# Informational asymmetry in a three-asset stock market

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## Introduction

In a financial market informational asymmetry is one of the main sources of the heterogeneity in investors' expectations: heterogeneous expectations contribute to make a stock market a complex system, with different agents which elaborate their own predictions about stock prices and interact to generate a macro-complex pattern for price dynamics. Investors with different information sets will have different expectations about the market, proposing different prices for a particular asset. In real markets investors can pursue active strategies acquiring information thanks to analysts' support: the goal is obviously to out-perform a passive strategy, for which it's possible to hold a market index without worrying about information on assets or concerning other market features. The idea of 'beating the market' exploiting current information and forming reliable expectations about future asset prices is appealing but it's also avoided by the efficient market hypothesis: according to the economists, stock prices reflect immediately the current publicly available information, following a stochastic process which is close to a random walk, so invalidating the performance of every active strategy.

Our model is willing to investigate the role of informational asymmetry on stock prices dynamics in a simple artificial three-asset stock market: we have two stocks (which pay no dividends) and an index which is defined as the weighted sum of the stock prices. The artificial market is created in a program on NetLogo, which is excellent to build ABMs and analyze the results. Agents in the simulation can acquire information at each step (instant of time in the simulation), asking their analyst if the stocks (and, as a consequence, the index) are underpriced or overpriced: an alpha parameter provided by the analyst will tell them how much is the mispricing. Then agents will buy the stock if it's underpriced and sell it if it's overpriced, forming their price expectations through the corresponding alpha parameter and a subjective component. An auction mechanism matches the selling(bid) and buying(ask) prices, completing trading and setting the actual price for each security. Moreover an arbitrageur is acting in the market looking at the index price and at the current weighted sum of the two stock prices. At each step of the simulation the arbitrageur is buying or selling the two stocks in order to avoid arbitrage opportunities for the investors (no-arbitrage condition); in this way we will observe the alignment of stock prices to the index price, satisfying one of the most important condition of market rationality. The goal of the simulation is to check what is the level of market efficiency under different levels of informational asymmetry. So we want to analyze stock prices time series from our simulation data, assuming an autoregressive process (for all securities) which depends only from the lagged returns.

## How the program works

Our program in NetLogo includes two types of agents: the investors who operate on the market on the basis of the information acquired at each time, and one arbitrageur who profits by temporary misalignments between index price and the weighted sum of stock prices (with weighs remain fixed over time).

In the first part of the program investors gather information at each step about the two securities (paying their analyst): this information is represented through an alpha parameter for both stocks, where alphas are values which describe the mispricing (in terms of percentage) of the stock price according to the analyst; the alpha parameter for the index is calculated as weighted sum of the two alphas. If alpha is positive then the asset is underpriced and the investor has to buy it; if alpha is negative the asset is overpriced and the investor will sell it. Alpha parameters are taken randomly according to a normal distribution with standard deviation represents the degree of informational asymmetry in the market (i.e. the different views of the analysts about the market). Every investor uses the alpha values so obtained to compute their buying or selling price, including a subjective and irrational component too.

In the second part of the program agents interact with each other in the auction mechanism. Vectors logB and logS are respectively filled with buying and selling prices of every investor. Moreover the arbitrageur, when an investor makes a buying or selling price for a security, proposes, separately, two different prices following two different arbitrage strategies:

1) he looks at the external data series of the corresponding security (created in Mathematica in the way are going to explain next) and he compares the exogenous price at that instant of time (tick in the simulation) with the current price in the simulation: if the last one is less then the exogenous one, he will buy the security, otherwise he will sell it, and the buying or selling price is always a random between the exogenous price and the actual one in the simulation. In this way all three asset prices are aligned to some external data, which can be built upon every kind of financial model.

2) he looks at the three asset prices in the simulation checking the validity, at each tick, of the no-arbitrage condition: the weighted sum of the two stock prices has to be equal to the index price. If this price is greater than the weighted sum the arbitrageur will buy the two stocks proposing two prices such to restore the equilibrium condition and to apply the same price changes to

both assets; if the index price is lower than the weighted sum of the stock prices the arbitrageur will sell the stocks, proposing again two prices such to restore the equilibrium condition and to apply the same price changes to both assets. With this operation the three markets (related to each asset) are linked together and the operations an investor can make on one asset can influence the price of another one.

So the arbitrageur interacts with the investors in the auction market for every asset. While we count the cash and the number of stocks owned by every investor, we chose to not count such variables for the arbitrageur, so simplifying the trades. Every time we have a possible matching in the book of some asset (the buying price is greater or equal than the selling price) the trade is concluded: we can have trading between two investors, between an investor and the arbitrageur and between two arbitrageurs.

**Why did we create external data with Mathematica?**

One important question to consider is why we create external data in a Mathematica notebook to relate them to the endogenous one produced in the simulation. If we let asset prices free to evolve without links to external data, they could move in a nonsensical way from an economic point of view. Creating time series data exogenous to the simulation we can start from common stochastic processes useful to describe asset dynamics and then relate these data to the prices which emerge during the simulation. This is possible through the work of the arbitrageur who guarantees the alignment of internal data to the exogenous one: so we get time series will reflect the fundamental processes chosen in Mathematica. The power of this approach is that we can construct data in Mathematica using any econometric model as a benchmark for our simulation. In this project we created data index price using a deterministic trend (a sine function which mimic a bull market period and then a recession period) plus an AR(1) process and then, for both stock prices, we added an ARMA(1,1) process to the this data series to simulate stocks data. This artifice is useful to create two time series which coevolve with the index stochastic process: such a behavior is quite realistic since we know that a security can be priced through the return of some market index plus a specific-firm component (which is in general unpredictable). So we are sure that our three asset 'move almost together' in the market as it could happen in a real market. We can easily change the stochastic process which describes the index dynamics or the stocks dynamics on the basis of other econometric models, so being able to test different hypothesis on our three-asset stock market: the possibilities to explore are almost infinite. Once our data are related to the exogenous one in Mathematica, we can observe how the agents' behavior in the simulation can affect, and slightly modify, the external main trend: then the question will be how the micro-interactions among investors (and the arbitrageur) in the market can shift prices' dynamics with respect to the exogenous data used as benchmark.

Here we show briefly the stochastic processes used to simulate the index and the stocks. Starting from the index we have

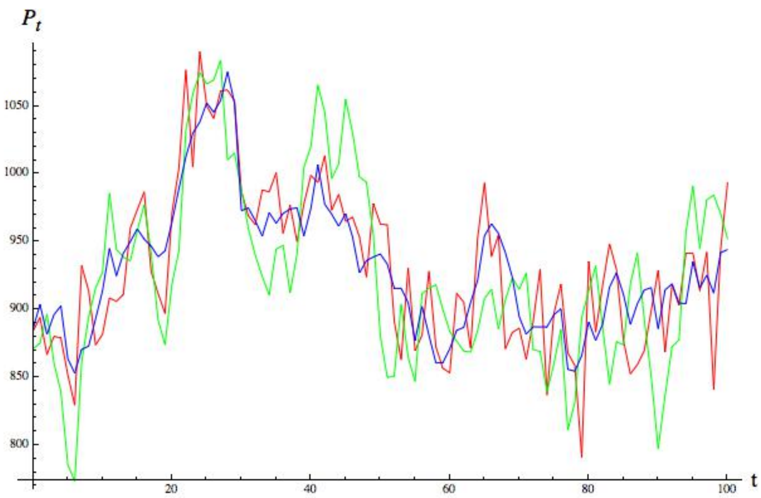
$$P_{I,t} = 100sin(\frac{2\pi t}{100}) + 0.9P_{I,t-1} + \epsilon_t$$

The AR(1) process used for the index presents an autocorrelation coefficient of 0.9, which means the process is close to a random walk (autocorrelation coefficient equal to 1): this follows from many previous works which show how the hypothesis of random walks for weighted index is not completely rejected. The variance of the white gaussian noise is 300, to produce significant fluctuations around the deterministic trend. Now the two stock price processes are

$$P_{1,t} = P_{I,t} - 0.2P_{1,t-1} + 3\nu_t$$

$$P_{2,t} = P_{I,t} + 0.3P_{2,t-1} + 2.5\xi_t$$

The fluctuations of the first stock around the index are described by an ARMA(1,1) process with an autocorrelation coefficient equal to -0.2 and a moving-average coefficient of 3, while the variance of the noise terms is 100. The fluctuations of the second stock are defined by an ARMA(1,1) process with autocrrelation coefficient of 0.3 and a moving average coefficient equal to 2.5, with a variance for the noise terms of 150. We show here the plot of all the exogenous data.




The blue line is the index, the red is the first stock and the green is the second stock

### One arbitrageur, more arbitrage operations

In the program we created only one arbitrageur because this solution was easier, in terms of the program code, than including different arbitrageurs can interact in the auction. In a real market obviously we have instead different arbitrageurs who compete among each other identifying arbitrage opportunities and proposing their selling or buying price, which can be different for every arbitrageur. Looking at the program we notice however that the arbitrageur's actions 'are called' several times in the simulation (in particular two times for every buying or selling price proposed from an investor), mimicking the existence of many arbitrageurs. Indeed, since we do not account for cash or the number of shares owned by the arbitrageur, the number of arbitrageurs is not relevant to fulfill the no-arbitrage condition; we have no need to distinguish the different arbitrageurs, so we can use only one representative agent which perform all the necessary arbitrage operations asking or bidding different prices (a random number is extracted from a uniform distribution such to define the arbitrageur greedy in pursue the arbitrage intervention).

### Parameters in the simulation

Here we present the list of parameters used in the program. NetLogo is very handy in visualizing different settings of parameters and changing them even during the simulation. We briefly describes the function of every single parameter in the program.

- **ninvestors:** the number of investors in the market. Let us notice that the program is built in order to distinguish the investors from the arbitrageur through the who number of an agent (assigned automatically from NetLogo when agents are created); the arbitrageur has a who number equal to ninvestors plus one (because is created after the investors and the who numbers are not breed-specific). So we cannot change the parameter ninvestors during the simulation, otherwise the program shows an error, no more being able to distinguish investors from the arbitrageur!

- **out-of-marketLevel:** the probability of testing the bankruptcy condition for assets. If a random between 0 and 1 is lower than the out-of-marketLevel we test if all asset prices are under a common level (here 500): if this is true the investor chooses to exit from the market, setting the out-of-market variable as true. We usually adopt a high out-of-marketLevel in the simulation (0.9 in data analysis). However, given arbitrage operations related to external data created in Mathematica, there are no chances some asset price can drop below the bankruptcy threshold. If we changed the external data or the bankruptcy threshold, the parameter out-of-marketLevel would become very important: we do not consider its role in our project, even maintaining it in the program for future works.

- **arbitrageLevel:** the probability with which the arbitrageur operates on the market to restore the no-arbitrage condition: an arbitrageLevel equal to 1 means the arbitrageur acts every time, an arbitrageLevel of 0 corresponds to the total absence of arbitrageur's intervention. It is very important to understand that this parameter entails only the arbitrage operations which look at the internal assets in the simulation: in other words, a high arbitrageLevel means the index price is well aligned to the weighted sum of the stocks prices. The arbitrage between exogenous data and simulation data (for each asset) is no affected (directly) from arbitrageLevel because we always want our time series are linked to the external series in Mathematica.

- **asymmetry-degree:** the standard deviation of the normal distribution of alpha parameters: it quantifies the informational asymmetry in the market. We are going to study the effect of this parameter on the no-arbitrage condition and on the stochastic processes for asset prices.

- **mean-information:** the mean of the normal distribution of alpha parameters. If the mean is positive agents will tend to acquire stocks, while if it is negative they will want to sell them more frequently. We chose to set mean-information to zero in our experiments, in order to present a neutral market situation in which assets can be considered underpriced or overpriced with the same probability.

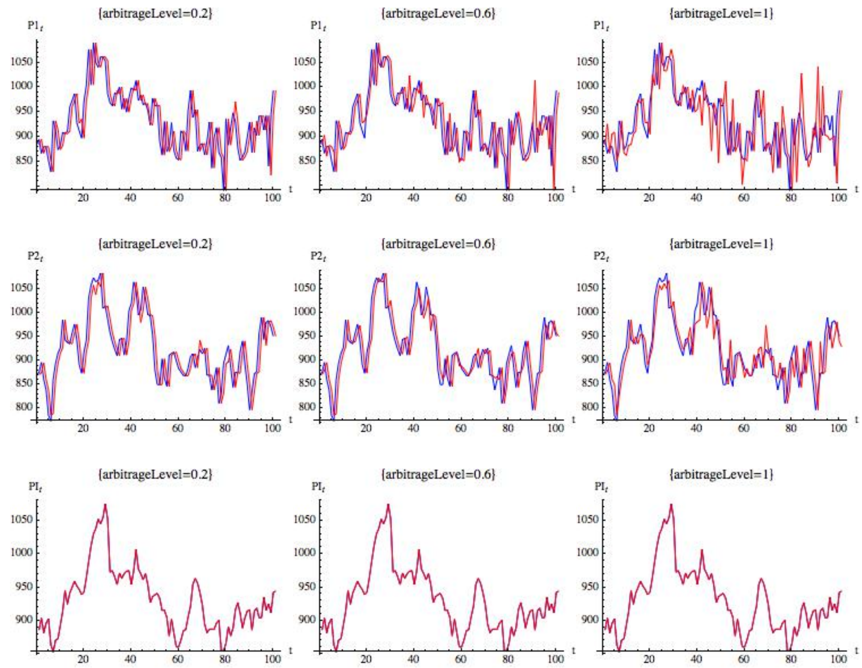
- **information-threshold:** the threshold of reliability of alpha parameters: if the alpha parameter of some asset is greater than the threshold the investor does not rely on the analyst and prefers to not participate to the auction of that asset (the boolean variable pass is set true). This parameter is strictly correlated to the asymmetry-degree because if the latter is much lower than the information-threshold (for instance one third or less), the probability of having alpha parameters under the threshold of reliability is very low and all the investors will participate almost every time to the auctions of all assets.

- **cost-of-information:** the cash payed from an investor to her analyst if she has gathered a reliable alpha parameter (i.e. under the information-threshold). This parameter has no influence in market dynamics because investors has no limits in borrowing to continue to acquire information or buying stocks. Nevertheless it will be crucial in evaluating the efficacy of trading strategies with different levels of asymmetry-degree and arbitrageLevel.

## Checking no-arbitrage condition: the role of the parameter arbitrageLevel

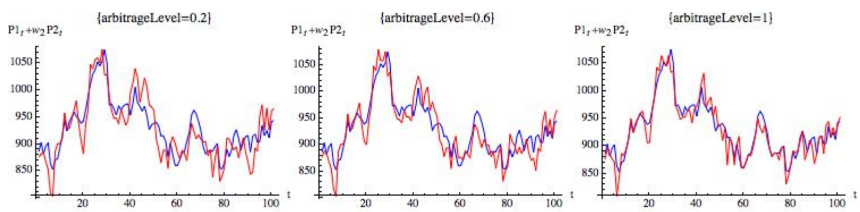
Let us check the validity of the no-arbitrage condition and study how the arbitrageLevel can influence all arbitrage operations in the market. Our analysis will be only qualitative in this section and this will be suffice for now. Remember the system we are studying is totally stochastic because agents (both investors and the arbitrageur) act on the basis of random numbers drawn at each tick in the simulation. So, to perform a complete and reliable statistics we have to run many simulations and store the results as a significative sample to analyze. Here instead, since we are working in qualitative way, we run one single simulation for different values of the parameter arbitrageLevel: we consider three regimes of arbitrage corresponding to three values of this parameter (0.2, 0.6, 1). The result of every simulation is stored collecting a data matrix which shows the lists of asset prices as columns (the first column is relative to the first stock, the second to the second stock and the third column is about the index data); the number of rows is the number of tick in the simulation, which depends on the size of external data previously created (here 101).

First of all we compare each asset time series to the external one, visualizing the results in the three different arbitrage regimes. Here we show the three different plots for every asset.

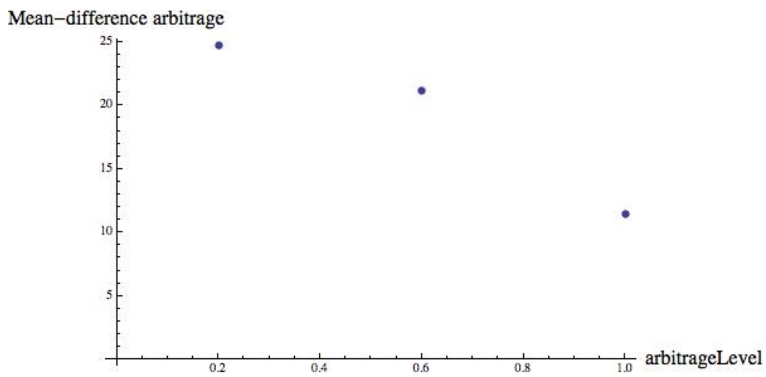



We can see from these plots how increasing the parameter arbitrageLevel the internal prices series regarding the stocks are less aligned to their exogenous counterpart defined in Mathematica. This is due to the fact that arbitrageLevel controls only the validity of the internal no-arbitrage condition; in general we speak about the no-arbitrage condition as the almost perfect alignment between index price and the weighted sum of the stock prices, omitting the adjective internal. The alignment between internal and external price series is instead always performed by the arbitrageur in the program (or, in a real market perspective, by other arbitrageurs): however the two different arbitrage operations clash each other because when the arbitrageur proposes frequently (i.e. high arbitrageLevel) buying or selling prices for the stocks to align their weighted sum to index price, he can easily contrast prices proposed by herself (or, in reality by another arbitrageur) to match the internal data series to exogenous ones. So a good alignment between index price and the weighted sum of stock prices correspond to a bad alignment between internal and external asset time series. The exception is the index and it is simple to understand why the latter is always lined up to the external source: the arbitrageur buys or sells only the stocks to restore the no-arbitrage condition inside the market but she does not operate on the index. So changing arbitrageLevel we do not find differences between index data series and this is important for our purposes, because we can be sure that the index dynamics is always the same for every parameters settings; the micro-interaction dynamics among agents can only affect the stocks prices.

Now let us look directly at the validity of the no-arbitrage condition in the market, with the three different values of arbitrageLevel. We can start from comparing the exogenous index data (which now we know coincides with the internal one) to the weighted sum of the two stock prices.




It is easy to check that increasing the arbitragLevel parameter the no-arbitrage condition becomes more consistent. We have then calculated the difference (in absolute value) between the two series in every plot (the exogenous data and the weighted sum of the stock prices) and we have computed the mean of these lists. So we can show the average difference between the index price and the weighted sum of stock prices.




It is straightforward that increasing the parameter arbitrageLevel the alignment (on average) between the index and the weighted sum of stock prices.

Resuming these qualitative results we can think about the arbitrageLevel as the parameter which guarantees the market rationality, in sense that increasing this parameter the arbitrage opportunities for an investor who looks at price series in our three-asset stock market become almost null. Now the question is what kind of arbitrage regime we can choose to perform a quantitative analysis of our simulation data. Remember however that high arbitrageLevel values will ensure very good alignment between the index and the weighted sum of the underlying stocks, but more freedom for stocks data to vary from the exogenous series.

## Returns analysis with different informational asymmetry levels

The main theme of the project is to discover the role of the asymmetry-degree parameter in determining asset dynamics. Indeed we are investigating how different levels of informational asymmetry can change the features of the stochastic processes which fit asset time series. Of course now we have to follow a quantitative method to produce significative results from a statistical point of view. We will analyze the price and return time series for 10 different values of the asymmetry-degree parameter (0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5) and we will repeat this analysis in two different completely opposite arbitrage regimes (arbitragLevel equal to 0.9 and to 0.1). The choice to compare two opposite arbitrage regimes is essential since we are looking at the effect of the arbitrage operations on the micro dynamics which is controlled by asymmetry-degree. Can arbitrage operations obscure the effect of different informational asymmetry levels? Having a look at the results in the previous section we can yet guess the answer, though not in quantitative way. With high arbitrageLevel values we know that stock series can vary significantly from the exogenous data and this enables the micro-level interactions among agents in the market are relevant to determine the macro trend of the stock price. Such a behavior is just what we expect from a complex system and then we can argue that high arbitrageLevel regimes are the most realistic and interesting for our purposes: the index price will be aligned to the weighted sum of the two stocks, and from the other hand, every single stock price process will be affected by micro dynamics (i.e. hopefully by asymmetry-degree).

Let us work setting the other simulation parameters as

| ninvestors | mean-information | out-of-marketLevel |
|---|---|---|
| 270 | 0 | 0.9 |

| information-threshold | cost-of-information |
|---|---|
| 10 | 1.0 |

We have chosen a high information-threshold in order that investors participate (almost) every time to the auction (this is much as true as the asymmetry-degree is lower as we have explained presenting the parameters). The cost-of-information parameter does not affect the asset dynamics, but only the cash of the investors; in future extensions of the program (for example including a debt constraint) this parameter could become important for the emergence of macro patterns. In order to store significant results, which cannot depend from stochastic fluctuations of the system, we performed, for every parameters setting (a value of asymmetry degree and a regime of arbitrageLevel), 50 simulations: this can be done creating loops (with For and If) in Mathematica, storing the results of all simulations. Taking the mean of the price values (for each asset) of different simulations at the same instant of time we built a new list of prices, which are now more reliable on a statistical basis. We can easily check the mean on 50 simulations is stable in terms of stochastic convergence. From these new data series we computed the list of returns and then the list of log returns (which are the logarithms of gross returns); we use log returns because are more handy for statistical analysis in the long-run period, since the compound log return over k periods is simply the sum of the k log returns. We will focus on the list of log returns so determined for every parameters setting.

### Return autocorrelation coefficients

First we look for a stochastic process which fit the log return series for every asset. In Mathematica we can use the command FindProcessParameters, which finds the parameter estimates of the process chosen to fit the data. The fit is executed using an estimator (there are some possibilities), namely minimizing some error function (or maximizing some fitness function). We do not worry about the error in the fit because this is a fit on a stochastic process and so the error should be estimated running different times the 'target' stochastic process (with different parameter settings) and comparing the data such obtained with the actual data. Let use FindProcessParameters assuming an AR(1) process: the command will compute the autocorrelation coefficient and the variance of the noise term, but we are obviously more interested about the autocorrelation coefficient. Here we summarize the results in a table, including also the data for the index (*as* is for asymmetry-degree).

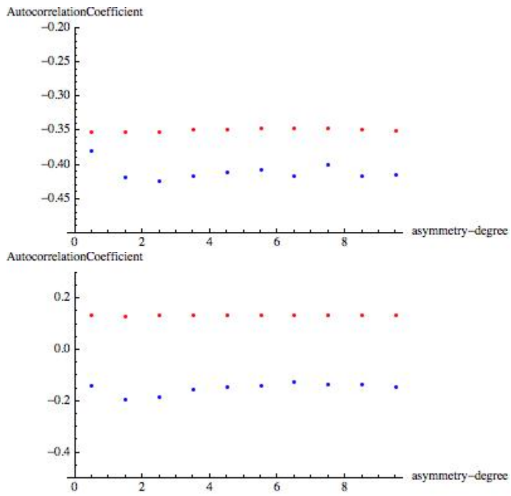
Autocorrelation coefficients of AR(1) process estimated with FindParametersProcess.

| | arbitrageLevel = 0.9 | arbitrageLevel = 0.1 |
|---|---|---|
| as = 0.5 | -0.3798 , -0.1396 , 0.0538 | -0.3527 , 0.134504 , 0.0538 |
| as = 1.5 | -0.4182 , -0.1921 , 0.0539 | -0.3515 , 0.1289 , 0.0538 |
| as = 2.5 | -0.4245 , -0.1834 , 0.0538 | -0.3520 , 0.1336 , 0.0537 |
| as = 3.5 | -0.4166 , -0.1522 , 0.0537 | -0.3490 , 0.1349 , 0.0538 |
| as = 4.5 | -0.4108 , -0.1440 , 0.0538 | -0.3483 , 0.1365 , 0.0538 |
| as = 5.5 | -0.4064 , -0.1414 , 0.0538 | -0.3470 , 0.1364 , 0.0538 |
| as = 6.5 | -0.4161 , -0.1255 , 0.0538 | -0.3469 , 0.1370 , 0.0537 |
| as = 7.5 | -0.3999 , -0.1358 , 0.0538 | -0.3466 , 0.1351 , 0.0538 |
| as = 8.5 | -0.4159 , -0.1357 , 0.0538 | -0.3481 , 0.1366 , 0.0538 |
| as = 9.5 | -0.4140 , -0.1446 , 0.0538 | -0.3502 , 0.1344 , 0.0538 |

The first two values are relative the first and the second stock, the third is for the index.

It is clear that the index autocorrelation coefficient remains almost constant over different arbitrage levels or informational asymmetry degrees: this arises from the behavior yet observed in the past section. The index is only affected by arbitrage intervention which aims to align the internal data to the external one, so the alignment is not stroked by other arbitrage operations and it results almost perfect. Therefore the index dynamics does not depend on the parameters arbitrageLevel and asymmetry-degree, as we have yet understood, and this implies a constant autocorrelation coefficient. However the magnitude of this coefficient is clearly smaller than that ones of the stock series: the reason is the nature of the exogenous data, because the index price process was through an AR(1) process, with autocorrelation coefficient 0f 0.9 that makes the process close to a random walk. A process similar to a random walk will present an autocorrelation coefficient for returns close to 0, as shown in our table. We will no more consider the index data for the next analysis because we have no interesting information from it, as just remarked. Now we plot the different autocorrelation coefficients for the stocks, showing the behaviors for the two different arbitrage regimes.

Autocorrelation coefficient for the log returns AR(1) process: different asymmetry-degree values



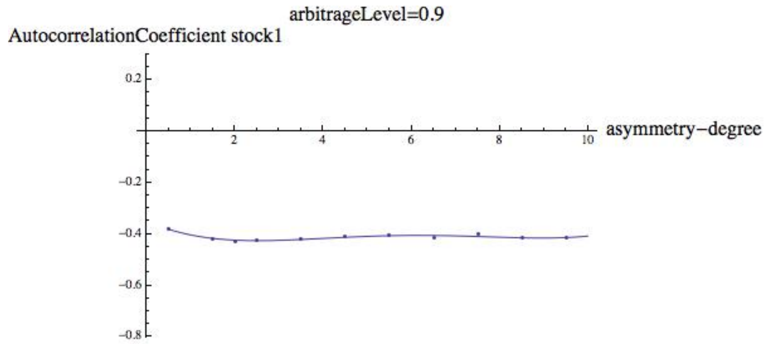

The blue points are for the first stock, while the red points define the second stock.

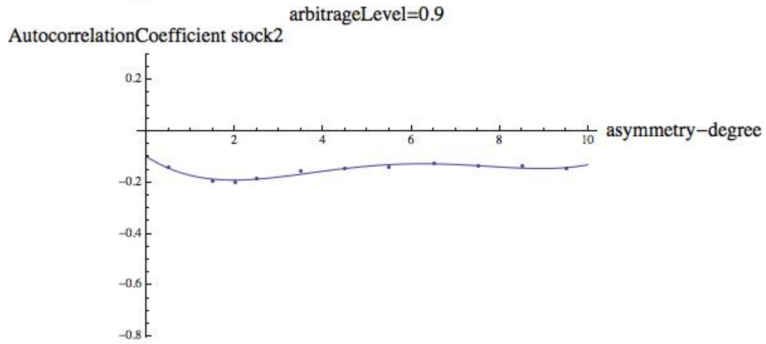
What we can notice from the plots is that under a low arbitrage regime (arbitrageLevel = 0.1) the autocorrelation coefficient for the log returns process is almost constant with different asymmetry-degree values: we have a further proof that when the stock price series are well aligned to the exogenous data, the effects of informational asymmetry does not emerge in the market. Instead, when arbitrageLevel is 0.9 it is possible to see a trend in the autocorrelation coefficients at different asymmetry-degree values: we recognize a sort of hole in the autocorrelation coefficient, which could approximately presents a minimum around a value of asymmetry-degree equal to 2. So we run again 50 simulations for this asymmetry-degree collecting the results as we did before and then we add this point to our plot. Now we make a fit, for both stocks, using a quadratic curve and finding these results

$$\beta_1 = -0.348767 - 0.0772481as + 0.0259131as^2 - 0.00326919as^3 + 0.000139162as^4$$

$$\beta_2 = -0.0981564 - 0.111482as + 0.0433855as^2 - 0.00578301as^3 + 0.000252748as^4$$

where $\beta_1$ and $\beta_2$ are the autocorrelation coefficients of the first and the second stock, while $as$ is the asymmetry-degree value. Now we can visualize the fit curve with the related points in these plots.



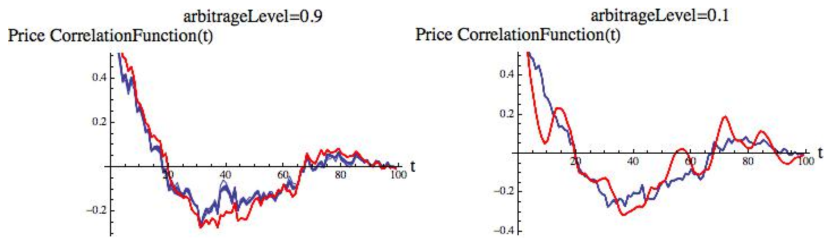

We can exploit the fit estimated to find the value of asymmetry-degree for which the autocorrelation coefficient is minimum: it is suffice to set the derivative of the two curves to zero, getting the following values.

| | stock1 | stock2 |
|---|---|---|
| asymmetry-degree minimum | 2.5 | 2.0 |

What is the meaning of the existence of a minimum in the autocorrelation coefficient for a particular asymmetry-degree value? When the autocorrelation coefficient is minimum the level of efficiency in the market is the lowest as possible and that implies stock prices reflect the current available information in the worst possible way, with respect to different informational asymmetry levels. So we can think about an optimal value of asymmetry-degree such to allow agents to perform active/forecasting strategies with best possible results (i.e. best profits).

**Prices' autocorrelation**

Another interesting feature to consider is the autocorrelation function of the price processes in these different parameter settings. We know that stock prices show in reality a momentum effect, which corresponds to positive autocorrelations on the short-period, while they are characterized by a mean-reverting component in the long run, which implies a negative correlations over large periods. These two components make the price the appearance of fluctuating around its fair value; in our market the different level of information leads the investors to buy or sell at what they consider the fair price (the price which perfectly reflect their known information), and we expect anyway the presence of these two components in prices' autocorrelation. Using the price lists previously created we apply the function CorrelationFunction, defined in Mathematica, at each instant of time and plot the results for both the stocks comparing the autocorrelation functions in the two arbitrage regimes. Notice that this is possible under the assumption that our stock price processes are stationary in weak sense, i.e. processes with mean constant over time and, in particular, with autocorrelation function which is independent from time translations. In the case of AR(1) processes, as those here assumed for stock prices, the weak stationarity is satisfied when the autocorrelation coefficient is lower, in module, to 1: this is of course always true in our experiments, so our stochastic price processes are always stationary in weak sense. Here we have the plots of the autocorrelation functions: since, varying the value of asymmetry-degree, the functions do not change significantly we plot them all together in a sort of bundle. If the bundle is thick the autocorrelation functions are very similar but not completely equal.




The plots show the two bundles of autocorrelation functions. Blue = first stock; Red = second stock.

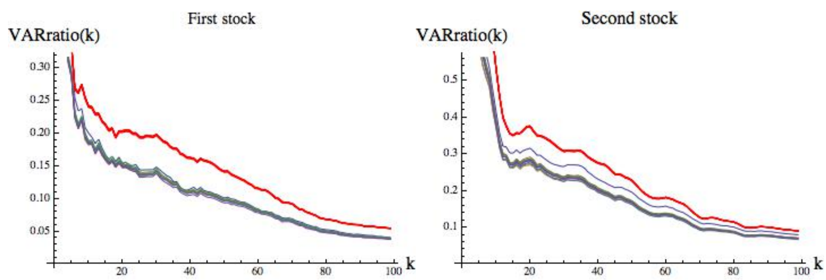
We can clearly see, in both arbitrage regimes, how the autocorrelation function is positive in the first periods (it becomes negative approximately at the 20th tick) showing the momentum effect. Next the autocorrelation is negative for almost 50 periods (mean-reverting component), and finally it turns positive (but with small values) again until the end of the simulation (tail momentum effect). When arbitrageLevel is 0.9 the functions are more irregular and this is not surprising because we have said that high arbitrageLevel values mean more freedom for stock prices, leading to more fluctuations for the autocorrelation. Indeed when arbitrageLevel is 0.1 the stock price series are almost perfectly aligned to the exogenous data and the correlation is more smooth; moreover both the bundles of the two stocks are very slight because difference in informational asymmetry does not affect the autocorrelation structure for the stock price.

**Variance ratio**

If the price process is a random walk the returns (or the log returns) are i.i.d. variables and the variance of the compound return (or log return) over k period is exactly k times the variance of the single-period return (or log return). So it is relevant to compute the variance ratio, which is defined as

$$\frac{Var(r_{t,k})}{Var(r_t)} = 1 + 2\sum_{i=1}^{k-1}(1 - \frac{i}{k})\rho(i)$$

where $\rho(i)$ is the autocorrelation coefficient at time i (starting from 0). It is clear that when negative autocorrelations are predominant in the process (over k periods) the variance ratio is less than 1, while it will be higher if positive correlations are more important. The more this ratio is close to 1 and the more the process is similar to a random walk and the market is efficient. So we can compare this ratio, as a function of k, under different parameter settings. First of all we show, for both stocks, the variance ratios at different values of asymmetry-degree (separately in the two arbitrage regimes); next we compare the variance ratios of a same stock in the two different arbitrage scenarios.



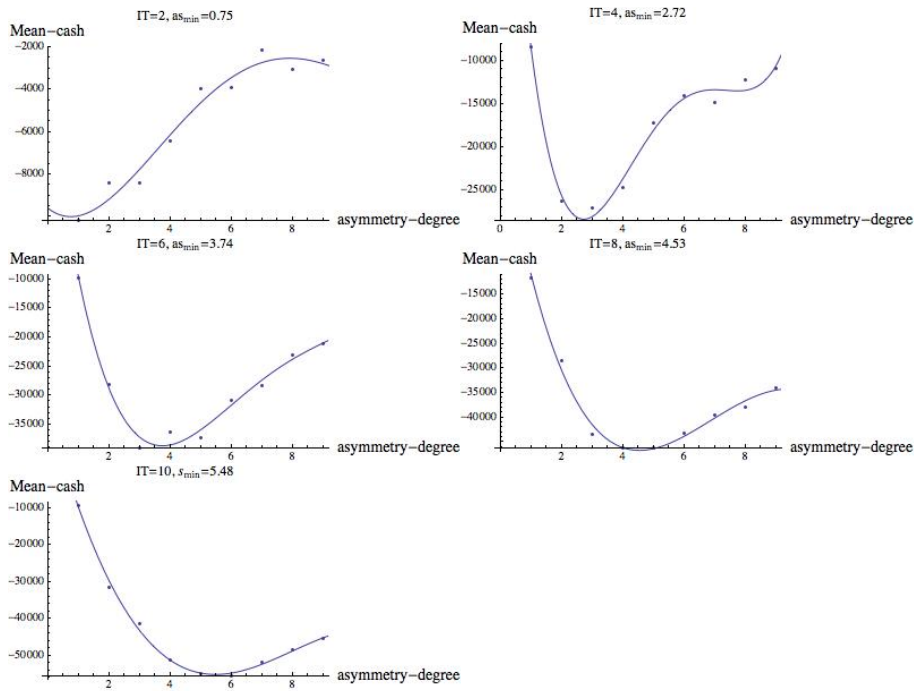

Red lines = VARratios(k) with arbitrageLevel = 0.1; Blue lines = VARratios(k) with arbitrageLevel = 0.9

The plots show how the VARratio(k) is always a decreasing function in k: stock prices do not follow a random walk and, moreover, are dominated by negative autocorrelation coefficients when we extend time horizon: this behavior supports the idea of a mean-reverting component which rules the long-period trend. Let us notice now two important things that can reveal essential to complete our analysis. First, the bundle of VARratio(k) functions in the case of high arbitrage regime (blue lines) is thick and irregular, and this a sign of that informational asymmetry affects stock prices dynamics (though with a very little magnitude). Second, the VARratios(k) in the low arbitrage regime (red lines) are always greater than the VARratios(k) of high arbitrage regime; so, increasing arbitrage intervention, to align the index price to the weighted sum of stock prices, entails the stock price processes become less correlated, i.e. less comparable to a random walk process. We can argue that augmenting the arbitrageLevel parameter makes the market efficiency worse, though this prevents arbitrage profits for investor (which is a condition for market rationality).

## Investors' profits with different informational asymmetry levels

Checking the magnitude of autocorrelation in stock price processes is useful to understand the degree of efficiency in our three-asset market, and in particular how much this degree is linked to the level of informational asymmetry. If the market is highly efficient any active strategy which wants to exploit new information about the assets to forecast market dynamics is inconsistent; it can be better to perform a passive strategy, holding a market index without acquire any information. In this model we cannot compare directly such a passive strategy (for example constructing the market portfolio for this three-asset market) with the active strategy performed by the investors and here we can briefly understand the reason. In our market **all** the investors pursue an active strategy, so though they have different information sets, market dynamics is only determined by this kind of agents (with the exception of the arbitrageur who is fundamental in every market). To compare passive investors with the active ones, we should at least include non-rational investors (which can be called random agents or 'noise traders') in order to include in the market an irrational component which contribute to asset dynamics. So only adding this kind of agents (and other kinds) our active rational investors can exploit correctly the new information acquired to make profitable strategies. However our aim is comparing the profits of our agents under different informational asymmetry levels: this can be done considering another simulation parameter which have no considered in the previous analysis, i.e. information-threshold. The role of information threshold is to push agents to not rely of all analyst' forecasts: if an asset alpha value is greater than the level of information-threshold the investor discards this value and she does not participate to the auction relative to that asset, and in this case she does not spend the cost-of-information value for the service. The implications for this kind of micro interaction between investors and their analyst has obvious consequences: high levels of information threshold will lead the investors to spend less in the market, and so, at the end of the simulation, their cash (which is a private variable of the breed investors) will be higher.

Let us explore how the cash of the investors, on average, can be affected by informational asymmetry, considering different regimes for information-threshold. Since at the end of each simulation the cash of every investor is stored in the list $list - cash$ we can use such a list and computing its mean. Here we do not run different simulations for a single parameters setting because we are yet considering a quantity which is a good estimator from a statical point view, i.e. the mean of the investors' cash. So it is suffice to run a single simulation for every couple of the parameters asymmetry-degree and information-threshold: we chose 5 values of information-threshold (2, 4, 6, 8, 10) and nine values of asymmetry-degree (from 1 to 9). We collect the results in 5 plots (one for each information-threshold value), yet including the fit curve (we used a quadratic form) through which we can estimate the value of asymmetry-degree where there is the minimum in cash. The simulations are performed with the same other parameter settings used in return analysis with the exception of arbitrageLevel which we set to 0.5, in order to work in a medium arbitrage regime (so the two opposite arbitrage operations are well balanced).




First of all it is crucial to understand why these plots show only negative cash values, in other terms, why at the end of the simulation investors will always have a very big debt. The answer lies in the nature of the proposing price mechanism: investors propose a selling price which is under the current price level and a buying price which is greater. This choice, which regards the expectations formation for the investors, leads necessarily to a spread between cash spend buying assets and cash gained selling them; at the end of the simulation the result will be strongly negative. However, decreasing information-threshold the magnitude of the final cash values is lowered because investors will tend to participate (and spend) less to the auction. All these features are further causes which make incomparable, in this model, such an active strategy with a passive one. But we have to remember that our goal is to investigate the effect of informational asymmetry on rational investors' profits and this can be done looking at the plots. We can recognize a very interesting pattern: in general the mean-cash curve (estimated through quadratic fit) decreases towards a minimum and then rises again for high levels in informational asymmetry. Using the equations of these curves (which do not show because they are related to different scales), it is very easy to estimate the value of asymmetry-degree in which we can find the minimum: plot labels include also these values, in each different information-threshold regime. As the information-threshold value becomes greater the minimum increase itself.

How can we interpret all these patterns? The fact that there is a minimum in the mean cash of investors is related to the role of asymmetry degree in the interactions among agents in the market. When asymmetry-degree is very low, agents make expectation forecast about asset prices which are very similar, so the competition in the auction is very high (buying and selling prices very close to each other): the trades an investor can complete are, on average, not so much. By contrast, when asymmetry-degree is high price expectations are very different from one other (great variance in expectations, or in analysts' views), and in the auction the matching possibilities of completing a trade are low: so again, on average, an investor will complete a limited number of market operations. In this two cases it is obvious how, with low completed trades, the final cash of an investor will be higher at the end of the simulation. The presence of a minimum proofs that there is an intermediate range (and in particular an optimal value) for asymmetry-degree such to allow agents to conclude a great number of trading operations, so lowering their final cash. Therefore we can say there is an optimal level of informational asymmetry which optimize the number of total trades in the market, and this level rises when information-threshold is increased. High information-threshold levels mean greater attitude for investors to avoid market trading operations: only a high level of informational asymmetry can contrast this effect.

# Conclusions

Our work has shown how the effects of informational asymmetry in market dynamics can emerge only when stock price series are free to vary form the exogenous data, i.e. when they can be affected by the trading actions of investors. This corresponds to high values of the parameter arbitrageLevel, which is a good thing for market rationality too. So in this regime we are sure about the absence of arbitrage opportunities and we can also see the effect of informational asymmetry in terms of market efficiency: high values of arbitragLevel guarantees a realistic behavior for our three-asset stock market, letting emerge different macro patterns in price dynamics, as usual in a complex system. In this setting we found an optimal value of asymmetry-degree which makes market efficiency the lowest possible: in this situation, if we included other noise traders (or even traders with different expectational models), our investors could be able to exploit the new information to make a profitable active strategy. In future extensions of the model, the inclusion of other agents in the market can allow us to compare different trading strategies, under different levels of informational asymmetry. However we have demonstrated how too low or too high values of informational asymmetry do not facilitate market trading operations: there is an optimal value of asymmetry-degree which maximizes the number of trades in the model. So, the idea of exploring different levels of informational asymmetry and finding those which guarantee the best forecasting power for investors and the highest trading volume, is very important to understand how a complex system such a stock market can significantly change with respect to the variety of information sets available to the agents.

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