brexit" scenario, that is UK crashing out from EU without any deal. Here, the parameters are constant we only doubled the probability of firms to leave and the period involved, moreover, we strongly cut immigration. Indeed, this scenario has been foreseen by BoE as the worst crisis after the second World War, even worst than 2007 financial crisis;
- *automation*: in this scenario, we decrease the job opportunities for low-skill workers keep other variables constant. This possibility mirrors nowadays scenario where the raise of artificial intelligence and robotic is bringing the same results in many countries that are now facing high rates of unemployment among their low qualified population;
- *wages' alignment*: another interesting case is simulating a Brexit where the wages' difference between high and low skill workers is smaller. Indeed, in this case we slightly cut the wage gap offered by the firms. High skill workers, both natives and immigrants, gain still more, on average, that low skill workers but the distance is reduced. Moreover, we keep the minimum wage and other parameters constant to the baseline scenario;
- *productivity alignment*: this possibility assumes a "smooth" Brexit but with immigrants and natives with the same productivity. This is a to show the results of cutting immigration when natives and immigrants have a homogeneous productivity;
- *less qualified immigrant population*: in UK, the population of EEA immigrants, that Is the most likely affected by the Brexit process, is highly qualified, on the contrary the overall immigrant population is more equilibrated since non-EU immigrants are mainly low skill workers. This scenario presents the results when the share of low and high skill workers among immigrants is equal;
- *increase minimum wage differences*: in this scenario, we act on the average minimum wage of households. Keeping the wage offered by the firms constant as well as the other parameters, we increase the minimum wage of native population and decrease the ones of immigrants, for both low and high skill workers.

10.1 Baseline Scenario

In order to simulate the "smooth" Brexit we compose our population with 40% of high skill workers, UK average graduating rate in 2017, that is close to the OECD average of graduate total population equal to 37%. On the other hand, we set immigrants with a share of 75% high skill because according to [Dustmann *et al.*(2012)Dustmann, Frattini and Preston] since 2005 only 25% of EEA immigrants fell into low skill job and the latter are the most likely influenced by Brexit.

Moreover, firms hire 55% high skill workers and 45% low skill ones, this is because

nowadays the labour demand for high skill workers is slightly higher. On the contrary the ratio of fired workers is split equally between high and low skill. At the beginning, in order to keep the model balanced, we set parameters in a equilibrated way (see Figure 23). Indeed, the minimum wages of natives are on average 1.1 and 0.8 for high and low skill respectively, both with a standard deviation of 0.1. At the same time, immigrants, have on average 0.1 less in the minimum wage with 0.7 for low skill and 1.0 for high skilled, even in this case, both with standard deviation of 0.1. We did this choice because there is a wide literature both theoretical and empirical that support the fact that immigrants have on average minimum wage lower than their native counterparties. Firms offer on average a wage that is the mean between the two groups of agents, they indeed propose a wage of 0.75 and 1.05 to low and high skill workers respectively, even in this case with a standard deviation of 0.1.

Furthermore, regarding high skilled workers, we set that immigrants have a little more productivity (1.2) with respect to their native counterparties (1.1). We did this choice because UK is a well-known hub for the brightest talent from all around Europe seeking for good job opportunities and so looks reasonable that high skill immigrants are more productive than native ones. On the contrary, for low skill households, we set that immigrants are 0.1 less productive than natives, we indeed set natives' productivity to 0.8 against 0.7. This is based on the assumption that low skill immigrants are more likely to emigrate to find better labour market conditions with respect to their country of origin where they are likely to face more difficulties to get a job. Anyway, for both natives and immigrants low skill workers the productivity is, on average, equal to their minimum wage (0.7 and 0.8), the same happen for high skill natives (1.1). Only high skill immigrants have a productivity (1.2) higher than their minimum wage (1.0) for the already above-mentioned reasons.

Regarding the Brexit process we set that during each period for 30 periods (2,5 years) there is a probability that each unemployed immigrant leaves the market as well as some firms can relocate their production. The transition period is between $80 < t < 110$, so that the model can show its initial evolution and find a pattern, then the Brexit shock is introduced, moreover from period 110 to 250 (13 years) we can show how the model reacts. Nevertheless, we decide to keep this period open for 2.5 years because it is the same length of the transitions process foreseen in the May's agreement. The probability of unemployment immigrants to stay is 80% (20% leave) and we set this parameter according to the estimations done by [Dustmann *et al.*(2005)Dustmann, Fabbri and Preston] that forecast a UK labour force reduction due to Brexit about 3% (1 million reduction out of a total population of 31 million). As far as firms are concerned, we decide that there is a likelihood of 0.1% to relocate abroad their production, this necessity came up after that several MNCs announced to move their production, for instance the last announce was by Honda that declared to close their UK's factories leaving more than 3,500 workers unemployed.

Number of natives	10,000
Number of firms	1,000
Number of immigrants	2,000
Consumption	0.9
High skill natives	40%
High skill immigrants	75%
High skill hire	60%
High skill fire	50%
Average initial workers	10 ± 2
Productivity high natives	1.1
Productivity low natives	0.8
Productivity high immigrants	1.2
Productivity low immigrants	0.7
Wage amount low skill	0.75 ±0. 1
Wage amount high skill	1.05 ± 0.1
Min. wage low Skill natives	0.8 ± 0.1
Min. wage high skill immigrants	1.0 ± 0.1
Min. wage low Skill immigrants	0.7 ± 0.1
Min. wage high skill natives	1.1 ± 0.1
Leaving period	2.5 years
Immigrants staying	80%
Firms staying	99.9%
Number of cycles	250

Figure 23: *Baseline scenario parameters*

We now summarize the main results of the model on macroeconomics variables, splitting the results in three parts which are:

- Pre-Brexit period ($t < 80$);
- Transition period ($80 <= t <= 110$);
- Post-Brexit period ($110 < t <= 250$).

We analyse in particular model results on:

- *unemployment rate*:

During the first part the average unemployment rate is 5%, but it decreases before the beginning of Brexit, indeed it is 4% at $t = 80$ (figure 24). During the transition period it remains constant with average value of 4% thereafter it starts to steadily increase going back to its initial level, so that, at the end of the period ($t = 250$) it is almost equal to its initial value (5.6%).
The initial decrease is due to the adaptation of firms and households to the labour market conditions, cutting the gap between expected and offered wages. During Brexit, the unemployment rate remains, on average, constant because from one side the adaptation of agents still acts to decrease it and on the other side there some firms exit the market leaving their workers unemployed. Moreover, some unemployed immigrants leave the market so that there are less people unemployed. Even if the total labour force is smaller, it has a balance effect on the unemployment rate too. What is interesting is that after the Brexit process the unemployment rate starts to increase slowly but constantly for the first time in the run. This happens because a part of the labour force leaves the market and creates a mismatch between skills demanded by firms and the ones offered by the population; the latter results in more difficulties for firms to hire the skill level workers they want, decreasing the number of total hiring and impacting on the unemployment rate negatively.
Overall, with these settings and according to the estimates carried out by [Lemos and Portes(2008)] it seems that the Brexit process only has a minor impact on unemployment rate on the long term. Anyway we have to consider that agents' adaption in this model brings to reduction on unemployment rate and that Brexit first balance it (during the transition period) and then offset this positive impact completely bringing to an increase (even if only of 0.7%) of unemployed households at the end of the run.
Moreover, if we split the unemployment rate by skill level (Figure 24) we see that high skill workers (red) and low skill ones (black) have a similar unemployment rate pattern till few periods after the transition period; while if we look only at the third part of the run they take opposite directions. What happens in our economy is even more clear: indeed high skill unemployment rate continues to decrease while it sharply increases for low skill workers. This happens because there is a shortage of high skill workers and this is due to unemployed immigrants leaving the market (immigrants are 75% high skilled), so that firms are not able to hire the high skill workers they would and, perhaps, they stop their assumption. This comes out because in our model, as happens in reality, each firm needs a certain number of high skill workers to survive and cannot just substitute them with low skill ones.

Figure 24: *Aggregate unemployment rate*

Figure 25: *Unemployment rate of high skill (red) and low skill black)*

These results are confirmed even if we look at the unemployment rate between natives (green) and immigrants (red) of figure 26. They have more or less the same percentage till the end of the transition period, natives' rate is more volatile because they are a smaller number (16% of population) and perhaps more sensible to variations. In the last part of the run we see that the distance between the two increases: we can explain that because immigrants are mainly high skilled (75%) so they can easily find a job, while natives face more difficulties because they are principally low skilled (60%). The high skill ratio on the population plays then a crucial role on the final unemployment rate;

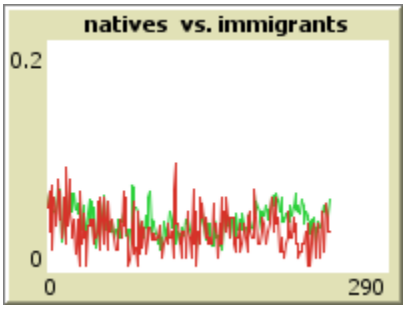

Figure 26: *Unemployment rate of immigrants (red) and natives (green)*

- *firms and population* the mismatch between skills demanded by firms and ones supplied by the population affects the unemployment rate with a twofold effect: firms decrease the number of hiring and moreover a higher number of firms close leaving its workers unemployed and with less probabilities to find a new job. The number of firms is 1000 at the beginning and 930 at the end but they increase in the first part of the model (1050 when t= 80), while they are constant during the transition period and drop after it.
  The number of firms clearly reflects the unemployment rate and the economic conditions. Indeed, the economic growth before Brexit increases the number of firms and thus decreases the unemployment rate. During the transition period some firms relocate their production abroad increasing the unemployment rate, but this is still balanced by the birth of new firms. In the last part there are more firm bankruptcies not replaced by new ones because of the mismatch between skill demand and supplied by labour market. As a matter of fact, a firm cannot start its production if it is no able to find in the labour market the skill that it needs.

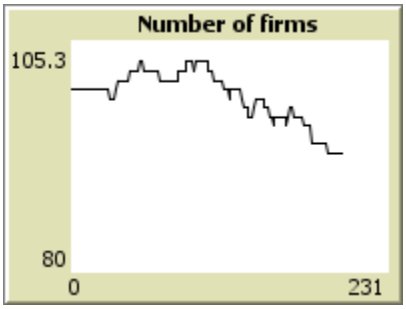

Figure 27: *Firms' turnover*

  For the same reasons, profits are, on average, positive during the first two parts of the run and negative during the last one.
  Profits are really volatile but on average almost null since the shock that affects firm's price has mean 0 and standard deviation 0.1, as well as the return of investment is on average the 10% that employed household do not consume each period.
  As a matter of fact, firms decrease on number, but they increase their size. At the beginning each firm had an average of 10 workers while at the end of the run there are 50 firms with more than 50 employees, 80 with more than 25, 620 between 25 and 10 while only 180 firms have 10 employees or less.

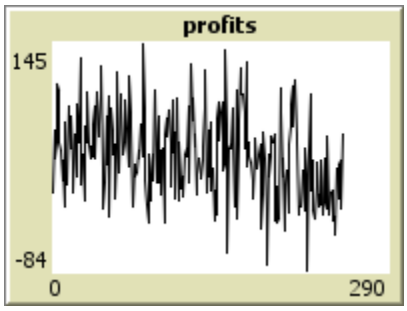

Figure 28: *Firms' profits*

With these parameters we got consistent results even with [Jafari and Britz(2018)] that estimate 1 million of labour force reduction with respect to a total labour force of 31 million, perhaps 3.2% less. In our model the initial population is 12,000, composed by 10,000 natives and 2,000 immigrants. At end the number of natives is the same while immigrants are 1,600: we lost then 20% of them and we got a total labour force reduction of 3.2%;

- *wages*: As above specified, firms must adapt their wages to the market conditions. This adaptation is consistent with the unemployment rate evolution. Indeed, since there is a shortage of high skill workers, firms have to increase their wages to hire them. This happens especially during the last part of the run, indeed at the end of our simulation high skill wage almost doubles (from an average of 1.05 to an average of 1.9). On the contrary, low skill workers must decrease slightly (from an average of 0.75 to 0.6) their minimum wage since they face more competition to get a job;

Figure 29: *Firms' wages to high skill (red) and low skill (black)*

- *price, demand and GDP*: during the entire run the price is stable and close to 1 for the following reasons:
  - the shock added to the firms' price has mean zero;
  - the price is given by the ratio of the aggregate demand and aggregate production, which are almost equal because households average wage

are close to their production and, during the whole run, they consume everything they get.

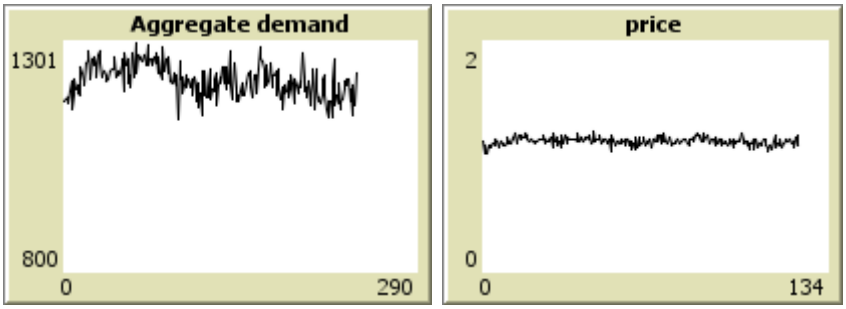

Figure 30: *Aggregate demand (left) and price (right)*

to firms and unemployment. Furthermore, since GDP is the sum of firms' production it slightly increases (4%) in the first part of the model because of a twofold effect: first, there are more firms and perhaps more employed households. For the same reason, it starts to decrease as soon as firm go bankrupt and the unemployment rate increases. Anyway since the number of firms decrease but their size increase and the unemployment rate increase even if just of 0.7% we do not see big changes even in the GDP that, at the end of the run, is 2% less than its initial values. Nevertheless, we have to remember that there is also an inner positive growth in the model without Brexit due to agents' adaptation that is completely offset. So, when we consider the losses of our economy we have to sum the potential growth that has not been realized yet plus the losses: the sum of these two can give us a complete view of the phenomena. Moreover, since the price is close to one we have barely the same results between GDP and real GDP, as well as the aggregate demand that is almost equal to the firms' production. These results are consistent with [Greenaway and Milner(2019)] that states that UK can continue to growth even after Brexit but not as much as it remains in the EU. We are then considering a huge loss of potential growth.

10.2 Australian type

Restrict immigration only to certain skill level, that are scarce in the internal market and can provide clear benefits to the local population, it is the recommendation made by several research.

For instance, [Armstrong and van de Ven(2016)] claimed that "The UK's post-Brexit immigration system could be decided by the UK on its own, as is done for example, by most countries outside the EU (except for some relatively minor provisions in trade agreements) or could be part of the negotiations with the EU. Access to the UK labour

market is valuable to migrants, especially from lower-income countries. In theory, the UK may therefore be able to trade-off some preferential access for EU citizens to the UK in return of benefits in other areas of the negotiations, such as trade". Furthermore, even the report completed by the Bank of England stated that "the evidence points in the direction of high-skilled migrants having a clear benefit to existing residents while the same is not true for lower-skilled migrants. As a result, a policy on work migration that provided greater access for higher-skilled migration while restricting access for lowers killed workers to enter the UK would be consistent with the available evidence."

For these reasons we decide to propose a scenario where only low skilled unemployed immigrants leave the market.

The parameters are exactly the same as in the baseline scenario (Figure 23), we only modify the skill type involved in leaving the market, indeed, here, other than firms, only low skilled unemployed immigrants can abandon the market with a certain probability each period (20%).

Moreover, we have to underline that our immigrant population is 75% high skill and it is only 16% of the total household population, so low skill immigrants represent only about 3,3% of the total population. Furthermore, only a fraction of them is unemployed so, since we are affecting only a small share of our households, even the shock to the market is expected to be lower. Actually, pursue this policy regarding immigration is almost like do not do Brexit at all, at least regarding the immigration issue, perhaps "block uncontrolled migration from the EU" was one of the core points supported by leavers. This is consistent with the statistics of UK gov that see, especially from the EU, a migration composed by mainly high skilled workers and whose net flow is already decreasing over time since 2010.

If we look at the results of the run, as expected, our labour force has only a little decrease. We start with 12,000 households, 10,000 natives and 2,000 immigrants and we lose "only" 98 immigrants during the run. Our total population decreases of 0.8% (in contrast with 3.3% in the baseline scenario).

This shock is not enough to destabilize our economy, this is clear in firms' turnover. Indeed, in this case there are some firms closing but this is balanced by new ones opening (they cannot open before because they were not able to find the skill needed in the market) and, at the end of the run, firms are 960, 4% less. Anyway, firms decrease because of Brexit, indeed some of them relocate their production abroad and this brings to a final slightly negative turnover.

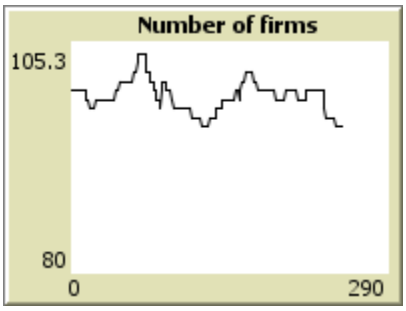

Figure 31: *Firms' turnover*

Even if firms increase their size. At the beginning each firm had an average of 10 workers while at the end of the run there are 51 firms with more than 50 employees, 90 with more than 25, 620 between 25 and 10 while only 200 firms have 10 employees or less., thus we have a concentration of firms.

This concentration during the run offset the negative turnover, indeed we can see that the unemployment rate slightly but constantly decreases over time.

The average unemployment rate is about 4%, but it is 5.5% at the beginning of the run and 3% at its end; this decrease is due to the adaptation of firms and households to the labour market conditions. Moreover, if we split the unemployment rate by skill type we see that high skill workers average unemployment rate (red line Figure 32) is a little lower (0.5% on average), anyway they are both decreasing over time. This is consistent with the parameters of the model; indeed, firms hire more high skill households (60% vs. 40%) but fire equally (50%). Perhaps they are even a minority in the total population but with more job opportunities.

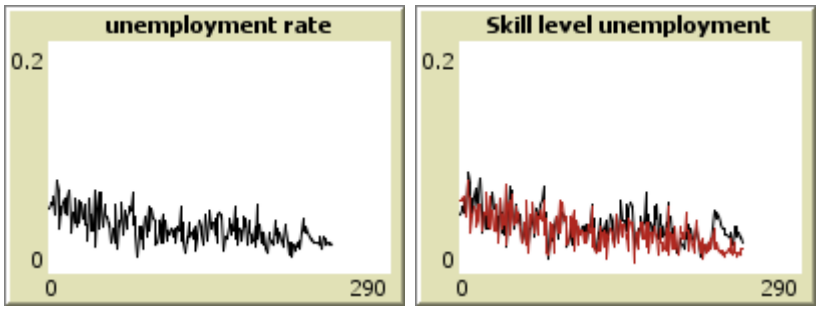

Figure 32: *Unemployment rate*

The fact that high skill households are a minority and with more vacancies comes out if we look at the wage evolution by skill level. As we said before, firms adapt their wages to the market conditions, competing to get the high skill workers necessary to run their business. This adaptation is consistent with the wages' evolution: firms have to increase their wages for high skill workers (from an average of 1.05 to an average

of 1.75). On the contrary, firms can decrease slightly (from an average of 0.75 to 0.70) their wage to low skill workers found easily in the market. The pattern of wage evolution explain that low-skill households can find a job without facing too many problems, but they have not such a bargaining power to increase their wage; perhaps, with these settings, we are close to an equilibrium between supply and demand in the low skill labour market. On the contrary, the shortage of high skill gives them a stronger bargaining power in the market, that allow to almost double their wage during the run. However, the latter is not strong as it was in the baseline model, where they were able to increase their wages up to 1.9. This is consistent with the setting of this scenario, indeed, the fact that high skill immigrants do not leave the market contribute to decrease the shortage.

Figure 33: *Firms' wages evolution to high skill (red) and low skill (black)*

Moreover, if we look at the GDP of our economy we retrieve the same results. Indeed, despite the small decrease of firms, the GDP is almost flat, it has a little increase of 2% during the entire run. This is correlated with the small decrease of unemployment rate, since firms have more workers and produce more.

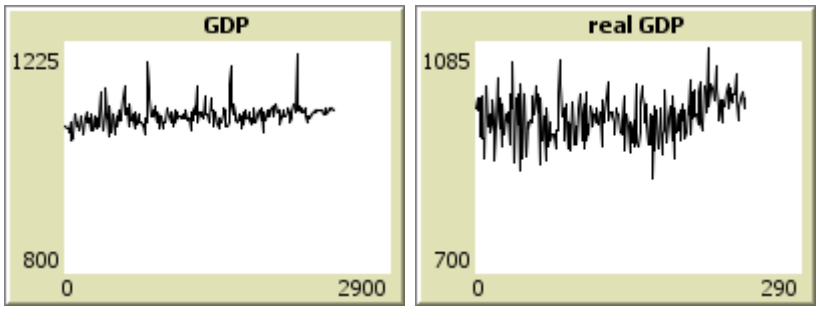

Figure 34: *GDP (left) and real GDP (right) evolution*

The real GDP follows the same path of GDP, it is a bit more volatile because of the shock added to each firm's price, but since it has mean 0 it does not affect the final long-term result.

The real GDP is close to GDP because the price stays really close to one during the entire simulation, it has only a 1% increase due to unemployment rate decrease (-0.7%). As specified above, the price is close to one because it is a clearing price and it is composed by the ratio of aggregate production and aggregate demand which are almost equal since households produce, on average, how much they are paid : everything they gain in their whole life flow into economy.

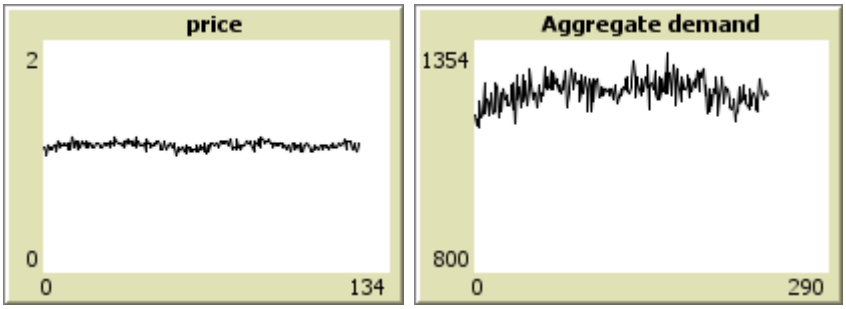

Figure 35: *Price (left) and Aggregate demand (right) path*

The results of this scenario are as we expected at the beginning, indeed the share of population concerned by Brexit, in this case, is not enough to unbalance the run of the model that instead keeps an equilibrium and a small macroeconomic growth (real GDP +1%, unemployment rate -0.7%).

10.3 Hard Brexit

During the last weeks, the Prime Minister Theresa May made clear what is her strategy about Brexit: her deal or no deal at all. The latter scares MPs so much that they are now trying to take control over the Brexit process in order to reach an extension of article 50 and postpone the deadline fixed now to the 29th of March.

The Bank of England carried out some analysis of the possible scenarios and according to them the no-deal case may drop national income to its lowest level from the Second World War; the government economists foreseen a GDP decrease up to 10.5%, unemployment rate up to 7.5% and house prices dropping of 30% over a five-year period compared to their pre-referendum analyses. This is even worst with respect to the 2007 financial crisis where house prices tumble of 17% and GDP shrank of 6.25%.

Since the likelihood of the no-deal scenario is increasing day by day as Brexit deadline getting closer and closer, we choose to include the no-deal case inside in our analysis. In order to assess the impact of UK crashing out of EU without any deal we decide to keep constant the parameters of the baseline model (Figure 23) changing only the probability of both firms and unemployed immigrants to leave as

well as the duration of the period involved. Moreover, we decided to double these probabilities, switching:

- transition period from 2.5 to 5 years, indeed it will take a very long time for the UK to renegotiate the 759 treaties (at least) with 168 countries already got now as part of the EU plus any new agreement with the continent;
- during the transition period firms leave the market relocating their productivity abroad with a probability of 0.2% (0.1% in the baseline scenario);
- unemployed immigrants leave the market with a probability of 40% (20% in the baseline scenario).

In our opinion, it seems ration to double the probabilities of both firms and households to leave because the estimates carried out by several and independent research foreseen a system stretched to its limit and in some parts almost collapse (i.e. Port of Dover collapse, UK running out of pharma and some foods).

In this scenario, we expect to see the same mechanisms happening in the "smooth" Brexit but exacerbate to another scale. This scenario is divided in three parts:

- Pre-Brexit phase ($t <= 70$);
- Brexit ($70 < t <= 130$)
- Post-Brexit ($130 < t <= 270$)

In this scenario, the unemployment rate starts close to 5% (Figure 36) and as in previous versions tends to decrease when the model run without any shock, indeed before Brexit (= 70) it is 4%. Furthermore, it starts to increase already in the Brexit process, indeed the decrease gained during the first part is already burn at the end of Brexit where the unemployed rate is back to its initial level (5%). Nevertheless, the worst happens later, indeed in the third part of the run the unemployed rate sharply increase, closing to 7.8% (really close to the result got from BoE that is 7.5%).

Figure 36: *Unemployment rate*

Moreover, if we split the unemployment rate by skill level (Figure 37) we see that high skill workers (red) and low skill ones (black) have a similar unemployment

rate pattern until Brexit. Thereafter they take opposite directions with high skill unemployment rate that continues to decrease and the one for low skill that strongly increase up to 17% at the end of the run. Splitting unemployment rate by skill level makes clear what happens in our economy, indeed high skill unemployment rate continues to decrease while it sharply increases for low skill workers because of high skill workers shortage. The latter is due to unemployed immigrants leaving the market (immigrants are 75% high skilled), so that firms are not able to hire the high skill workers they would like to and perhaps they stop all their assumption.

Figure 37: *Unemployment rate by skill level*

This is the same thing that happens in our baseline model but here the shortage is bigger as well as its consequences to the economy, consistently with our new settings. Indeed, since immigrants have more probabilities to leave the market the final number of high skill households will be reduced. Moreover, since firms have higher probabilities to relocate they will leave more workers unemployed, the high skill ones will be easily reabsorbed by the market while low skill face more competition finding difficult to get a job increasing the overall unemployment rate.

Due to higher probabilities, more immigrants leave the market with respect to our baseline scenario, indeed, at the end of the run only 980 remain while 1020 left. Our total labour force shrank of 8.5%, losing more than half of the total immigrant population. We even lost 25% of the initial firms, indeed they are 750 at the end of the run (1000 when it starts). After the initial increase, firms start to close for two reasons, first, some of them leave the market due to Brexit, perhaps about 150, while the others 100 closes because of high costs. Indeed, firms have to compete with each other and increase a lot their wages to high skill workers in order to get the quantity they need from the market and this increases their costs. Indeed, since there is a shortage of high skill workers, firms have to increase their wages from an average of 1.05 to an average of 2.2, this is the first scenario where high skill workers are able to more than double their wage. On the contrary, they can decrease slightly their wage to low skill workers (from an average of 0.75 to 0.65). The latter is not anyway sufficient to offset the increase in high skill.

Figure 38: *Firms' wages for high skill (red) and low skill (black)*

Moreover, the firms that bankrupt are not replaced by new ones for the same reason that we found in the baseline model , i.e. the mismatch between skill demand and supply. As a matter of fact, a firm cannot start its production if it is no able to find in the labour market the skill that it needs. In particular, in this case, firms are not able to find the number of high skill workers they need and so they stop the whole hiring process as well as the opening of new firms.

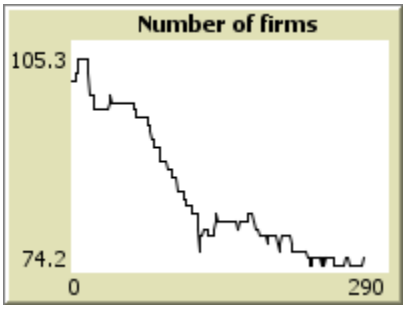

Figure 39: *Firms' turnover*

Anyway the decrease of firms do not necessarily means a decrease of production or increase of unemployment rate. Perhaps, as we showed in the other scenarios, in this model there is a small tendency of firm concentration, indeed usually most of them increase their size and this offset their reduction. Even in this scenario firms increase their average size, at the end of the run we have 40 firms with more than 50 employees, 55 with more than 25, 490 between 25 and 10 while only 165 firms have 10 employees or less.

Nevertheless, the increase of size is not enough to counterbalance the loss of production or the increase of unemployment rate, the remaining firms are not able to absorb entirely the labour force left unemployed but just a part of it.

Another consequence of the decrease of labour force and the problems faced by firms in the labour market is the decrease of GDP. Indeed, since the number of employed households determines the total productivity as the unemployment rate

increase we will see a decrease of the GDP. In particular, the GDP slightly increases (2%) before Brexit (when unemployment rate was decreasing) and it starts sharply to decrease during the transition period as well as it does in the third part of the run but more smoothly. The heavy drop during the second part (Brexit) is caused by the firms that suddenly stop their production, leave their workers unemployed and the market. On the other hand, the drop in the final part is due to the firms that close because are not able to afford costs and cannot be replaced by new ones. Yet with the existing firms that are not able to absorb the labour force that remain unemployed for the above-mentioned reasons.

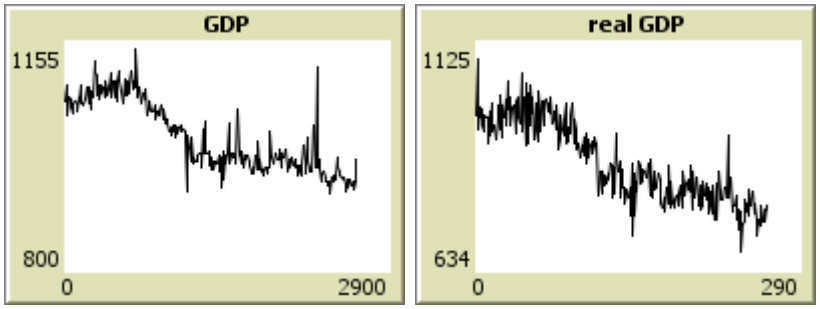

Figure 40: *GDP (left) and real GDP (right) evolution*

Nevertheless, this is the first scenario where the price is not constant. Indeed, the production of firms drops faster than the aggregate demand. This is because unemployed households still consume their subsidy but their production is null; the same happened even in the baseline scenario with the difference that the unemployment rate did not increase enough to affect the price. In this case, the unemployment rate almost double during the run and it has a clear effect on price, that slightly but constantly increase with a final inflation rate of 6.9%. The latter is consistent with the estimate done by [Kierzenkowski *et al.*(2016)Kierzenkowski, Pain, Rusticelli and Zwart] that foreseen an inflation rate of 6.5% in the worst-case scenario.

The rate of inflation plus the decrease of production affect the real GDP that drops about 12%.

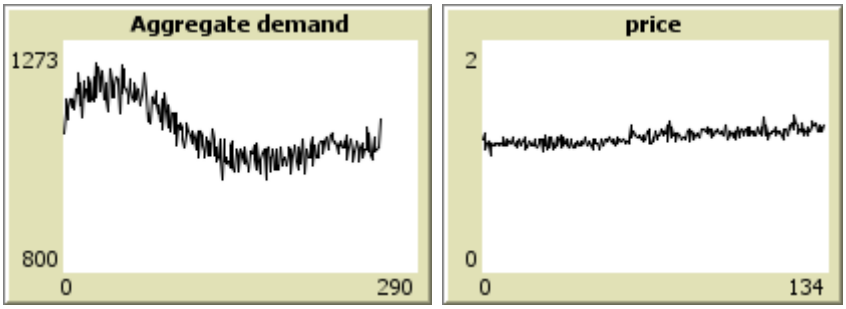

Figure 41: *Demand (left) and price (right) evolution*

As we mentioned at the beginning, in this scenario we can see the same frictions happened in the baseline scenario but with another magnitude. Indeed, the shortage of high skilled workers is higher, and this means that the gap between demand and supply of labour is bigger than before. The latter has a big impact on both unemployment rate and GDP, and moreover we can even see an effect on price with an inflation rate close to 7%. Nevertheless, labour force shrinks of 8.5%, resulting in a twofold effect on the economy. First, it decreases the demand because immigrants consume their income somewhere else and second, the overall production drops. Since the overall production is even affected by the increase of unemployment it decreases faster than the demand with a catastrophic effect on the real GDP.

10.4 Rise of automation

During the last few years the rise of artificial intelligence and automation brought more than one concern to policy makers. Indeed, a consistent amount of jobs is easily substituted by machine now, and this rate is foreseen to increase even more in the next years. This means that the it is likely to see a change in the skills demanded. In general, the workers that face more risks are the low skilled ones since they can be replaced easier.

Furthermore, low skilled workers are already the weaker category in the labour market, they are still the majority of the population but for a minority of jobs. This clearly emerges in our model where they are hardest hit and face more difficulties during the Brexit process.

Moreover, low skilled households (without a degree) voted strongly in favour of Brexit (GCSE or lower educated voted 70% to leave), but their vote could turn into a double sword edge. Indeed, in the case of rise of robotic and automation, low skilled workers which are already negatively affected by Brexit, risk to find themselves in even worst conditions. For these reasons, we decide to propose it as fourth scenario. The parameters are the same as in the baseline scenario (Figure 23), the only thing that changes is the ratio of high skilled workers with respect to low skilled ones demanded by the firms during the hiring process. We increased the latter to 70% high skill and 30% low-skill, while it was 60% high skill and 40% low skill in the baseline scenario.

The results that come out from this experiment are interesting and they can give us a more complete view of the phenomena, moreover, they can be analysed in three parts which are:

- Pre-Brexit phase ($t <= 50$);
- Brexit ($50 < t <= 80$)
- Post-Brexit ($80 < t <= 250$)

First, we look at the unemployment rate (Figure 42): it starts with usual value of 5%

and it remains flat during the first two parts of the run (pre-Brexit and Brexit). Few periods after (t= 90), it starts sharply to increase, and it ends up to a value of almost 25%.

Figure 42: *Unemployment rate*

What is interesting to see is that the model is in equilibrium until the end of Brexit. Thereafter, the migration of a part of the labour force creates a mismatch between skill supplied by the population and the ones demanded by the firms, so that firms are not able to hire the ratio of skills they need so that they stop entirely the hiring process. Moreover, this does not happen immediately but the worst consequences due to Brexit appear as a second momentum, almost one year after the end of the transition period.

What happens in the labour market is even more clear if we split the unemployment rate between high and low skills (figure 43).

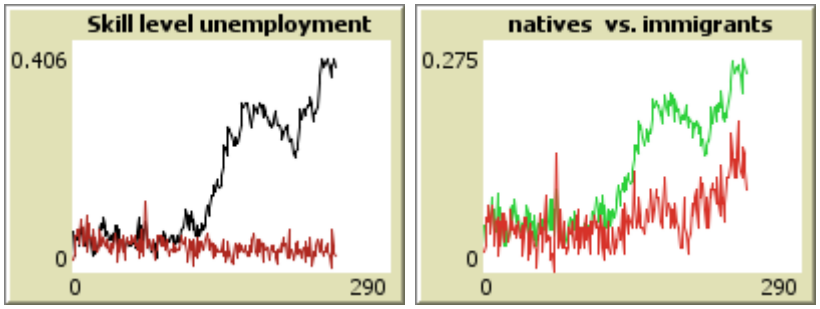

Figure 43: *Unemployment rate divided by skill level (left) and population (right)*

Indeed, high skilled workers unemployment rate slowly but constantly decrease, they reach almost full employment at the end of the run (unemployment rate $< 2\%$). On the contrary low skilled workers face more than one difficulty, their unemployment rate rises strongly after Brexit leaving almost 40% of our low skilled households unemployed at the end of the run. This happens because if firms do not find the ratio of high skills they need in the market, they do not even hire the low skills part.

We can retrieve the same conclusions if we look at the unemployment rate evolution between natives and immigrants (Figure 43). Here, natives, which are mostly low skill

(60%) and represent 86% of our initial population, have an unemployment rate whose path is closer to the one of overall low skilled workers. On the contrary, immigrants which are mostly high skill (75%) has a lower unemployment rate. However, even their unemployment rate increases after Brexit, probably due to the fact that several high skills leave the market during the transition period leaving a more equilibrated immigrant population.

The unemployment rate increase is strongly correlated to firms' turnover. Indeed, the number of firms rises during pre-Brexit phase. It comes back close to its initial level when finishes the transition period ($t = 80$) and strongly drop later, leaving only 801 firms at the end of the period, so that, the number of firms decreases of about 20% during the entire cycle, a result close to the no-deal scenario.

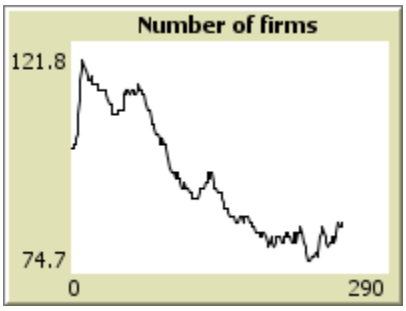

Figure 44: *Unemployment rate divided by skill level (left) and population (right)*

Since firms are 1007 when Brexit ends it means that what really affects their turnover is not the fact that some of them leave during the transition period but the labour market conditions they face later.

Nevertheless, new firms cannot be born because they do not find the skills they need to start (shortage of high skilled workers) in the market and perhaps those which go bankrupt are not replaced. When firms go bankrupt they leave their workers unemployed, and if the remaining firms are not able to absorb these workers shortly we have an increase in the unemployment rate. That is exactly what happens during our run, where unemployment rate and firm bankruptcy are strongly correlated for the above-mentioned reasons.

The wages' evolution even confirms our results, the high request of high skilled workers with respect to their share of total population allows them to triplicate their initial wage (from an average of 1.05 to 3.20). On the contrary, low skilled workers are less paid, from an average of 0.65 to 0.45, because they are so many, that they have to compete a lot to get a job. The wage of low skill workers could decrease even more but we set that firms cannot paid their workers less than the subsidy amount (0.35) plus a certain amount (0.1), this means minimum wage and it is exactly the value we got at the end: nobody would go to work if can earn more with an unemployment

subsidy.

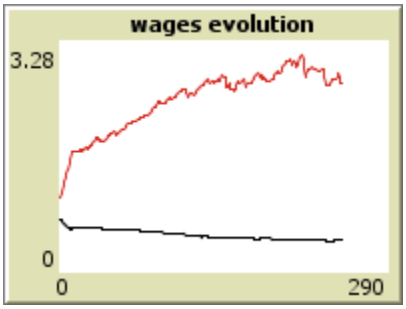

Figure 45: *Firms' wages evolution*

During this scenario we start with 2,000 immigrants but we have only 1546 at the end of the run because 454 of them left during Brexit. Moreover, our total labour force decreases of 3.8% with a final unemployment rate of 25%. Surprisingly, the demand for goods slightly increase during the run.

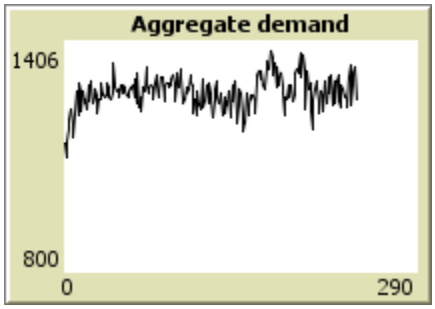

Figure 46: *Aggregate demand evolution*

The reason behind the increase of the demand is related to the wages' pattern. Indeed, even if we have less labour force and less employed people that negatively affects the demand, the huge increase of high skill wages completely offset this situation, leaving the demand of goods almost steady. Indeed, the consume of high skilled workers increases strongly as their wages keep the demand constant. Moreover, the wage of low skilled workers is not that far from the subsidy amount, that is entirely consumed.

The decrease of firms and increase of unemployment rate reflect the GDP's path.

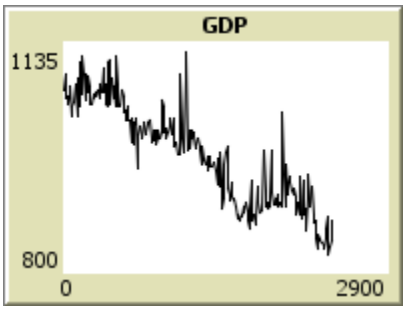

Figure 47: *GDP evolution*

Firm production starts to decrease during Brexit and never stops, it lost more than 20% of its initial value at the end of the run. Anyway, this figure is not trustable because this model does not take in account the production done by potential artificial intelligence or automation that along with Brexit is the cause of the unemployment rate increases and firm bankruptcy.

The almost flat demand together with the drop of production bring to an increase of price, the inflation rate is 20%, the price starts around 1 and close about 1.20.

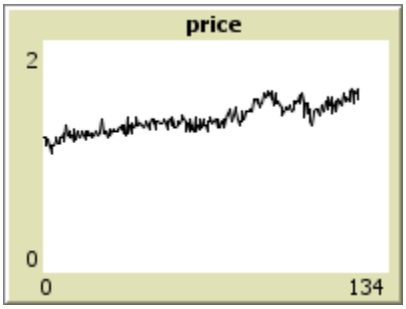

Figure 48: *Price evolution*

Yet, even this result is biased by the fall of firm production, since the clearing price is given by the demand divided by production. The same happens to real GDP, that due to price increase, falls more than the nominal GDP.

Figure 49: *Real GDP evolution*

This scenario shows the negatives consequences that an economy can suffer with an increase of automation together with a shock in the labour market like Brexit. Moreover, it sheds the light on the importance of training the labour force to get the skills demanded by the market, a topic that policy makers should take into account more seriously according to [Arntz *et al.*(2016)Arntz, Gregory and Zierahn].

10.5 Wages alignment

For our fifth scenario, in order to test model sensitiveness to the parameters, we carried out an experiment where high and low skilled workers gain, on average, the same wage at the beginning. The parameters are those set in the baseline scenario (Figure 50) with the only difference that we change the wage amount offered by the firms. Wages were set to start, on average, at 1.05 for high skilled workers and 0.75 for low skilled workers, while now we set both at 0.90, their average. This experiment is important to test agents' adaptation to the market conditions and the impact on the final outcome.

In this case, not surprisingly, we got similar results of the baseline model. Indeed, the main differences with previous results are just at the beginning of the run (pre-Brexit) where the agents have still to adapt. First, the unemployment rate starts 2% higher (7%) than the baseline model (5%). This is because high skilled households have an expected minimum wage of 1.1 if natives, and 1.0 if immigrants, so a share of them do not accept the wages offered by firms. These frictions increase the unemployment rate during the first few runs, and reaches the peak of 9.2% when $t = 15$, thereafter agents' adaptation comes out.

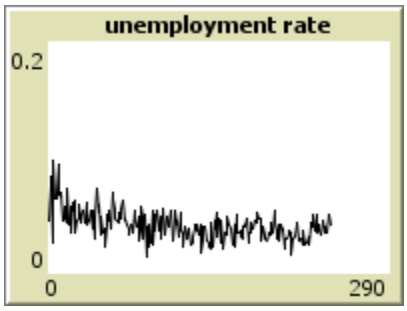

Figure 50: *Unemployment rate evolution*

The unemployment rate then starts to decrease because of our agents adapt their wage, from one side firms increase their wages for high skilled workers, on the other hand high skilled workers decrease their expectation closing the gap created at the beginning. Unemployment rate has the same level as in the baseline scenario at the beginning of Brexit (4%) and it has the same evolution after. Indeed, the mismatch of supply and demand in the labour market offsets agents' adaption and keeps the unemployment rate close to its pre-Brexit level (4%). Since the gap between salaries has been closed before the shock, the model evolves in the same way as before and this is clearly visible if we split the unemployment rate by skill level (Figure 51). Moreover, each run of the model has a random part that affects the final results, perhaps we will never have a run that is exactly the same, anyway we can recognize the same path and evolution among variables.

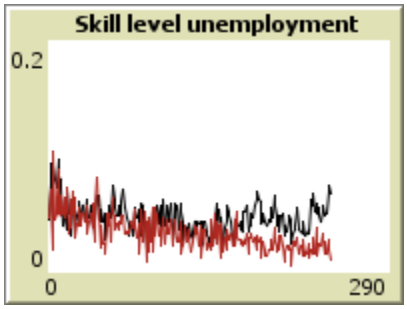

Figure 51: *Unemployment rate evolution by skill level*

The agents' adaptation is clearly visible in the evolution of wages (Figure 52). Indeed, we see that the wages of high and low skilled workers start at the same level but then they take quickly opposite directions. After 24 cycles (2 years),the average high skill wage is already 1.3 (39% increase), while the ones for low skill are at 0.65 (27% decrease). Anyway, since at the beginning even high skilled households decrease expectations, the final value of their salary is a little lower (1.75) than in the baseline scenario (1.9).

Figure 52: *Firms' wages evolution for high skill (red) and low skill (black)*

Looking at the speed adjustment of our agents, it seems properly to question if it is trustable or not, indeed someone might think that they adjust too quickly with respect to the evolution of the model. This open a big question about Agent-Based Model parameters tuning, that is one of the trickiest things to be implemented since it is not easy to map perfectly agents' behaviour. Anyway, our model is an oversimplification of a labour market whose tuning can be still improved, so we let this for further developments, however we consider that these settings are enough to capture the main forces that drive the process.

The effect of flattering wages is visible even in firms' turnover. Firm number increases strongly (9.9%) at the beginning of the run, almost up to 1,100 during the Brexit period. Firms rose because they can pay high skilled workers less than they do usually, and this means less costs for the same production and perhaps more profits that give birth to new firms. Once that agents' adaptation fills the gap and the economy pass gets through Brexit the turnover switches negative and for the already above-mentioned reasons it follows the same path as the baseline model.

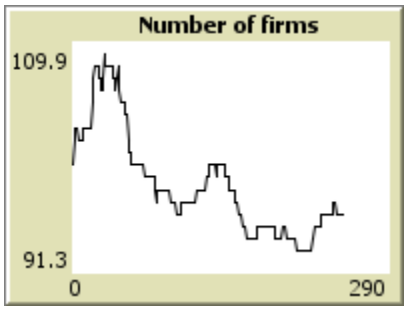

Figure 53: *Firms' turnover*

Initially, the fact that firms gain because of the lower wages is clear if we look at their profits. What happens is that perhaps is more difficult for a firm to find the workers it needs, but in case it gets positive results, the firm has a huge costs

advantage, indeed firms' profits reach the highest peak (2,830) of the whole run at the beginning before agents' adaptation close this gap.

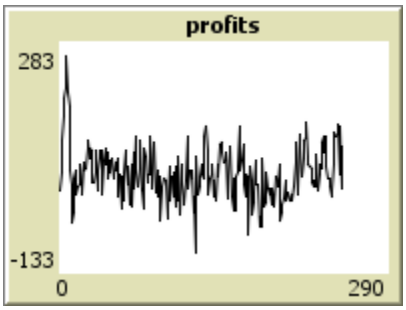

Figure 54: *Firms' profit evolution*

Even the price has a jump in the first part of the run because of the initial wage settings. It acts with a twofold effect: first, a higher rate of unemployment means an overall decrease of consumption and, second, if households get lower wage they consume less and this negatively affects the demand for goods. As we showed for the other variables, once the agents' adaption get through the initial phase the price is constant and close to 1.

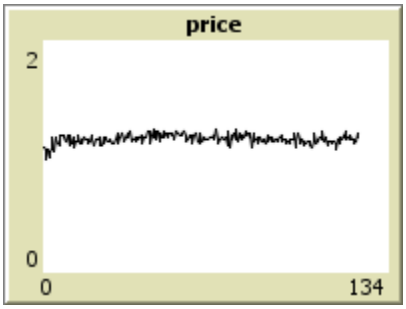

Figure 55: *Price evolution*

This experiment sheds the lights on the how agents' adaptation works and affect the results of the model. Indeed, at the beginning the change of setting affects all ours variable but once that agents adapt to the market conditions the model shows the same mechanisms of the baseline version. It is important to mention that even the speed of agents to adapt themselves affects significantly the outcome of the model and perhaps one of the future developments to be implement for further research is a better and slower agents' adaptation.

10.6 Less skilled immigrant population

In the next scenario we present a case where our population has a homogenous skill ratio. Indeed, we set that immigrants have the same probability to be high skill as natives have, 40%. In other words, we change the composition of immigrant population, moving the high skilled workers from a majority (75%) to a minority (40%). We keep the other settings of the baseline scenario constant (Figure 23), so that immigrants and natives have still differences in productivity and expected minimum wage. This experiment tries to carry out the hypothetical case where immigrants have not complemented skills with respect to natives, they have indeed, on average, the same skillset. This is not the case of Brexit since EEA immigrants are mainly high skilled while UK population is still low skilled majority (baseline scenario) but it can be considered as the whole non-native population in UK, without any distinction between EU and non-EU immigrants.

As expected, in this case the unemployment rate is a little higher (7%) than the initial rate of the baseline scenario (5%), this is because we have a lower number of high skilled workers in our population and this implies that firms find more difficult to get the skill ratio they need to complete the hiring process (Figure 56). Since immigrants are only a fraction of total population the impact on the final unemployment rate is "only" 2%.

Figure 56: *Unemployment rate*

Moreover, with the evolution of the model and before Brexit, the unemployment rate slightly decreases due to agents' adaptation. Indeed, since high skilled workers are scarce in the market, firms have to increase their wages in order to hire as much as possible. In the pre-Brexit period ($t < 80$) the wages paid to high skilled workers double, starting from an average of 1.05 it reaches 2.10 at $t = 80$ (Figure 57). The full employment of high workers allows firms to hire even more low skilled households thus decreasing the unemployment rate to 4% at $t = 80$.

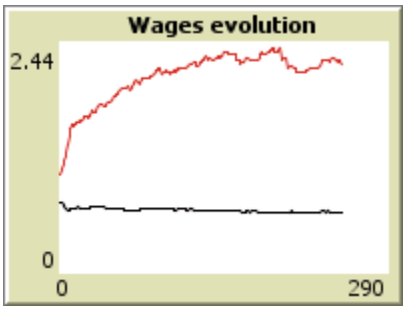

Figure 57: *Wages evolution*

Furthermore, the unemployment rate remains stable during Brexit ($80 < t < 110$), while it starts to rapidly changes straight after. Indeed, as soon as the transition period finishes the unemployment rate of our model sharply increases, closing at its highest level at the end of the run (8%).

It happens that the shock that hits the labour market is enough to block firms' hiring process in the next period. Indeed, our labour force decreases of 2.5% and this affect firms with a twofold effect. First, it shrinks the demand for goods (Figure 58) and second gets harder to find the high skilled workers they need to complete the hiring process.

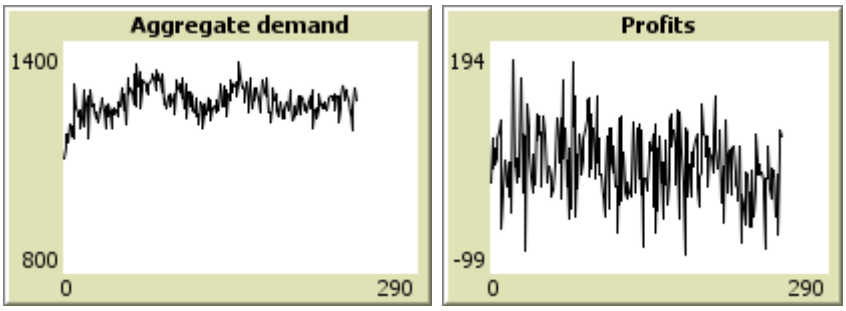

Figure 58: *Demand for goods (left) and profits of firms (right)*

The decrease of the demand affects negatively firms' profits (Figure 58): since firms have to increase their wages to high skills workers they increase their costs, on the contrary the revenues decrease with the demand, and this implies a negative firms' turnover (Figure 59). Moreover, new firms cannot arise because they do not find in the market the high skilled workers necessary to start their activity. The latter affects negatively both the turnover of firms because the ones that bankrupt are not replaced by new ones, and the unemployment rate because the remaining firms are able to hire only a part of the workers left unemployed.

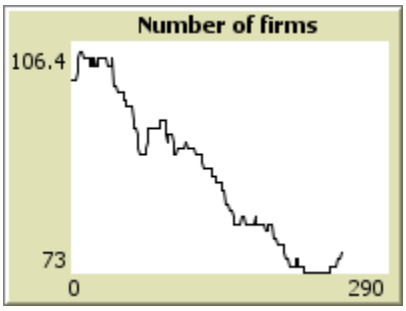

Figure 59: *Turnover of firms*

If we look at the unemployment rate split by skill level these results are even more clear. Indeed, in the first part they follow more or less the same path while after Brexit they take opposite directions (Figure 60). The unemployment of high skilled continues to decrease, reaching almost the full employment at the end of the run (2%), on the other hand since there are few high skilled workers available on the market it becomes really difficult for firms complete the hiring process. Therefore, the unemployment rate of low skilled workers strongly increases because they are left unemployed by bankruptcy firms and not anymore hired by the others.

Figure 60: *Unemployment rate of high skill (red) and low skill (black) households*

The major cause is therefore the high rate of firm bankruptcy and the consequences are evident even in the global price and GDP. As a matter of fact, firms decrease on number, but they increase their size. At the beginning each firm had an average of 10 workers while at the end of the run there are 40 firms with more than 50 employees, 60 with more than 25, 520 between 25 and 10 while only 110 firms have 10 employees or less. Anyway, this concentration is not enough to replace the negative turnover that brings firms to decrease of 27% during the entire run. This implies an decrease of GDP of 2.5% and a 4% inflation rate (Figure 61).

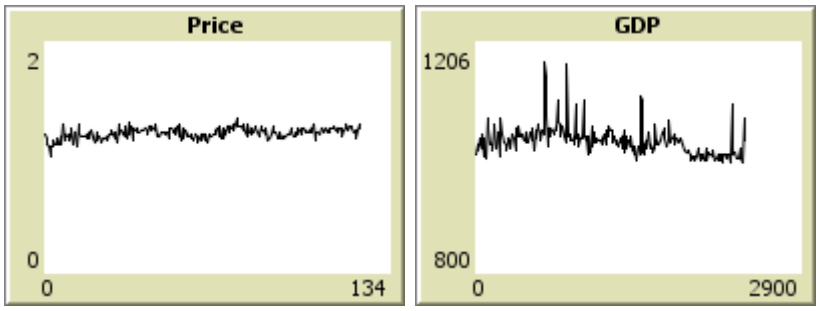

Figure 61: *Price (left) and GDP (right) evolution*

The combination of inflation and GDP recession brings the real GDP to tumble even more (Figure 62). Indeed, it reaches its peak exactly before Brexit and its minimum at the end of the run, with and overall fall of 8%.

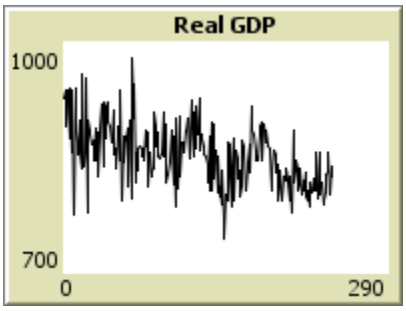

Figure 62: *Real GDP evolution*

The difference with the baseline scenario comes out if we look at the unemployment rate of immigrants against natives (Figure 63). Indeed, since in the baseline case immigrants are mainly high skilled their unemployment rate was decreasing for the entire process. On the contrary, since in this scenario they have the same skill ratio of natives they face the same problems with an unemployment rate than strongly increase in the third part of the run.

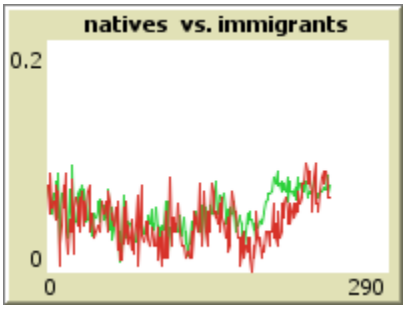

Figure 63: *Unemployment rate of native (green) and immigrants (red)*

This scenario reflects the problems that can arise in an economy where a shock on its population brings a negative firms' turnover in combination with a decrease of demand. The effects are similar to the baseline scenario even if our immigrant population is less skilled with the only difference that the entire population is negatively affected. This is because even if we remove more low skilled workers than before, in any case we decrease the number of high skilled workers already scarce in the market as well as the demand, resulting in worse negative consequences on our economy.

10.7 Increasing expectations

As last experiment, we propose an economy where the expected minimum wages of high and low skilled workers is higher. Indeed, we increase up to 0.8 and 0.9 the average expected minimum wage of low skilled immigrants and natives respectively. Nevertheless, we rise to 1.1 the average minimum wage of high skilled immigrants and up to 1.2 the one of high skilled natives. The other settings are equal to the baseline scenario (Figure 23), so the average wage offered by the firms is equal to 0.75 for low skilled workers while it is 1.05 for high skilled ones. In other words, with this settings, immigrants have the same expectations that natives had previously while native expectations are now higher from the ones offered in the labour market. We propose this scenario to see how an economy where the expectations of workers are, on average, higher than the possibilities offered by firms can react to such a shock as Brexit.

The effect of the new settings is immediately clear if we look at the unemployment rate (Figure 64), its level at the beginning is 18% against the usual 5%. This is due to households' expectations that do not match with the wages offered by firms, they prefer to stay unemployed and get the subsidy instead of working under these conditions. Thereafter, its level sharply drops thanks to the agents' adaptation that impacts the unemployment rate with a twofold effect. First, firms that are not able to find their workers increase their wages, on the other hand households decrease their expectations if unemployed for some periods.

Figure 64: *Unemployment rate*

The unemployment rate decreases up to the 4% at the beginning of Brexit ($t = 80$), that is the same level of the baseline scenario. Indeed, from $t = 80$ onwards this scenario has the same results as the first case we presented, that is the increasing of unemployment rate due to the mismatch of skills in the labour market. As before, firms are not able to find the amount of high skilled workers they need stopping the hiring process. Moreover, new firms cannot open for the same reasons and the high cost for high skilled workers bring several firms to bankruptcy (Figure 65).

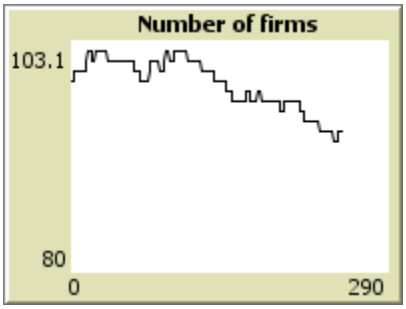

Figure 65: *Turnover of firms*

The path of unemployment rate is clear if we focus on its evolution in the first part of the run. At the beginning, when the model still needs to find its balance, 73% of low skilled households and 60% of high skilled ones are unemployed. Anyway, they adjust quickly their expectations and the model find its usual path before Brexit and this is the reason why the results are exactly the same in the second and third part of the run. The results could be totally different if during the transition period there are still differences between the expectations of firms and workers, indeed this could bring to the economy even worst consequences due to bigger shortage of high skilled workers.

Figure 66: *Unemployment rate of high skill (red) and low skill (black) households in the first part of the run*

model finds its equilibrium before Brexit even because firms increase quickly their wages (Figure 67), it especially happens for high skilled workers. Nevertheless, the

effect is less important for low skilled workers because they are less requested so that the adaptation is less effective. Furthermore, in the second and third part of the run, the wages evolution presents the same results as the baseline scenario, indeed high skilled workers end with the same average wage (1.9); the same happens for low skilled households that at the end of the run have an average wage of 0.65.

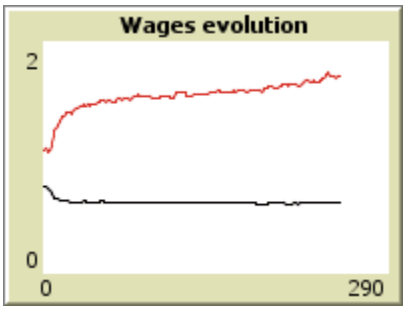

Figure 67: *Wages evolution*

Even the GDP evolution is consistent with the previous results (Figure 68): it increases in the first part of the run as the unemployment rate does, it is stable during Brexit and slightly decreases after it. Initially, it sharply increases because of agents adaptation that brings to a decrease of the unemployment rate, firms hire more workers and so their production is higher. On the other hand, in the second and third part it follows the same path of the baseline scenario, the final slight decrease is indeed correlated with the increase of unemployment rate and negative firms turnover due to the shortage of high skilled workers.

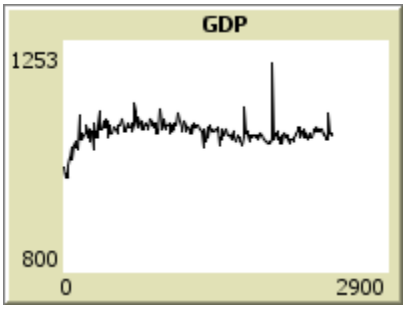

Figure 68: *GDP evolution*

The same happens to the price (Figure 69), it slightly decreases at the beginning because firms production increases more than the demand due to sharp decrease of the unemployment rate. Moreover, the price slightly but constantly increases until the end of Brexit as our economy experiences economic growth. Moreover, the same initial mechanism happens at the end where the production of firms decrease is offset by an

even higher decrease of demand, this causes a small decrease of price. Anyway, the oscillations of price are never more than 2%, it is quite stable during the entire run.

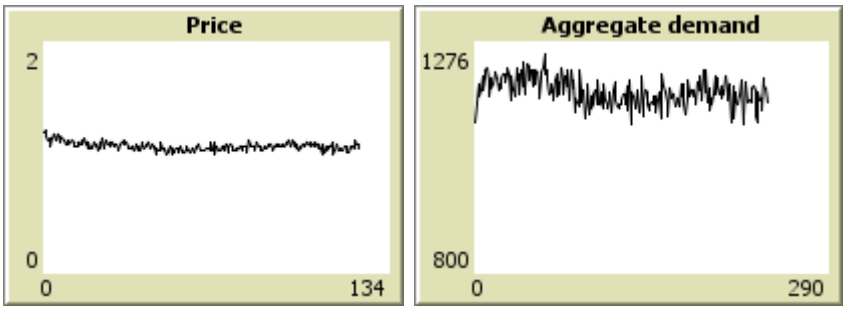

Figure 69: *Price (left) and demand for goods (right) evolution*

The real GDP evolution is more marked since price and GDP move in opposite directions. Indeed, the initial price decrease and GDP increase bring real GDP to a boost of 25% in the first part of the run. The same happens in the second part, while at the end both price and GDP decrease. Anyway, the price is more stable than the GDP and this reflects final real GDP decreasing.

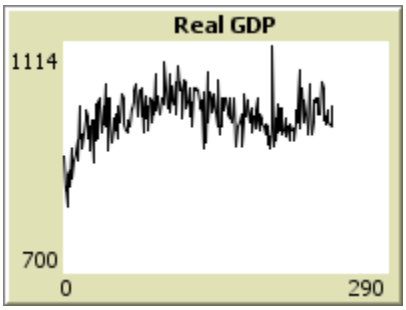

Figure 70: *Real GDP evolution*

At the same time, profits of firms slightly decrease during the entire run, especially after Brexit. This happens for the same reason that brings a negative turnover of firms. Indeed, the shortage of high skilled workers brings costs to increase too much with respect to revenues, firms are not then able to cover their expenses and bankrupt. The costs increase as wages do, the ones for high skilled workers almost double while for low skilled they only decrease a little.

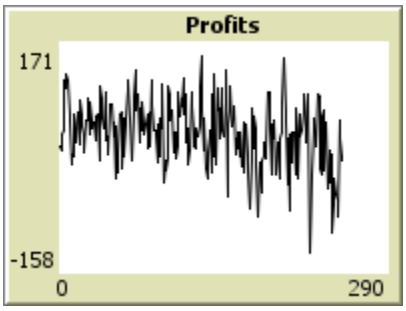

Figure 71: *Profits evolution*

This scenario sheds the lights on the evolution of a labour market where household expectations are way higher than the wages offered by firms. Anyway, since the model reaches its equilibrium before the happening of Brexit, we have the same results of the baseline scenario from $t = 80$ onwards. The economic consequences could be indeed amplified if Brexit happens exactly in a period where there is a gap of expectations. Indeed, the latter may bring to an even more shortage of high skilled workers, this would bring our economy in a worst recession. Nevertheless, the agent adaptation is quite quick in this model and a better tuning might be applied in further development.

11 Conclusions

In our work we try to assess the economic impact of immigration. In particular, we refer to Brexit as a study,

In Chapter 2, we have described the real-world phenomena that have inspired the research: Brexit, its possible outcomes and consequences.

In Chapter 3, we analyzed the literature related to the economic impact of immigration, in particular we focused on the effect on wages, unemployment rate and public budget.

In Chapter 4, we have presented the main characteristics of Agent-Based Models (ABMs). In particular, we have described the differences with respect to Equation-Based Models and the situations where ABMs are most beneficial.

In Chapter 5, we reported the first version of our Model. Although it is quite simple, it already represents an economic cycle with stable prices and economic growth. Moreover, it represents some basic economic interactions happening in a labor market where firms decide to hire or fire workers based on their previous profits.

In Chapter 6, we have described the second version of our Model. In particular, we added some powerful characteristics of Agent-Based Models such as heterogeneity among our agents and their ability to adapt at the market conditions with the evolution of the model.

In Chapter 7, we examined the third version of the model that represents a dual skill labor market. Indeed, we divided our households between high and low skill, with differences in average minimum wages expectations and productivity.

In Chapter 8, we presented the results of the Model III and its sensitivity of main parameters. The main results are that high skill households have a lower rate of unemployment because they are a population minority and they have more job opportunities. For the same reason they are able to increase slightly their average wage; the opposite happens to low skill workers. Moreover, our economy experienced a small economic growth with GDP increase and overall unemployment rate decrease due to adaptation of agents during the run. Nevertheless, the model turns out to be robust to the variation of the initial parameters.

In Chapter 9, we analyzed the final version of the model. In this version, we added immigrants as third category of agents. Moreover, in order to simulate Brexit, we set that through a part of the run unemployed immigrants and firms have a probability to leave the market.

Finally, in Chapter 10, we described the results of the Model IV with seven different scenarios. Each scenario tries to carry out a different outcome both of Brexit and its economic consequences. In particular we analyzed the following scenarios:

i) due to higher probabilities we presented as baseline model a “smooth” Brexit;

ii) “hard-Brexit”;

iii) “Australian model”;

iv) increase of automation;

v) alignment of wages;

vi) less skilled immigrant population;

vii) increase of wage expectations.

The main result of the Baseline scenario is that cutting immigration our economy experienced a shortage of high skilled workers. The shortage of high skill workers affects our economy with a twofold effect. First, firms that are not able to find the high skill workers they need stop entirely the hiring process and for the same reason new firms cannot arise. Second, firms have to increase their wages in order to hire the few high skill workers available on the market and this means an increase of costs. The consequences are a negative turnover of firms, increasing of unemployment rate and economic recession. In particular, the worst consequences are faced by low skilled households. The magnitude of these effects can be smaller of bigger based on the different scenario. In particular, these effects are amplified in the "hard-Brexit" due to higher probabilities of firms and agents to leave the market. Even the rise of automaton boosts the shortage of high skill workers that intensify the above-mentioned consequences on our economy. On the contrary, the impact is smoother in the case of "Australian model" and less skilled immigrant population since the shortage is less evident.

12 Further developments

Further developments include a better tuning of agent adaptation, indeed sometimes agents adapt to the market conditions too quickly with respect to changes in parameters. Moreover, we should include a dynamic productivity of firms in order to capture technological changes. Nevertheless, we can include new categories of agents in order to get a more realistic representation of our economy. In particular, we should add the financial market in order to have a better control of investment cycle that now it is not taken into account. Moreover, the presence of the public administration could sheds the light on the effect of immigration on taxes and public budget.

Appendices

A Model I Code

```
Breed [ households household ]
Breed [ firms firm ]
Globals [ quantity  price priceList  totalSubsidies
          singleSubsidy  productivity consume demand ]
Firms-own [  myWages mySubsidies nEmployee production
             revenues costs profits myPrice wageAmount subsidyAmount ]
Households-own [ worker myFirm myWage mySubsidy minWage  ]

to setup
  clear-all
  setup-households
  setup-firms
  reset-ticks
  set consume  0.9
  set productivity 1
end

to setup-households
  create-households 100 [
    setxy random-xcor random-ycor
    set shape "person"
  ]
end


to setup-firms
  create-firms 10 [
    setxy random-xcor random-ycor
    set shape "house"
   ]
end

to go
  firstEmployment
  wageSetting
```

```
  minWageSetting
  hiringFiring
  firmWages
  generalSubsidy
  firmSubsidies
  firmProduction
  HouseholdsIncome
  householdsDemand
  clearing
  firmsProfit
  closing
tick
end

to firstEmployment
  if ticks = 0  [
   ask households [ ifelse random-float 10 > 3
    [ set  myFirm one-of firms ]
    [ set myFirm nobody ]
   ]
  ]

end


to hiringFiring
  if ticks > 0 [
   ask firms [
    if profits > 0 [
      ifelse count  households with [ myFirm = nobody ] = 0 [] [
        ask one-of households with
        [ myFirm = nobody and wageAmount > minWage ]
         [ set myfirm  myself ]
        ]
      ]
    if profits < 0  [
     ask one-of households with [ myFirm =  myself ]
      [ set myFirm nobody ]
     ]
    ]
```

```
   ]

end

to wageSetting
  ask firms [ set wageAmount 0.8 + random-float 0.4 ]

end

to minWageSetting
  ask households [ set minWage random-normal 1 0.1 ]

end

to subsidySetting
  ask firms [ set subsidyAmount wageAmount * 0.5 ]

end

to firmWages
  ask firms [
   set nEmployee count households with [ myself = myFirm ]
   set myWages wageAmount * nEmployee
  ]

end




to generalSubsidy
  set totalSubsidies  ( count households with
  [ myFirm = nobody ] * [ subsidyAmount ] of firms )
  set singleSubsidy totalSubsidies / sum [ nEmployee ] of firms

end

to firmSubsidies
  ask firms [
   set mySubsidies singleSubsidy * nEmployee
```

```
  ]

end

to firmProduction
  ask firms [ set production nEmployee * productivity ]

end

to householdsIncome
  ask households [
   if myFirm != nobody  [ set myWage wageAmount ]
   if myFirm = nobody  [ sey mySubsidy subsidyAmount ]
   ]

end


to householdsDemand
  set demand sum [ myWage ] of households * consume +
             sum [ mySubsidy ] of households

end

to clearing
  set price demand  / sum [ production ] of firms

end


to firmsProfit
  ask firms [
   set myPrice price + random-normal 0 0.2
   set revenues myPrice * production
   set costs myWages + mySubsidies
   set profits revenues - costs
  ]

end
```

```
to closing
  ask firms with [ profits < -2.5 ] [
    ask households with [ myFirm = myself ] [ set myFirm nobody ]
    die
  ]

end
```

B Model II Code

```
Breed [ households household ]
Breed [ firms firm ]
Globals [ quantity  price priceList  totalSubsidies singleSubsidy
          subsidyAmount productivity consume demand ]
Firms-own [  myWages mySubsidies nEmployee production  revenues
             costs profits myPrice wageAmount k notEnoughWorkers gotAllWorkers ]
Households-own [ worker myFirm myWage mySubsidy minWage unemployedTime  ]

to setup
  clear-all
  setup-households
  setup-firms
  reset-ticks
  set subsidyAmount  0.5
  set consume  0.9
  set productivity 1
end

to setup-households
  create-households 100 [
    setxy random-xcor random-ycor
    set shape "person"
    set unemployedTime 0
  ]
end


to setup-firms
  create-firms 10 [
    setxy random-xcor random-ycor
    set shape "house"
    set NotEnoughWorkers 0
    set gotAllWorkers 0
  ]
end

to go
  firstEmployment
```

```
  wageSetting
  minWageSetting
  changeExpectations
  changeWages
  firmWages
  wantedWorkers
  hiringFiring
  generalSubsidy
  firmSubsidies
  firmProduction
  HouseholdsIncome
  householdsDemand
  clearing
  firmsProfit
  closing
tick
end

to firstEmployment
  if ticks = 0  [
    ask households [
     ifelse random-float 10 > 3
      [ set  myFirm one-of firms ]
      [ set myFirm nobody ]
     ]
    ]

end

to wageSetting
  ask firms [
   set wageAmount 0.8 + random-float 0.4
   ]
end


to minWageSetting
  ask households [
   set minWage  0.8 + random-float 0.4
   ]
```

```
end


to changeExpectations
  ask households  [
    if myFirm = nobody [ set unemployedTime unemployedTime + 1 ]
    if unemployedTime = 3 [ set minWage 0.9 * minWage ]
    if unemployedTime = 4 [ set minWage 0.9 * minWage ]
    if unemployedTime >= 5 [ set minWage 0.8 * minWage ]
    ]
end


to firmWages
  ask firms [
    set nEmployee count households with [ myself = myFirm ]
    set myWages wageAmount * nEmployee
    ]

end

to changeWages
  ask firms [
    if gotAllWorkers = 3 [ set wageAmount wageAmount * 0.9 ]
    if gotAllworkers = 4 [ set wageAmount wageAmount * 0.9 ]
    if gotAllWorkers >= 5 [ set wageAmount wageAmount * 0.8 ]
    if notEnoughWorkers = 2 [ set wageAmount wageAmount * 1.1 ]
    if notEnoughWorkers = 3 [ set wageAmount wageAmount * 1.1 ]
    if notEnoughWorkers >= 4 [ set wageAmount wageAmount * 1.2 ]
  ]
end



to wantedWorkers
  if ticks > 0 [
    ask firms [
      if profits / production > 0 [ set k 1 ]
      if profits / production > 0.1 [ set k 2 ]
      if profits / production > 0.2 [ set k 3 ]
```

```
      if profits / production > 0.4 [ set k 4 ]
   ]
  ]
end


to hiringFiring
  if ticks > 0 [
         ask firms [
           let potentialWorkers count households with [
             myFirm = nobody and
             minWage <= [ wageAmount ] of myself
              ]
           let kk min list k potentialWorkers
           ifelse kk = k
            [ set gotAllWorkers gotAllworkers + 1 ]
            [ set gotAllWorkers 0 ]
           ifelse kk < k
            [set notEnoughWorkers NotEnoughWorkers + 1 ]
            [ set NotEnoughWorkers 0 ]
            if profits > 0 and potentialWorkers > 0 [
             ask n-of kk households with [
             minWage <= [ wageAmount ] of myself
             ]
             [ set myfirm  myself
               set myWage [ wageAmount ] of myself
               set unemployedTime 0 ]
             ]

              if profits < 0  [
               ask one-of households with [ myFirm =  myself ]
                [ set myFirm nobody ]
            ]
          ]
        ]
end



to generalSubsidy
```

```
  set totalSubsidies  ( count households with
    [ myFirm = nobody ] * subsidyAmount )
  set singleSubsidy totalSubsidies / sum [ nEmployee ] of firms

end

to firmSubsidies
  ask firms [
   set mySubsidies singleSubsidy * nEmployee
   ]

end

to firmProduction
  ask firms [
   set production nEmployee * productivity
   ]

end

to householdsIncome
  ask households [
   if myFirm != nobody
   [ set myWage  [ wageAmount ] of myFirm ]
   if myFirm = nobody  [set mySubsidy subsidyAmount ]
   ]

end

to householdsDemand
  set demand sum [ myWage ] of households * consume +
             sum [ mySubsidy ] of households

end

to clearing
  set price demand  / sum [production] of firms

end
```

```
to firmsProfit
  ask firms [
    set myPrice price + random-normal 0 0.2
    set revenues myPrice * production
    set costs myWages + mySubsidies
    set profits revenues - costs
  ]

end

to closing
  ask firms with  [ profits < -5 ]
   [ ask households with
     [ myFirm = myself ]
      [ set myFirm nobody ]
    die
   ]

end
```

C Model III Code

```
Breed [ households household ]
Breed [ firms firm ]
Globals [ quantity price totalSubsidies singleSubsidy
          subsidyAmount demand ]

firms-own [ myWages mySubsidies nHighEmployee nLowEmployee
            production revenues costs profits myPrice
            lowWageAmount highWageAmount hireTot hireLow
            hireHigh notEnoughLowWorkers notEnoughHighWorkers
            fireLow fireHigh fireTot gotAllLowWorkers gotAllHighWorkers ]

Households-own [ worker myFirm myWage mySubsidy minWage
                 unemployedTime householdType ]

to setup
  clear-all
  reset-ticks
  setup-households
  setup-firms
  hiring
  set subsidyAmount  0.5
end

to setup-households                                  ;
  create-households nHouseholds
  [
    setxy random-xcor random-ycor
    set shape "person"
    set unemployedTime 0
    set myFirm nobody
    ifelse random-float 100 > highSkilledHouseholdOnPopulation
    [ set householdType 1
      set color green
      set minWage random-normal avMinWageLowSkill stDevMinWageLowSkill ]
    [ set householdType 0
      set color red
      set minWage random-normal avMinWageHighSkill stDevMinWageHighSkill ]
  ]
```

```
end


to setup-firms
  create-firms nFirms
  [
    setxy random-xcor random-ycor
    set shape "house"
    set color blue
    set NotEnoughLowWorkers 0
    set NotEnoughHighWorkers 0
    set gotAllLowWorkers 0
    set gotAllHighWorkers 0
    set profits 0.1
    set highWageAmount random-normal avHighWageAmount stDevHighWageAmount
    set lowWageAmount  random-normal avLowWageAmount stDevLowWageAmount
    set hireTot round random-normal averageWorkers stDevAverageWorkers
    set hireHigh  round ( hireTot * highSkilledWorkersRatio / 100 )
    set hireLow hireTot - hireHigh
  ]
end

to go

  if count households with [ myFirm = nobody ] = nHouseholds [ stop ]
  if ticks > 0 [ wantedWorkers ]
  if ticks > 0 [ opening ]
  if ticks > 0 [ cuttingWorkers ]
  if ticks > 0 [ closing ]
  if ticks > 0 [ hiring ]
  if ticks > 0 [ firing ]
  changeExpectations
  changeWages
  firmWages
  if not any? firms [ stop ]
  generalSubsidy
  firmSubsidies
  firmProduction
  HouseholdsIncome
  householdsDemand
```

```
  clearing
  firmsProfit
  if ticks > 250 [stop ]
tick


end

to wantedWorkers
    ask firms [
     if production > 0 [
      if profits / production > 0.1 [ set hireTot  2 ]
      if profits / production > 0.2 [ set hireTot  3 ]
      if profits / production > 0.3 [ set hireTot  4 ]
      if profits / production > 0.5 [ set hireTot  5 ]
      set hireHigh  round ( hireTot * highSkilledWorkersRatio / 100 )
      set hireLoW hireTot - hireHigh
   ]
  ]
end



to opening

  ask firms [
    if profits >  0.5 * production  [
      hatch 1
      [
       set shape "house"
       setxy random-xcor random-ycor
       set color blue
       set NotEnoughLowWorkers 0
       set NotEnoughHighWorkers 0
       set gotAllLowWorkers 0
       set gotAllHighWorkers 0
       set profits 0.1
       set highWageAmount random-normal avHighWageAmount stDevHighWageAmount
       set lowWageAmount  random-normal avLowWageAmount stDevLowWageAmount
       set hireTot round random-normal averageWorkers stDevAverageWorkers
```

```
        set hireHigh  round ( hireTot * highSkilledWorkersRatio / 100 )
        set hireLow hireTot - hireHigh
        let potentialLowWorkers count households with [
         myFirm = nobody and
         minWage <= [ lowWageAmount ] of myself and
         householdType = 1
         ]
        let potentialHighWorkers count households with [
         myFirm = nobody and
         minWage <= [ highWageAmount ] of myself and
         householdType = 0 ]
        ifelse potentialHighWorkers < hireHigh or
               potentialLowWorkers < hireLow
         [ die ]
         [ ask n-of hireLow households with [
          minWage <= [ lowWageAmount ] of myself and
          householdType = 1 and
          myFirm = nobody
         ]
           [ set myfirm  myself
             set myWage [ lowWageAmount ] of myself
             set unemployedTime 0
             ]
          ask n-of hireHigh households with [
           minWage <= [ highWageAmount ] of myself and
           householdType = 0 and
           myFirm = nobody
          ]
           [ set myfirm  myself
             set myWage [ highWageAmount ] of myself
             set unemployedTime 0
           ]
          ]
         ]
        ]
       ]
end
```

```
to cuttingWorkers
   ask firms [
    if production > 0 [
      if profits / production < -0.1 [ set fireTot  2 ]
      if profits / production < -0.2 [ set fireTot  3 ]
      if profits / production < -0.3 [ set fireTot  4 ]
      if profits / production < -0.5 [ set fireTot  5 ]
      set FireHigh  round ( fireTot * highSkilledWorkersRatio / 100 )
      set fireLoW fireTot - fireHigh
   ]
  ]

end
```

```
 to closing
  ask firms [
    if production > 0 [
    if  ( profits / production ) < -0.75 [
     ask households with [myFirm = myself] [set myFirm nobody]
     die
    ]
   ]
  ]

end
```

```
to hiring

  ask firms [
    if profits > 0 [
     let potentialLowWorkers count households with [
      myFirm = nobody and
      minWage <= [ lowWageAmount ] of myself and
```

```
 householdType = 1
]
let LL min list hireLow potentialLowWorkers
ifelse LL = hireLow
[ set gotAllLowWorkers gotAllLowworkers + 1 ]
[ set gotAllLowWorkers 0 ]
ifelse LL < hireLow
[ set notEnoughLowWorkers NotEnoughLowWorkers + 1 ]
[set NotEnoughLowWorkers 0 ]
if profits > 0 and potentialLowWorkers > 0 [
 ask n-of LL households with [
  minWage <= [ lowWageAmount ] of myself and
  householdType = 1 and
  myFirm = nobody
 ]
 [ set myfirm  myself
   set myWage [ lowWageAmount ] of myself
   set unemployedTime 0
 ]
]
let potentialHighWorkers count households with [
 myFirm = nobody and
 minWage <= [ highWageAmount ] of myself and
 householdType = 0
 ]
let HH min list hireHigh potentialHighWorkers
ifelse HH = hireHigh
[ set gotAllHighWorkers gotAllHighworkers + 1 ]
[ set gotAllHighWorkers 0 ]
ifelse HH < hireHigh
[ set notEnoughHighWorkers NotEnoughHighWorkers + 1 ]
[set NotEnoughHighWorkers 0 ]
if profits > 0 and potentialHighWorkers > 0 [
 ask n-of HH households with [
  minWage <= [ highWageAmount ] of myself and
  householdType = 0 and
  myFirm = nobody
  ]
[ set myfirm  myself
  set myWage [ highWageAmount ] of myself
```

```
      set unemployedTime 0
     ]
    ]
   ]
  ]
end



to firing

  ask firms [
    if profits < 0 [
     set gotAllHighWorkers 0
     set NotEnoughHighWorkers 0
     set gotAllLowWorkers 0
     set NotEnoughLowWorkers 0
     let myHighWorkers count households with [
      myFirm = myself and
      householdType = 0
      ]
     let FH min list fireHigh myHighWorkers
     ifelse  FH = myHighWorkers and FH > 0
      [ ask n-of (FH - 1) households with [
       myFirm =  myself and
       householdType = 0
       ]
       [ set myFirm nobody ]
      ]
     [ ask n-of FH  households  with [
      myFirm =  myself and
      householdType = 0
      ]
      [ set myFirm nobody ]
     ]
    let myLowWorkers count households with [
     myFirm = myself and
     householdType = 1
     ]
    let FL min list fireLow myLowWorkers
```

```
    ifelse  FL = myLowWorkers and FL > 0
      [ ask n-of (FL - 1) households with
        [myFirm =  myself and householdType = 1 ]
        [ set myFirm nobody ]
      ]
      [ ask n-of FL households with
      [myFirm =  myself and householdType = 1 ]
      [ set myFirm nobody ]
     ]
    ]
   ]


end


to changeExpectations

  ask households  [
    if myFirm = nobody
     [ set unemployedTime unemployedTime + 1 ]
    if unemployedTime = 3 [ set minWage 0.9 * minWage ]
    if unemployedTime = 4 [ set minWage 0.9 * minWage ]
    if unemployedTime >= 5 and minWage > subsidyAmount
     [ set minWage 0.8 * minWage ]
  ]

end
```

```
to firmWages

  ask firms [
    set nHighEmployee count households with
    [ myself = myFirm and  householdType = 0 ]
    set nLowEmployee  count households with
    [ myself = myFirm and  householdType = 1 ]
    set myWages  ( ( lowWageAmount * nLowEmployee ) +
                   ( highWageAmount * nHighEmployee ) )
    if  nHighEmployee = 0 and  nLowEmployee = 0 [ die ]
    if  nHighEmployee + nLowEmployee > 10 and
        nHighEmployee + nLowEmployee <= 25
        [ set color grey ]
    if  nHighEmployee + nLowEmployee > 25 and
        nHighEmployee + nLowEmployee <= 50
        [ set color brown ]
    if  nHighEmployee + nLowEmployee > 50
        [ set color white ]

  ]

end

to changeWages

  ask firms [
    if gotAllLowWorkers = 3
    [ set LowWageAmount LoWWageAmount * 0.9 ]
    if gotAllHighWorkers = 3
    [ set highWageAmount highWageAmount * 0.9 ]
    if gotAllLowworkers = 4
    [ set lowWageAmount lowWageAmount * 0.9 ]
    if gotAllHighWorkers = 4
    [ set highWageAmount highWageAmount * 0.9 ]
    if gotAllLowWorkers >= 5 and lowWageAmount > subsidyAmount
    [ set lowWageAmount lowWageAmount * 0.8 ]
    if gotAllHighWorkers >= 5 and highWageAmount > subsidyAmount
    [ set highWageAmount highWageAmount * 0.8 ]
    if notEnoughLowWorkers = 2
```

```
    [ set lowWageAmount lowWageAmount * 1.1 ]
    if notEnoughHighWorkers = 2
    [ set highWageAmount highWageAmount * 1.1 ]
    if notEnoughLowWorkers = 3
    [ set lowWageAmount lowWageAmount * 1.1 ]
    if notEnoughHighWorkers = 3
    [ set highWageAmount highWageAmount * 1.1 ]
    if notEnoughLoWWorkers >= 4 and lowWageAmount <  4
    [ set lowWageAmount lowWageAmount * 1.2 ]
    if notEnoughHighWorkers >= 4 and highWageAmount < 5
    [ set highWageAmount highWageAmount * 1.2 ]
  ]

end



to generalSubsidy

  set totalSubsidies
  ( count households with [ myFirm = nobody ] * subsidyAmount )
  set singleSubsidy
  totalSubsidies / (sum [ nHighEmployee ] of firms +
                    sum [ nLowEmployee ] of firms )

end

to firmSubsidies

  ask firms [
    set mySubsidies singleSubsidy *
    ( nHighEmployee + nLowEmployee )

 end

to firmProduction

  ask firms [
    set production   ( nHighEmployee * productivityHigh +
                       nLowEmployee * productivityLow )
```

```
  ]

end

to householdsIncome

  ask households [
    ifelse myFirm = nobody
    [ set mySubsidy subsidyAmount ]
    [ set mySubsidy 0 ]
  ]

end

to householdsDemand

  set demand  ( ( sum [ myWage ] of households * consumption ) +
                  sum [ mySubsidy ] of households )

end

to clearing

  set price demand  / sum [ production ] of firms

end


to firmsProfit
  ask firms [
    set myPrice price + random-normal 0 0.1
    set revenues myPrice * production
    set costs myWages + mySubsidies
    set profits revenues - costs
   ; show highWageAmount
  ]

end
```

D Model IV Code

```
Breed [ natives native ]
Breed [ firms firm ]
Breed [ immigrants immigrant ]
Globals [ quantity price totalSubsidies singleSubsidy
          subsidyAmount demand households ]

firms-own [  myWages mySubsidies nHighNatEmployee nLowNatEmployee
             production nHighImmEmployee nLowImmEmployee
             revenues costs profits myPrice lowWageAmount
             highWageAmount  hireTot hireLow hireHigh
             notEnoughLowWorkers notEnoughHighWorkers fireLow
             fireHigh fireTot gotAllLowWorkers gotAllHighWorkers  ]

natives-own [ myFirm myWage mySubsidy minWage unemployedTime householdType ]
immigrants-own [ myFirm  myWage mySubsidy minWage unemployedTime householdType]

to setup
  clear-all
  reset-ticks
  setup-natives
  setup-immigrants
  setup-firms
  hiring
  set subsidyAmount  0.5
end

to setup-natives                                ;
  create-natives nNatives
  [
    setxy random-xcor random-ycor
    set shape "person"
    set unemployedTime 0
    set myFirm nobody
    ifelse random-float 100 > highSkillnatives
    [ set householdType 1
      set color green
      set minWage random-normal avMinWageLowSkill stDevMinWageLowSkill ]
    [ set householdType 0
```

```
      set color red
      set minWage random-normal avMinWageHighSkill stDevMinWageHighSkill ]


  ]
end

to setup-immigrants
    create-immigrants nImmigrants
  [
    setxy random-xcor random-ycor
    set shape "person"
    set unemployedTime 0
    set myFirm nobody
    ifelse random-float 100 > highSkillImmigrants
    [ set householdType 1
      set color yellow
      set minWage random-normal avMinWageLowSkillImm stDevMinWageLowSkillImm ]
    [ set householdType 0
      set color white
      set minWage random-normal avMinWageHighSkillImm stDevMinWageHighSkillImm ]


  ]

  set households (turtle-set natives immigrants)
end



to setup-firms
  create-firms nFirms
  [
    setxy random-xcor random-ycor
    set shape "house"
    set color blue
    set NotEnoughLowWorkers 0
    set NotEnoughHighWorkers 0
    set gotAllLowWorkers 0
    set gotAllHighWorkers 0
```

```
    set profits 0.1
    set highWageAmount random-normal avHighWageAmount stDevHighWageAmount
    set lowWageAmount  random-normal avLowWageAmount stDevLowWageAmount
    set hireTot round random-normal averageWorkers stDevAverageWorkers
    set hireHigh  round ( hireTot * highSkilledWorkersRatio / 100 )
    set hireLow hireTot - hireHigh


  ]
end



to go

  if ticks > 0 [ wantedWorkers ]
  if ticks > 0 [ opening ]
  if ticks > 0 [ cuttingWorkers ]
  if ticks > 0 [ closing ]
  if ticks > 0 [ hiring ]
  if ticks > 0 [ firing ]
  changeExpectations
  changeWages
  firmWages
  if not any? firms [ stop ]
  generalSubsidy
  firmSubsidies
  firmProduction
  HouseholdsIncome
  householdsDemand
  clearing
  firmsProfit
  if ticks > 50 and ticks < 80 [ leaving ]
  if ticks > 200 [stop ]
tick


end

to wantedWorkers
```

```
    ask firms [
     if production > 0 [
      if profits / production > 0.1 [ set hireTot  2 ]
      if profits / production > 0.2 [ set hireTot  3 ]
      if profits / production > 0.3 [ set hireTot  4 ]
      if profits / production > 0.5 [ set hireTot  5 ]
      set hireHigh  round ( hireTot * highSkilledWorkersRatio / 100 )
      set hireLoW hireTot - hireHigh
   ]
  ]
end



to opening

  ask firms [
    if profits >  0.5 * production  [
      hatch 1
      [
       set shape "house"
       setxy random-xcor random-ycor
       set color blue
       set NotEnoughLowWorkers 0
       set NotEnoughHighWorkers 0
       set gotAllLowWorkers 0
       set gotAllHighWorkers 0
       set profits 0.1
       set highWageAmount random-normal avHighWageAmount stDevHighWageAmount
       set lowWageAmount  random-normal avLowWageAmount stDevLowWageAmount
       set hireTot round random-normal averageWorkers stDevAverageWorkers
       set hireHigh  round ( hireTot * highSkilledWorkersRatio / 100 )
       set hireLow hireTot - hireHigh
       let potentialLowWorkers count households with [
        myFirm = nobody and
        minWage <= [ lowWageAmount ] of myself and
        householdType = 1
        ]
       let potentialHighWorkers count households with [
        myFirm = nobody and
```

```
        minWage <= [ highWageAmount ] of myself and
        householdType = 0
        ]
       ifelse potentialHighWorkers < hireHigh or potentialLowWorkers < hireLow
        [ die ]
        [ ask n-of hireLow households with [
          minWage <= [ lowWageAmount ] of myself and
          householdType = 1 and
          myFirm = nobody
          ]
          [ set myfirm  myself
            set myWage [ lowWageAmount ] of myself
            set unemployedTime 0 ]
          ask n-of hireHigh households with [
            minWage <= [ highWageAmount ] of myself and
            householdType = 0 and
            myFirm = nobody
          ]
          [ set myfirm  myself
            set myWage [ highWageAmount ] of myself
            set unemployedTime 0  ]
        ]
      ]
     ]
    ]
end



to cuttingWorkers
   ask firms [
    if production > 0  [
      if profits / production < -0.1 [ set fireTot  2 ]
      if profits / production < -0.2 [ set fireTot  3 ]
      if profits / production < -0.3 [ set fireTot  4 ]
      if profits / production < -0.5 [ set fireTot  5 ]
      set FireHigh  round ( fireTot * ( fireHighskillWorkersRatio ) / 100 )
      set fireLoW fireTot - fireHigh
   ]
  ]
```

```
end



 to closing
  ask firms [
    if production > 0 [
    if  ( profits / production ) < -0.60 [
     ask households with [myFirm = myself]
     [set myFirm nobody]
     die
    ]
   ]
  ]

end



to hiring

  ask firms [
    if profits > 0 [
     let potentialLowWorkers count households with [
      myFirm = nobody and
      minWage <= [ lowWageAmount ] of myself and
      householdType = 1
      ]
     let LL min list hireLow potentialLowWorkers
     show hireLow show potentialLowWorkers show LL
     ifelse LL = hireLow
      [ set gotAllLowWorkers gotAllLowworkers + 1 ]
      [ set gotAllLowWorkers 0 ]
     ifelse LL < hireLow
      [ set notEnoughLowWorkers NotEnoughLowWorkers + 1 ]
      [set NotEnoughLowWorkers 0 ]
```

```
     if profits > 0 and potentialLowWorkers > 0 [
      ask n-of LL households with [
       minWage <= [ lowWageAmount ] of myself and
       householdType = 1 and
       myFirm = nobody ]
      [ set myfirm  myself
        set myWage [ lowWageAmount ] of myself
        set unemployedTime 0
      ]
     ]
     let potentialHighWorkers count households with [
      myFirm = nobody and
      minWage <= [ highWageAmount ] of myself and
      householdType = 0
      ]
     let HH min list hireHigh potentialHighWorkers
    ; show hireHigh show potentialLowWorkers show HH
     ifelse HH = hireHigh
      [ set gotAllHighWorkers gotAllHighworkers + 1 ]
      [ set gotAllHighWorkers 0 ]
     ifelse HH < hireHigh
      [ set notEnoughHighWorkers NotEnoughHighWorkers + 1 ]
      [set NotEnoughHighWorkers 0 ]
     if profits > 0 and potentialHighWorkers > 0 [
      ask n-of HH households with [
        minWage <= [ highWageAmount ] of myself and
        householdType = 0 and
        myFirm = nobody]
     [ set myfirm  myself
       set myWage [ highWageAmount ] of myself
       set unemployedTime 0
     ]
    ]
   ]
  ]
end



to firing
```

```
  ask firms [
    if profits < 0 [
     set gotAllHighWorkers 0
     set NotEnoughHighWorkers 0
     set gotAllLowWorkers 0
     set NotEnoughLowWorkers 0
     let myHighWorkers count households with
      [ myFirm = myself and  householdType = 0 ]
     let FH min list fireHigh myHighWorkers
     let FH min list fireHigh myHighWorkers
      ;show FH
     ifelse  FH = myHighWorkers and FH > 0
      [ ask n-of (FH - 1) households with
        [ myFirm =  myself and householdType = 0 ]
        [ set myFirm nobody ]
      ]
      [ ask n-of FH  households  with
        [ myFirm =  myself and householdType = 0 ]
        [ set myFirm nobody ]
      ]
     let myLowWorkers count households with
      [ myFirm = myself and  householdType = 1 ]
     let FL min list fireLow myLowWorkers
      ifelse  FL = myLowWorkers and FL > 0
      [  ask n-of (FL - 1) households with
        [myFirm =  myself and householdType = 1 ]
        [ set myFirm nobody ]
      ]
      [ ask n-of FL households with
        [myFirm =  myself and householdType = 1 ]
        [ set myFirm nobody ]
      ]
     ]
    ]


end
```

```
to changeExpectations

  ask households  [
    if myFirm = nobody [ set unemployedTime unemployedTime + 1 ]
    if minWage > subsidyAmount + 0.1 [
    if unemployedTime = 3 [ set minWage 0.9 * minWage ]
    if unemployedTime = 4 [ set minWage 0.9 * minWage ]
    if unemployedTime >= 5 and minWage > subsidyAmount
      [ set minWage 0.8 * minWage ]
    ]
  ]

end


to firmWages

  ask firms [
    set nHighNatEmployee count natives with
    [ myself = myFirm and  householdType = 0 ]
    set nLowNatEmployee  count natives with
    [ myself = myFirm and  householdType = 1 ]
    set nHighImmEmployee count immigrants with
    [ myself = myFirm and  householdType = 0 ]
    set nLowImmEmployee  count immigrants with
    [ myself = myFirm and  householdType = 1 ]

     set myWages  ( ( lowWageAmount * nLowNatEmployee ) +
                    (lowWageAmount * nLowImmEmployee ) +
```

```
                    ( highWageAmount * nHighNatEmployee ) +
                    (highWageAmount * nHighImmEmployee)  )

    if  nHighNatEmployee + nLowNatEmployee +
        nLowImmEmployee + nHighImmEmployee > 10 and
        nHighNatEmployee + nLowNatEmployee +
        nLowImmEmployee + nHighImmEmployee  <= 25
         [ set color grey ]
    if  nHighNatEmployee + nLowNatEmployee +
        nLowImmEmployee + nHighImmEmployee  > 25 and
        nHighNatEmployee + nLowNatEmployee +
        nLowImmEmployee + nHighImmEmployee  <= 50
         [ set color brown ]
    if  nHighNatEmployee + nLowNatEmployee +
        nLowImmEmployee + nHighImmEmployee > 50
         [ set color white ]

 ]
 ; ask firms [show nLowEmployee show nHighEmployee    ]

end

to changeWages

  ask firms [
    if gotAllLowWorkers = 3
    [ set LowWageAmount LoWWageAmount * 0.9 ]
    if gotAllHighWorkers = 3
    [ set highWageAmount highWageAmount * 0.9 ]
    if gotAllLowworkers = 4
    [ set lowWageAmount lowWageAmount * 0.9 ]
    if gotAllHighWorkers = 4
    [ set highWageAmount highWageAmount * 0.9 ]
    if gotAllLowWorkers >= 5 and lowWageAmount > subsidyAmount
    [ set lowWageAmount lowWageAmount * 0.8 ]
    if gotAllHighWorkers >= 5 and highWageAmount > subsidyAmount
    [ set highWageAmount highWageAmount * 0.8 ]
    if notEnoughLowWorkers = 2
    [ set lowWageAmount lowWageAmount * 1.1 ]
    if notEnoughHighWorkers = 2
```

```
    [ set highWageAmount highWageAmount * 1.1 ]
    if notEnoughLowWorkers = 3
    [ set lowWageAmount lowWageAmount * 1.1 ]
    if notEnoughHighWorkers = 3
    [ set highWageAmount highWageAmount * 1.1 ]
    if notEnoughLoWWorkers >= 4 and lowWageAmount <  4
    [ set lowWageAmount lowWageAmount * 1.2 ]
    if notEnoughHighWorkers >= 4 and highWageAmount < 5
    [ set highWageAmount highWageAmount * 1.2 ]
  ]

end



to generalSubsidy

  set totalSubsidies  ( count households with [
                        myFirm = nobody ] * subsidyAmount )
 ;show totalSubsidies; total amount of subsidies to be paid ;
  set singleSubsidy totalSubsidies /
     (sum [ nHighNatEmployee ] of firms +
      sum [ nLowNatEmployee ] of firms +
      sum [ nHighImmEmployee ] of firms +
      sum [ nLowImmEmployee ] of firms )
 ;show singleSubsidy

end

to firmSubsidies

  ask firms [
    set mySubsidies singleSubsidy * (
     nHighNatEmployee + nLowNatEmployee +
     nHighImmEmployee + nLowImmEmployee )       ; firms pay a subsidiy for each emplo
  ]
  ;ask firms [show mySubsidies ]

  end
```

```
to firmProduction

  ask firms [
    set production   ( ( nHighNatEmployee * productivityHigh ) +
                       ( nLowNatEmployee * productivityLow  )+
                       ( nLowImmEmployee * productivityLowImm ) +
                       ( nHighImmEmployee * productivityHighImm  ) )
  ]
; ask firms [show production]

end

to householdsIncome

  ask households [
    ifelse myFirm = nobody
    [ set mySubsidy subsidyAmount ]
    [ set mySubsidy 0 ]                  ; set unemployed households
  ]
 ;ask households [ show myWage show mySubsidy]

end

to householdsDemand

  set demand  (
   ( sum [ myWage ] of households * consumption ) +
     sum [ mySubsidy ] of households ) +
    ( sum [ myWage ] of households / 100 * random-normal 11 2 )
 ;show demand

end

to clearing

  set price demand  / sum [ production ] of firms             ;clearing price
  ;show price
end
```

```
to firmsProfit
  ask firms [
    set myPrice price + random-normal 0 0.1     ; shock to each firm's price
    set revenues myPrice * production
    set costs myWages + mySubsidies   ; show costs show myPrice
    set profits revenues - costs
   ; show highwageamount
  ]

end



to leaving

  ask immigrants with [ myFirm = nobody ] [
    if random-float 100 > immigrantStaying
     [ die ]
  ]
  ask firms [
    if random-float 100 > firmStaying [
     ask households with [myFirm = myself]
      [set myFirm nobody]
      die
    ]
  ]
end
```

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