

The tendency of the rate of profit to fall

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1. Introduction

In this work we aim to simulate with NetLogo the functioning of a Marxian economy. In particular, we analyse one of the most controversial issue raised by Marx: the tendency of the rate of profit to fall.

Starting from a very simple economy, the user has several possibilities for making the model more realistic: an exogenous salary vs. an endogenous salary, the possibility of a change in the technique of production and the presence of technological shocks. In this way it is possible to isolate the effect of the different combinations on the equilibrium salary, average profit, rate of exploitation and several other variables.

In our model the rate of profit has the tendency to fall for two reasons: the increasing bargaining power of workers due to the increase in employment, and the increase in the ratio between dead labour (machines) and living labour (workers) used, in average, for the production process.

In this framework we inspect also the role of the political consciousness of the working class, and how this affects the equilibrium / disequilibrium conditions of the economy.

This paper is organized in the following way: in the next section we summarize the main topics of Marx's theory that will be used in our model; then we explain the NetLogo code; and finally we select and comment some particular combination of choices displaying interesting results.

The Labor Theory of Value

Marx, following the steps of Ricardo, uses in his work the Labor Theory of Value, according to which the value of a good depends on the quantity of labor used for its production (L). Let us not consider for the moment the fixed capital and let us pretend that only labor is used in the production. The amount of labor incorporated in a good can be divided into two parts: the amount of necessary labor (meaning the amount of labor necessary to produce the salary goods for the reproduction of the workers), denoted V, and the amount of Surplus-Values, SV, defined exactly as the difference between the labor used in the production and the necessary labor.

$$SV = L - V$$

From this simple formula we immediately see an important issue in the theory of Marx: the conflictual nature of distribution. In fact, given a fixed technology and therefore a fixed amount of labor needed to produce a good, if the salary goes up the profit of the capitalist (at this stage of the discussion the surplus-value SV) goes down. The profit is therefore *residual*, and determined politically, depending on the relative strength of Workers and Capitalists. Its value is the difference between the length of the work day and the time necessary to produce the goods that constitutes the salary, bargained in the labor market, and can be summarized in the rate of surplus-value ($s = SV / V$), also called “Rate of Exploitation” since it gives the measure of how much unpaid work the worker does, therefore how much he is, using Marx's terminology, exploited by the employer.

For example, a surplus-value rate of 100% means that, in an eight-hours workday, four hours are of necessary labor and four hours of surplus-labor. Taken for given the legislation regarding the maximum length of a work day, one of the main factor that influences the surplus-value rate is the rate of unemployment: in fact, as the unemployment reduces, the bargaining power of the employer reduces as well, and therefore the salary increases, reducing the surplus-value rate. This fact led Marx to theorize the existence of a “Reserve Army of Labor”, a level of structural unemployment necessary for the system in order to keep wages low.

Capital mobility and average rate of profit

Let us now consider the fixed capital as well, C . The value of the machines that the capitalist needs to buy to produce it is, as for any other good, the amount of labor necessary for their production. The capital invested by the entrepreneur will therefore be the sum of the value of the fixed capital and the value of the salary goods, $C + V$, and the rate of profit will be calculated as the ratio of the surplus value on this total amount of investment: $r = SV / (C + V)$.

We therefore have:

$$\text{Value of production} = C + V + SV = C + V + sV = C + V + r(C + V)$$

But will a good be sold according to its value? Not exactly. Here Marx moves beyond Ricardo, and introduces the concept of Production Prices.

Note in fact that the surplus-value, the source of profit, is generated only by the “living labor” employed in the production, that is V . The “dead labor”, the labor incorporated into the fixed capital, simply transmits its value into the value of the final good. But this means that the profit will be higher the greater number of workers are used in the production. Using the Marxian terminology of “Organic Composition of Capital” to define the ratio between dead labor and living labor, C/V , it could seem that we have a situation in which the industries with a lower organic composition of capital will have the higher rate of profit.

Consider the following example:

Industry	C	V	SV	Value	r
I	80	20	20	120	0,2
II	60	40	40	140	0,4
III	40	60	60	160	0,6

We can see that investing the same capital (100) in the three different sectors, the profit will be different: 20, 40 and 60 respectively.

But this clearly cannot be an equilibrium situation in an economy that allow the mobility of capitals: in fact the capitalists that invested in the industries less profitable will move their capital to industry III. This will result in a permanent increase of supply in industry III that, for a given demand, will reduce the price of the good *without affecting its value* (in fact the technique did not change, and therefore neither the necessary labor for the production), and consequently affecting the rate of profit. At the same time capital moving out from sectors I and II will push up the prices, increasing the unitary profit of the good produced. This process will keep on until the same rate of profit (\bar{r}) is obtained in all sector of production. The resulting prices are called Production Prices, and correspond to the invested capital plus the average surplus-value (\bar{SV})

$$\text{Production Prices} = C + V + \bar{SV} = C + V + \bar{r}(C + V)$$

Good will not be sold at their value, but at their price of production (apart for any temporary oscillation of supply or demand, of course), that could be higher, equal or lower the real value of the goods. This means that in this process of profits equalization there are capitals to which competition assigns a profit higher than the surplus-value produced in their industry, at the expense of others that, using a lower organic composition, see part of their surplus-value taken away.

In the previous example:

Industry	C	V	SV	Value	\bar{SV}	Price of Production
I	80	20	20	120	40	140
II	60	40	40	140	40	140
III	40	60	60	160	40	140
Sum			120	420	120	420

Superprofits and increase in productivity

As we said in the previous section, the competition among capitals has as a result a leveling of profits among the different branches of production. Nevertheless, within a specific industry, the production conditions are hardly homogeneous. It is possible that a specific firm (let us call it X) is able, thanks to a higher labor productivity or other characteristics, to produce the same amount of good of an other firm using less capital (or, equivalently, producing a higher amount of products with the same capital). The unitary price of production for the firm X will therefore be less than the unitary production price of a firm that uses average production conditions (let us call this second firm Y): let us say for example that both firms invest 100 of capital and that the average profit, determined on the aggregate, is 20. The production price of the *aggregate* production of both firm X and Y will be 120. But firm X with 100 of capital is able to produce 1000 units of goods, while firm Y only 500. The single good produced by X will therefore have an individual production price of $120/1000=0,12$, while the single good produced by Y will be worth $120/500=0,24$, the double. Within the same branch of production we can therefore have homogeneous goods produced in more favorable conditions and goods produced in less favorable conditions. But the competition among different firms will prevent that a homogeneous good will be sold at different prices. It will have to arise therefore a common market price, that will depend on the specific conditions of supply and demand but that will have as maximum level the unitary production price of the good produced at the worst production conditions and as minimum level the unitary production price of the good produced at the best production conditions. It is therefore clear that this market production price includes a “superprofit” for the firms that, within a specific branch of production, use the most favorable production conditions.

Two more things deserve to be underlined about these superprofits: first, they are usually ephemeral, since they depend on reproducible production conditions and competition pressures so that these will be eventually copied and used by all the firms; second, the techniques that allow for a higher productivity usually require an increase in fixed capital, resulting therefore in a higher organic composition of capital. This last point allow us to introduce one of the most controversial results obtained by Marx: the tendency of the rate of profit to fall.

The tendency of the rate of profit to fall

The argument behind the tendency of the rate of profit to fall is rather simple: the competition gives to the firms the incentive to increase their organic composition of capital in order to extract greater profit from their competitors. But *on the aggregate* this will lead to an *aggregate* greater organic composition of capital and therefore on a reduction of the average rate of profit. In fact the average rate of profit is determined as the *aggregate* super-value produced in the economy over the sum of the aggregate investment, divided in the sum of aggregate fixed capital and aggregate variable capital:

$$r = \frac{SV}{C+V}$$

If we divide both the denominator and the numerator for the sum of the total variable capital V we obtain

$$r = \frac{\frac{SV}{V}}{1 + \frac{C}{V}}$$

From this formula it is clear that the average rate of profit is inversely related with the aggregate organic composition of capital. In fact, as we said before, the surplus-value derives only from the living labor moved by the capitalist. An average, general technique that uses relatively more dead labor will clearly result in a lower rate of profit.

It must be pointed out that a decreasing rate of profit will not necessarily result in a falling *mass* of profits: in fact the mass of profit is the result of the product between the rate of profit and the capital invested. If the capital invested raises more than the fall of the rate, the mass of profit will raise. For example, if in period 1 the amount of capital invested is 100 and the rate of profit is 0.2, the resulting mass of profit will be 20; if the next period the rate of profit is fallen to 0.1 but the entrepreneur increased his investments to 200, the mass of profit will be stable at 20.

The fall of the rate of profit is for Marx simply a tendency, and not an exact and deterministic rule, because in a capitalist economy there are many counteracting factor that slow the fall or even permit a temporary rise. We here consider only three of them:

- Raise in exploitation rate: as it is clear from the formula above, the rate of profit is directly related with the surplus-value rate. For example a legislation that increases the workday hours, for a given V, increases the exploitation rate. Or, for a given length of the workday, a technological increase in productivity that reduces the necessary labor needed for the production of the salary goods reduces V (even if the physical goods received by the workers do not decrease) and therefore increases
- Decrease in salary: if the relative bargaining power of workers and capitalist shifts in favor of the capitalists (for example because of raising unemployment), this may result in a decrease of the salary, with a consequent increase of the exploitation rate. Note that this is different from the previous point, since it will result in an actual decrease of the purchasing power of the worker.
- Productivity shock in constant capital: if the productivity in the production of capital good lowers C, this of course results in a decrease in the organic composition of capital and gives the rate of profit a release.

It is important to point out that Marx did not predict the necessary, deterministic fall of the capitalist system. The counteracting factors can keep the system going theoretically forever. What Marx predicts is a series of crisis in the system, which could eventually bring to a *political* collapse of the system, once the Class Consciousness of the working class has reached a point so high to provoke a Revolution.

2. The Model

The model has two kind of agents: entrepreneurs and workers. Entrepreneurs make decisions about the investment to do any turn and about the change in the technique used in their firms. Workers make decision whether to go on strike or not if the employer threatens to fire some of them.

Initial situation and basic rules

At the beginning the prices of production of the factors (machines and labor) are given. The price of capital (c) will change only sporadically as the result of an exogenous technological shock. The price of labour (vt) on the other hand is endogenous and a positive function of the number of workers employed in the production. In the first period it is assumed to be at its lowest possible level, the historically contingent subsistence wage (v).

Also the length of the workday is given and common to all firms, and it is a multiple of the subsistence wage v.

Therefore, once the salary of the period is determined, the exploitation rate (s) is given as well.

Entrepreneurs

In the first period the entrepreneur does not really have a choice. He has a given endowment of capital and a randomly assigned technique (intended as combination of machines, K, and workers, L necessary for the production), so he just buys the maximum number of machines and workers and starts the production, which will be worth of course:

$$cK + v_t L + s v_t L$$

Since we do not want to focus on capital mobility we take the process of equalization of the rates of profit described above as given. Therefore, once all the firms ended their production, we determine the average rate of profit (r) as the ratio between the sum of all the surplus-values produced over the sum of all the capitals invested. The prices at which the firms will sell their product will be:

$$cK + v_t L + r(cK + v_t L)$$

From the second period onwards the capitalist does have a choice: to decide how to use the profit of the precedent period. He has therefore to choose which percentage of the previous profit he wants to use for his own consumption (and therefore “burn”) and which percentage he wants to invest in increased production. The choice will have some stochastic component (representing special needs of the entrepreneur, like “I have to buy a house”) but will depend somehow on the variation of the rate of profit.

Once the investment decisions are made, the capitalist buys the new machines and hires the new workers.

He has anyway an other choice: to change the technique he uses to produce, implementing a more capital intensive one. To simulate the increase in productivity that this will create (as described above) this will grant him some turns of “superprofits”, above the average rate r . Also this decision will depend on the floating of the rate of profit or the mass of profit. Anyway, the entrepreneur will not always be able to implement the change in productivity. In fact the adoption of a more capital intensive technique will result in some workers getting fired, and this may be opposed by the workers.

Workers

The worker initially walks around town and it is absorbed by the firms who hire workers.

Each worker has many level of “class consciousness” that he can reach. The initial level is random, while further changes are determined by the simulation.

Once the entrepreneur decides to fire someone, the workers of that firm can call a strike. The strike will start and have success depending on the level of class consciousness of the workers of the firm (for example there is a threshold exceeded when workers have reached a high level of class consciousness).

The factors that can bring up or down the level of class consciousness of a worker are: the physical proximity with a worker with a higher or lower level of class consciousness and being fired.

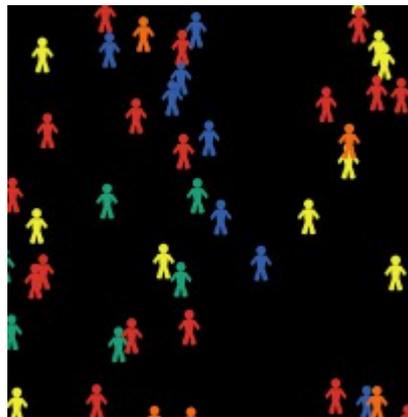
3. The code

The model represents a small open economy with a fixed population. This means that the amount of fixed capital is unconstrained (it can be bought in international markets) but the labour factor is limited.

To setup

At time zero, the firms and the workers are created and placed randomly in the world. The cost of capital and the subsistence salary are exogenously determined, and every firm has the same initial level of capital, equal to 100. Each firm has a different level of technique, meaning the ratio between the number of workers (tl , necessarily an integer) and amount of capital (tk , continuous variable) necessary for the production. A firm's technique determines if the firm is capital intensive or labour intensive.

If the Class Struggle button is active we equipped the worker with a random level of class consciousness, bounded between 0 and 1. The different levels of class consciousness are associated with five different colours through which the workers can be represented. By moving from a low level of class consciousness to a high level workers are blue, green, yellow, orange and red.



To go

During each period, a different mix of procedures are activated, according to the bottoms in the main interface.

Class struggle

A new level of class consciousness is assigned every period, always represented as a spectrum of five colours.

There are basically two determinants of class consciousness in this phase: first of all, the class consciousness of every worker can be increased by the presence of a certain amount of workers around him with a higher level of class consciousness;

```
ask workers [if ((count workers in-radius 1 with [color = 15]) / (count workers in-radius 1)) > 0.2 [set consc consc * 1.1]]
```

on the other hand, if a worker is surrounded by agents with a lower level of class consciousness, his class consciousness decreases.

```
ask workers [if ((sum [consc] of workers in-radius 1) / (count workers in-radius 1)) < consc [set consc consc * 0.95]]
```

Shocks

According to the choice made on the button “Shock”, we can have random reduction in the price of a unit of capital, in the subsistence wage, or in both.

```
if Shock = "on subsistence salary" [if (random-float 1) < 0.1 [set v v * 0.97]]
if Shock = "on cost of capital" [if (random-float 1) < 0.1 [set c c * 0.90]]
if Shock = "on both" [if (random-float 1) < 0.1 [set v v * 0.97] if (random-float 1) < 0.1 [set c c * 0.90]]
```

Technique change

In the model it is possible to keep the level of technique constant over time or it is possible to make firms try to become more capital intensive at the purpose of gaining superprofits.

If the latter is the same, the decision to change the technique of production can be determined randomly

```
to rand_change ; firms change randomly technique of production
  ask firms [if (random-float 1) < 0.1 [set tk tk + 3 set technique (tk) / (t1)]]
end
```

or it can be determined endogenously, as the reaction of the firm to a decrease in profit (the situation caught by the dummy variable “loss”). If the bottom Class Struggle is active, the workers can try to prevent this change, which would result in some of them getting fired. The “struggle” starts if the level of class consciousness is high enough, and it has a random chance of success, influenced by the amount of “reds” (highest level of class consciousness) presents in the firm. If Concertation is off, a battle won prevents the change in technique. On the other hand, if the button Concertation is on then a victory of the “struggle” for the workers will not result in a stop of the technique change, but in a splitting up of the resulting superprofits between the firm and the workers themselves.

In both cases, if a “struggle” is lost, the workers will lose part of their class consciousness.

```
if-else Class_Struggle [ask firms [ if (loss = 1) [set tk tk + 3 set technique (tk) / (t1)
  if ((sum [consc] of workers-here with [employed = 1]) / (count workers-here with [employed = 1])) > 0.5
    [ifelse (random-float 1) < (0.4 + 0.4 * (count workers with [color = 15]) / (count workers) )
      [ifelse Concertation
        [set consc 1]
        [set tk tk - 3 set technique (tk) / (t1) set loss 0]]
      [ask workers-here with [employed = 1] [set consc consc * 0.90]]]]]]]]
```

Determination of salary

The salary, can be determined exogenously (set at the subsistence level, “v”) or endogenously, according to the following formula:

```
set vt ( v * (1 + (1 - u)) )
```

where “u” is the rate of unemployment.

If Concertation is active then the workers in different firms will receive different salaries, according to the “firm – level contracts”, and therefore the average salary (“sal”) can be determined as following

```
if (u != 1) [if Concertation [set sal vt + ((sum [bonus] of firms) / (count workers with [employed = 1]))]]
```

According to the current level of salary, the exploitation rate is determined.

Production

During each period, given the price of factors and the amount of available resources, the firms set the desired amount of production.

Basically every firm solves the following maximization problem:

$$\max_{(K,L)} Kc + Lv_t + sLv_t$$

$$Kc + Lv_t \leq \text{capital}$$

subject to $\frac{K}{L} = \text{technique}$

The result of this maximization is the number of workers and the amount of capital that the firm wants to employ in this period. But it may be the case that the firm wants to hire a number of workers greater than the number of currently unemployed people. In this case, the firm maintains its current production.

```
ifelse (count workers with [employed = 0]) > (int ( capital / ( vt + technique * c )) - hired)
  [set L int ( capital / ( vt + technique * c )) set K (technique * L) set inv 1]
  [set inv 0]
```

As a consequence, we determine the value of the production and the respective, given the rate of exploitation, plus-value generated.

If the firm has decided to employ a number of workers greater than last period, then it hires the nearest unemployed workers and it makes them move inside itself.

```
[ask min-n-of (L - hired) workers with [employed = 0] [distance myself] [move-to myself set employed 1]]
```

If, on the other hand, the firm wants to reduce its personnel (because either of the increase in the general salary or of a change in production technique), then obviously it has to fire someone of its employed workers. The fired workers now become unemployed again and they are put outside the firm randomly all over the world. If Class Struggle is active, this will increase the class consciousness of the fired worker.

```
[ask n-of (hired - L) workers-here with [employed = 1] [setxy random-xcor random-ycor set employed 0
  if Class_Struggle [if-else (consc * 1.3) < 1 [set consc consc * 1.3] [set consc 1]]]]
```

Realization

After the production process every firm sells its product in the market at its price of production.

We have here two possible scenarios: usually every firm receives the same average rate of profit with respect to the capital employed; however, if a firm has become more capital intensive at the beginning of the period, then it receives a plus-profit (equal to 10% of “normal” profit). As we mentioned before, if the workers have won a dispute when Concertation is active, this super-profit must be divided with the workers

```
if-else (loss = 1 ) [ifelse (conc = 1)
  [set bonus 0.05 * (pp - (lcapital + dcapital)) set profit (pp - (lcapital + dcapital)) * 1.05 set conc 0]
  [set profit (pp - (lcapital + dcapital)) * 1.1 ]]
```

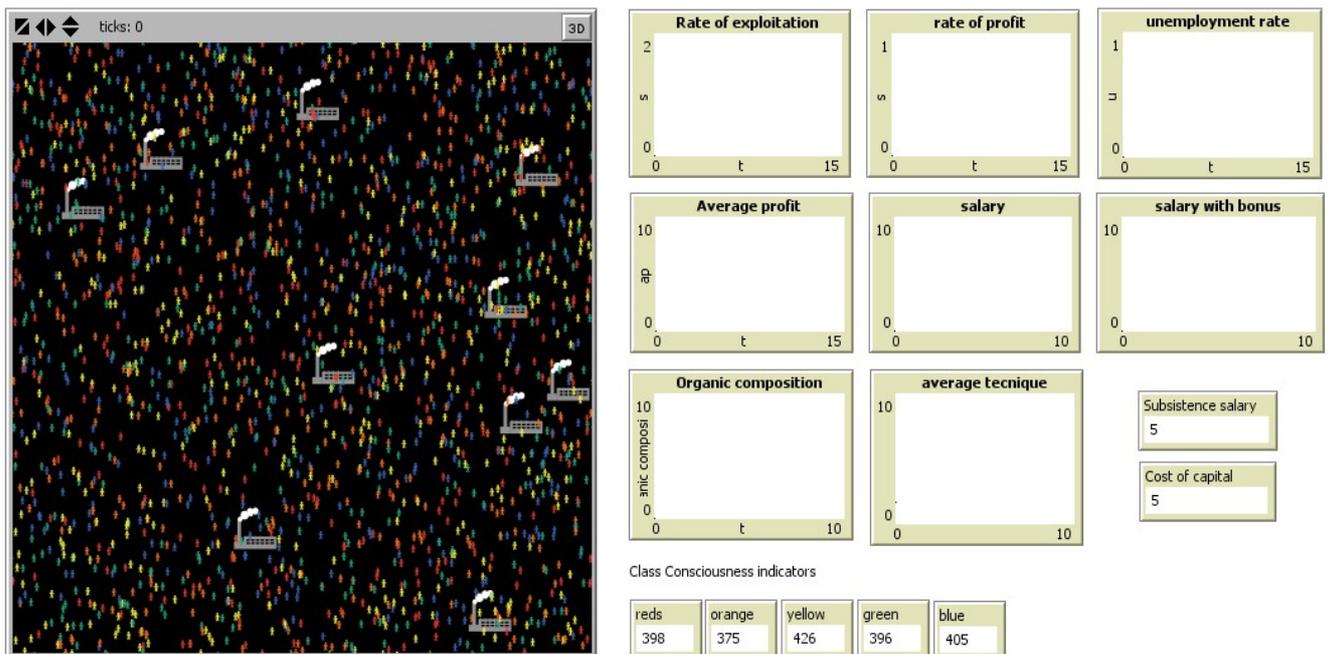
Investment

After the realization of production, the firms decide how much of the present profit will be used to increase the capital of the next period. This kind of decision is determined randomly for each firm. Anyway, if the firm has not been able to hire the amount of workers he wanted in the Production procedure, it will consume all its current profit.

```
ask firms
  [ifelse (inv = 1)
    [set invest (random-float 1 * profit)]
    [set invest 0]
  set capital (lcapital + dcapital + inventories + invest)]
```

Graphs and counters

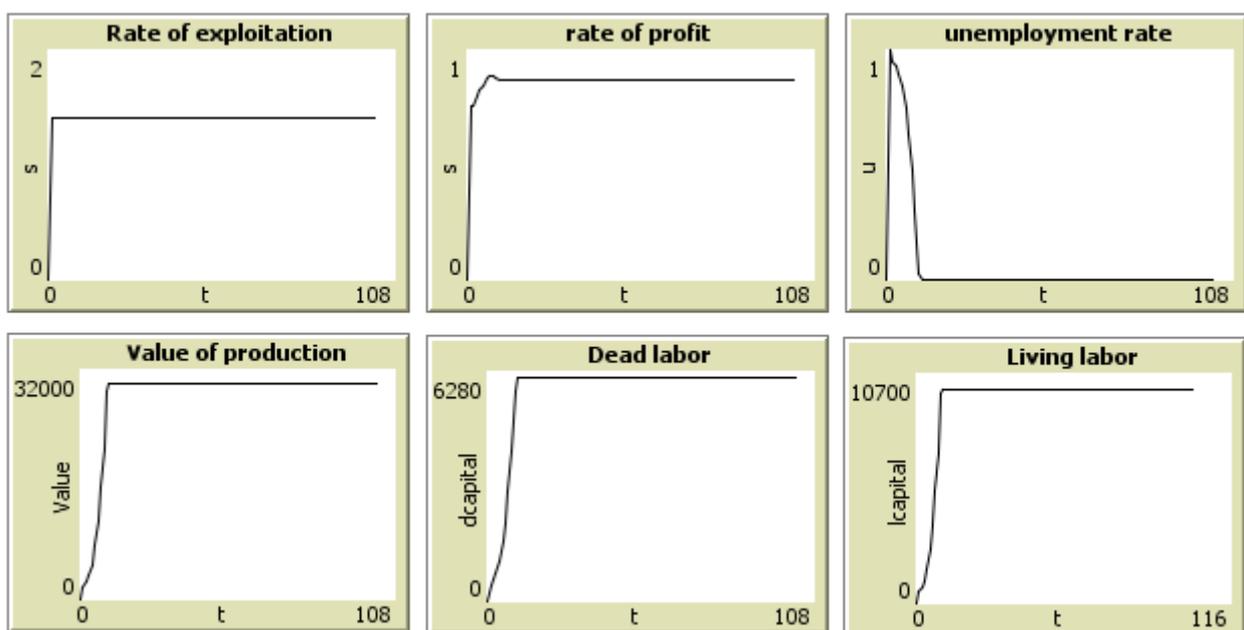
With the help of the graphs in the main interface the user can keep track of the evolution in time of the most important variables in the economy: the rate of exploitation, the rate of profit, the unemployment rate, the average profit, the salary (with and without bonus), the organic composition of capital, the average technique and the value of the inputs. Moreover, it is possible to inspect the evolution of the class consciousness among workers by means of the relative counters.



4. Results

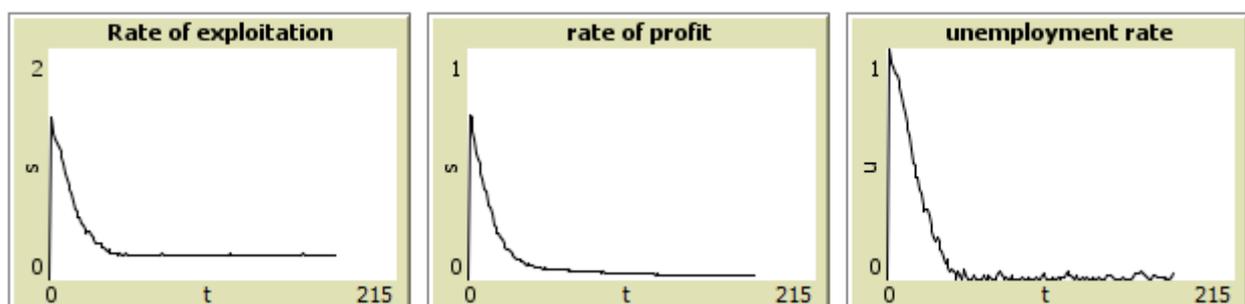
We start by presenting the simplest model with exogenous variables and predetermined dynamics. We then move to more sophisticated models by adding endogenous processes and shocks. In the last models we show how the introduction of the class struggle affects the results and how these change when concertation is allowed.

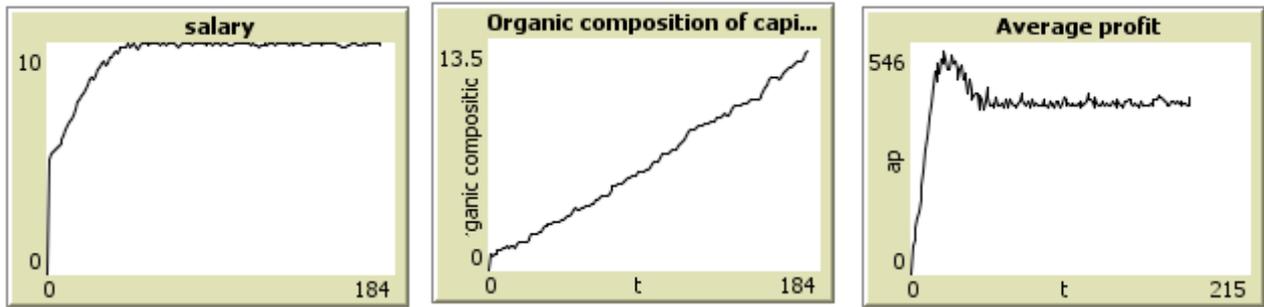
Exogenous salary, no shocks and no technique change in production



This is our simplest model where the salary is fixed equal to the subsistence salary which is exogenous. Firms hire almost immediately all the labour force making the unemployment rate shrink to zero. Since the salary is not affected by the unemployment rate, the rate of exploitation remains constant where the workday is twelve hours but the workers are paid always 5 hours. The rate of profit increases when new workers are hired and then stays constant because both the rate of exploitation and the organic composition of capital remain constant since the latter do not change due to exogenous shocks.

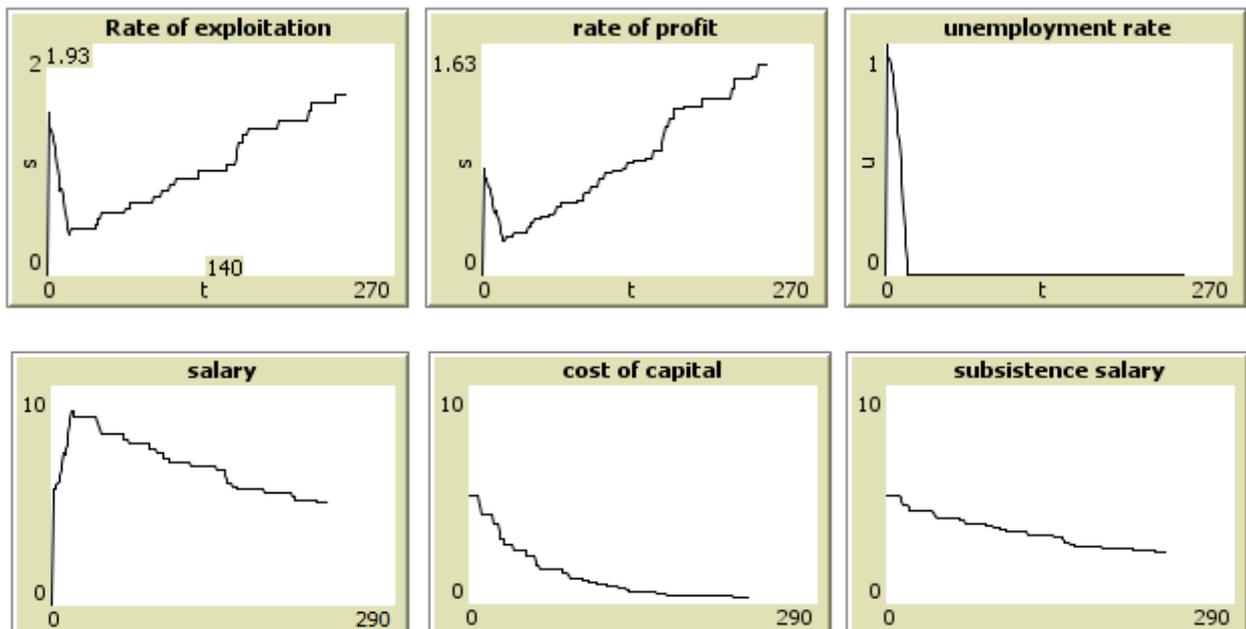
Endogenous salary, no shocks and random technique change





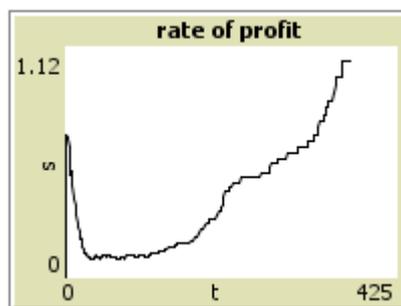
In this model the salary is endogenously determined and it is an increasing function of the employment rate, hence when firms start to hire workers the salary gradually goes up till the maximum level allowed of ten hours. The increase in salary leads to an equivalent decrease in the rate of exploitation, which stabilizes to a value of two because there are still two hours of the workday which are not paid. In contrast with the previous model, the rate of profit tends to zero because it follows the gradual decrease of the exploitation rate. The random technique change makes the technique to become more capital intensive, hence the organic composition of capital has a positive trend. This last result gives an additional reason for the tendency of the rate of profit to fall. An other effect of the random change in technique is that firms occasionally fire workers which are substituted by more capital, and the unemployment rate fluctuates around a level close to zero. The average profit increases when firms hire workers but decreases when unemployment is low and salary close to the maximum. The average profit then stabilizes and fluctuates because of fluctuations in the salary explained by fluctuations in the unemployment rate.

Endogenous salary, shocks on subsistence salary and cost of capital without technique change

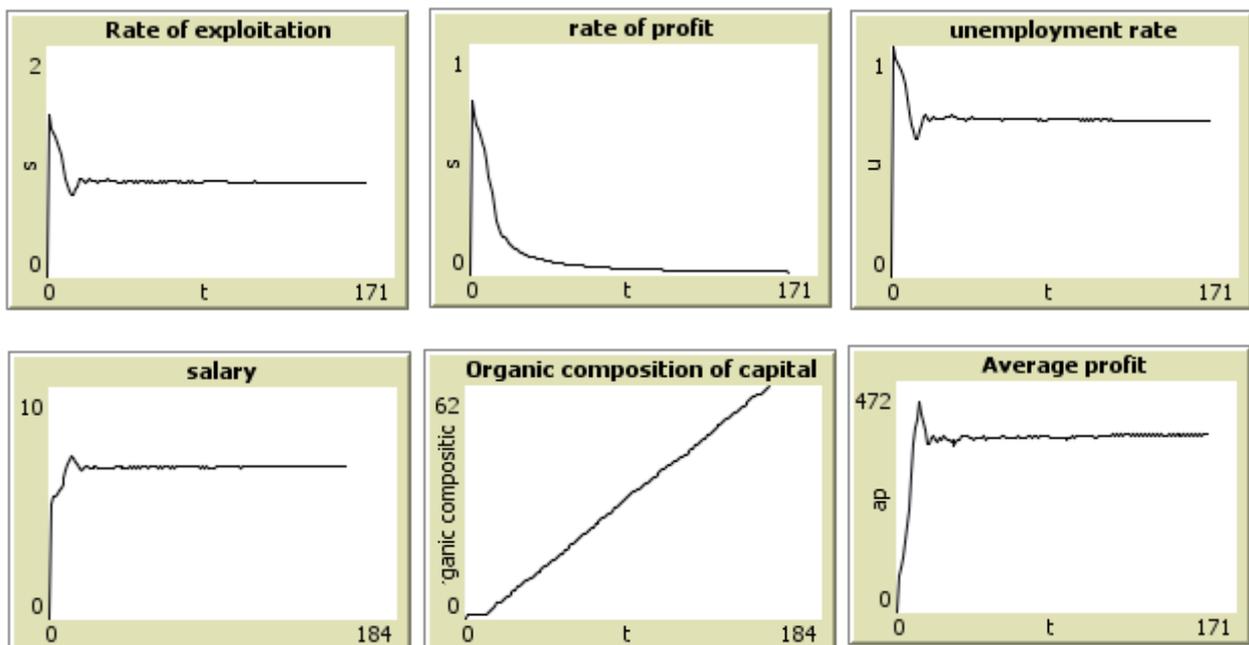


This model introduces shocks to the subsistence salary and to the cost of capital. As a consequence, the salary first rises until full employment and then gradually decreases according to the devaluation of the subsistence salary. The rate of exploitation follows the opposite dynamic. The striking different with respect to the previous models is that the rate of profit increases toward infinite. After a first contraction due to the higher salary, there are two forces on it: both the devaluation of subsistence salary and the devaluation of the cost of capital push it to infinite.

By introducing a random technique change all the variables follow the same patters but the rate of profit shows a temporary stabilization and then skyrockets. This can be explained by the increase in the organic composition of capital which has a negative effect on the rate of profit. Effect which is later neutralized by the devaluation of the cost of inputs. Note however that the sudden increase in the rate of profit happens in the long run, almost at 170 periods, hence it is negligible.

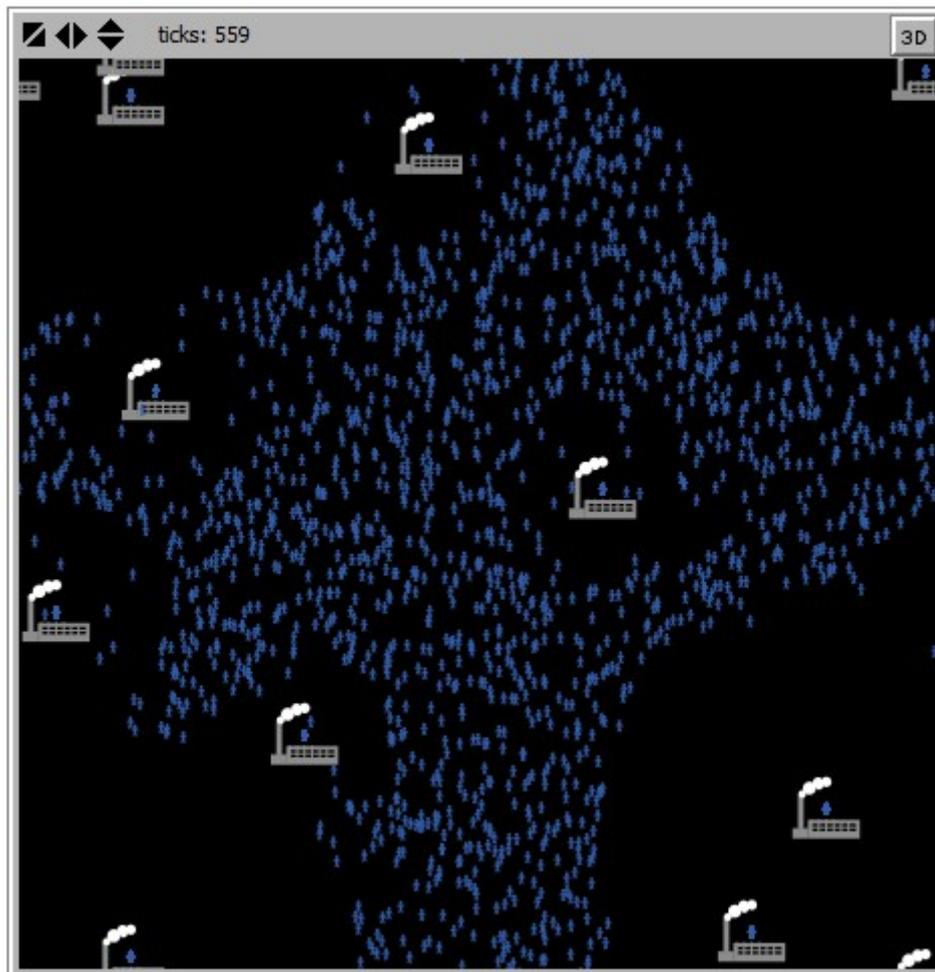


Endogenous salary, no shocks and endogenous technique change



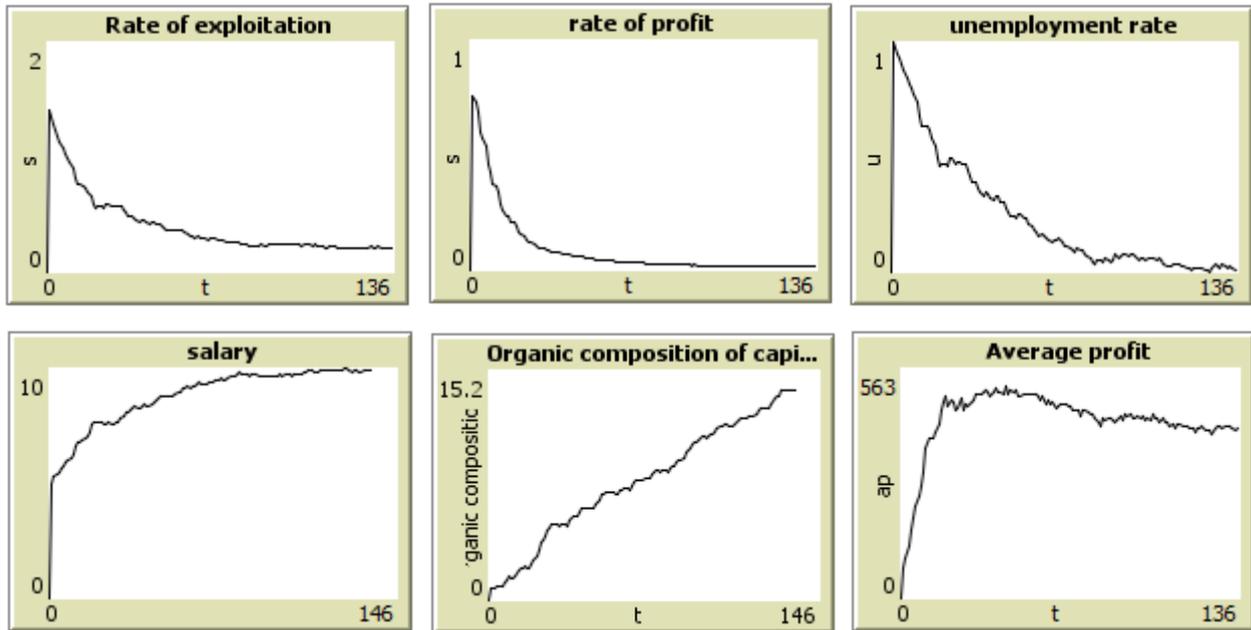
This model introduces an endogenous technique change where firms decide to move toward a more capital intensive technique when their profit decreases. The striking difference compared to the previous models is

the unemployment rate which stabilizes at a relatively high level. The reason is that firms start hiring workers facing an increase in profit but as soon as the salary increases beyond a threshold, they find more profitable to substitute the labour force with capital. The consequence is that salary stabilizes after a small initial increase, the rate of exploitation follows the inverse dynamic and the average profit remains stable even if the rate of profit gradually shrinks because of the increase of the organic composition of capital. The following picture illustrates the steady state with a high unemployment rate.



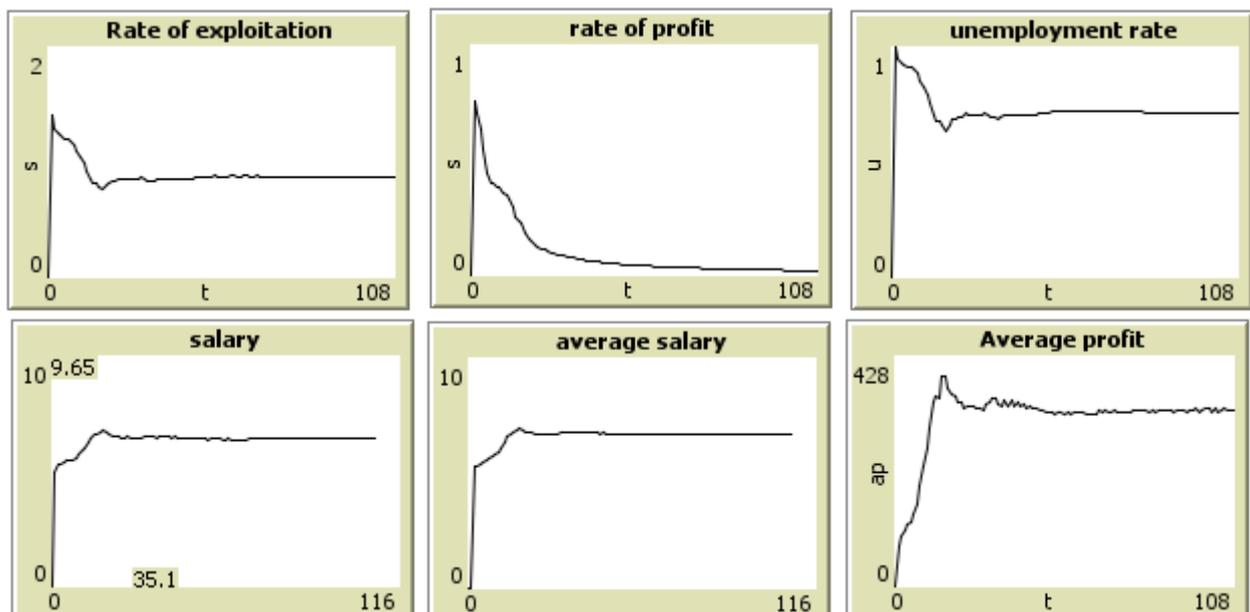
We now illustrate models which allow for the class struggle. We decided to build this scenarios on the model with endogenous technique change because it is the only one with an equilibrium characterized by a high rate of unemployment. After the introduction of the class struggle we introduce the concertation.

Endogenous salary, no shocks, endogenous technique change with class struggle



The introduction of the class struggle changes completely the previous results. When the firms want to change the production technique toward a more capital intensive combination, there is the possibility of facing the worker union, hence workers can success in stopping the technique change. As a result the unemployment rate decreases close to full employment, with continuous fluctuations. The path of the unemployment rate is similar to the model without class struggle where the technique change was random while in this model it is exogenous but the randomness arises from the result of the class struggle. The organic composition of capital is lower compared to the previous case and the salary is close to its maximum, following the tendency of the unemployment rate to fall. The rate of exploitation stabilizes at two, the rate of profit goes to zero and the average profit decreases gradually.

Endogenous salary, no shocks, endogenous technique change with class struggle and concertation



By introducing the concertation we annihilate the effect of the class struggle on the unemployment rate, with some modifications. Instead of blocking the technique change, workers accept to share part of the superprofit with the firms. For this reason the unemployment rate stabilizes at a high level as if there was no class struggle. There is a slight difference between the average salary (which includes the average bonus obtained) and the salary without bonus from which it is possible to deduce an average bonus of 0.1.